

Turning Model of a Skid-Steering Unmanned Ground Vehicle under Steady State Conditions Based on Magic Formula

Jun Han^{1, a}, Guoquan Ren^{1, b}, Dongwei Li^{1, c} and Ziyong Jia^{1, d}

¹Mechanical Engineering College, Shijiazhuang 050003, China.

^ahanjun_wy@163.com, ^b709955574@qq.com,

^c12ldw@163.com, ^d2432282264@qq.com

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Abstract. In recent decades, Unmanned Ground Vehicles (UGVs), as a part of the Army's Future Combat Systems (FCS), have become a hot research area in national defense science and technology. In order to study the influence factors of steering performance, turning model of a six-wheeled skid-steering UGV is established under steady state conditions based on the Magic Formula. With the turning model, the vehicle middle axle's position, which is one of the influence factors of steering performance, is analysed by the method of numerical analysis. The results proves that when the middle axle's position eccentricity is under one fifth of the distance between the first and the last axles, the steering performance is steadily, and the design is most reasonable. This paper is significative for the structural design of six-wheeled skid-steering UGV.

1. Introduction

Skid-steering UGVs are usually used in military field, such as these UGVs in American Army : Crusher, SMSS, MULE and so on (As shown in Fig 1). As a part of the Army's Future Combat Systems (FCS), the research on the structural design and motion control has become a hot research area[1-4]. Furthermore, either the structural design or the motion control require an accurate steering dynamics model. But skid-steering turning mode is different from the Ackermann mode, the wheels skid and slid heavily when the skid-steering vehicle is turning. Because of the sliding, the mechanical properties of the wheels is very complex, which leads to the difficulty in dynamic model research.

Yang used a simple friction model to describe the mechanical effect, then established the turning model of a skid-steering unmanned six-wheeled vehicle. But the simple friction model cannot describe the complex tire performances[4]. To solve the problem, Yan provided a effective method based on the famous Magic Formula model to build the turning mode[5]. The Magic Formula is proposed by Pacejka and Bakker in 1987, and it's a semi-empirical tire model, which can describe the tire performances accurately[6].

In this paper, firstly, the slip angle and the longitudinal slip ratio of each tire are calculated when a six-wheeled skid-steering UGV is turning. Then the vertical load forces of each tire are also figured up by the vertical load equilibrium relationship of tires. Furthermore, the longitudinal forces and lateral forces can be calculated by the Magic Formula. Finally, the turning model can be translated by the dynamic equilibrium equation under steady state conditions. By the numerical analysis of the turning model, the paper analyzes the effect of middle axis' position no the vehicle

steering performance. The conclusion provides reference significance to the structural design of six-wheeled skid-steering UGVs.



Fig.1 Skid-steering UGVs
(Crusher, SMSS and MULE)

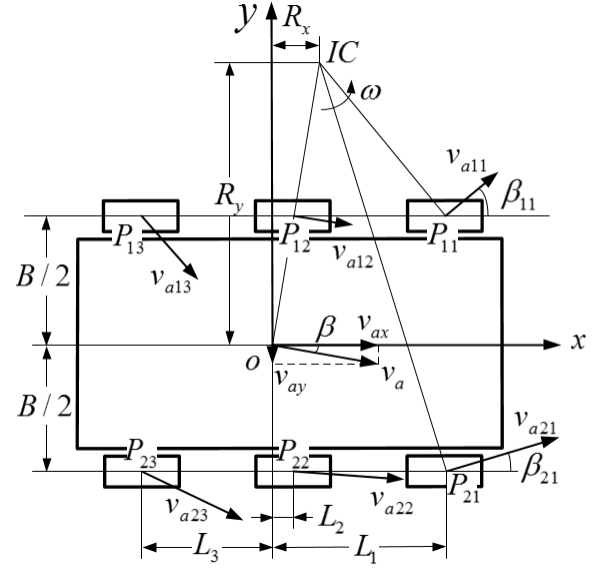


Fig.2 UG V coordinate
system &. Kinematic analysis

2. Kinematic Analysis

To describe the turning steering of a six-wheeled skid-steering UGV, corresponding coordinate system is established. As shown in Fig.2, the xoy coordinate system is follow-up coordinate system fixed on centroid of the UGV. In order to establish the kinematics and dynamics model of UGV, ignoring secondary factors, it is necessary to make the following assumptions:

- (1) The UGV is symmetrical about its transverse and longitudinal symmetry plane;
- (2) The UGV is steering steady on firm road;
- (3) Ignore the pitch and roll motion of the vehicle;
- (4) Each tire always contacts with the ground, and that is a point contact;
- (5) Ignore the deformation of the tire.

As shown in Fig.2, when the vehicle is turning under steady state, the instantaneous steering center of the UGV is point IC , and the turning angular velocity is ω , while the turning radius is R . Assume that the coordinate of in the follow-up coordinate system is (R_x, R_y) , then we have:

$$R = \sqrt{R_x^2 + R_y^2} \quad (1)$$

The absolute velocity of the UGV center is $v_a = \omega R$, and its components in the follow-up coordinate system are:

$$\begin{cases} v_{ax} = \omega R_y \\ v_{ay} = -\omega R_x \end{cases} \quad (2)$$

As shown in Fig.2, the wheels are marked as ij ($i = 1, 2$ and $j = 1, 2, 3$), and assume the tires ij are constant with the ground at points P_{ij} . Analyse the movement of P_{ij} , we find the absolute velocities are $v_{a1j} = \omega R_{1j}$, and the distances between P_{ij} and IC are marked as R_{ij} that:

$$R_{ij} = \sqrt{(R_x - L_j)^2 + [R_y + (-1)^i B/2]^2} \quad (3)$$

Where L_j are the horizontal coordinates of P_{ij} in the coordinate system, and L_j also can reflect the position of each wheel axle. Then the components of v_{aij} are:

$$\begin{cases} v_{axij} = v_{aij} \cos \beta_{ij} = \omega[R_y + (-1)^i B/2] \\ v_{ayij} = v_{aij} \sin \beta_{ij} = \omega(L_j - R_x) \end{cases} \quad (4)$$

And β_{ij} are the slip angles of the wheels ij . From equation (4), we know that:

$$\tan \beta_{ij} = \frac{v_{ayij}}{v_{axij}} = \frac{L_j - R_x}{R_y + (-1)^i B/2} \quad (5)$$

Assume the wheels ij are rolling forward with rotating speed ω_{ij} , and the radius of wheels are r . Then we can define the longitudinal slip ratio of each tires as K_{ij} that:

$$K_{ij} = \frac{v_{axij} - r\omega_{ij}}{v_{axij}} = 1 - \frac{r\omega_{ij}}{\omega[R_y + (-1)^i B/2]} \quad (6)$$

3. Calculation of Vertical Load Forces

To get the longitudinal forces and lateral forces of each tires, the vertical load must be calculated. Due to the influence of the centrifugal force, the vertical load on the two sides change a lot. As shown in Fig.3, N_1 and N_2 are the composite forces of two sides, and $G a_x / g$ is the centrifugal force, which comes into being when turning. Then we have:

$$\begin{cases} N_1 + N_2 = G \\ N_1 \frac{B}{2} + \frac{G a_x}{g} h - N_2 \frac{B}{2} = 0 \end{cases} \quad (7)$$

Where h is the height of the center of mass from the ground, and a_x is the lateral acceleration. According to the kinematic relationship, a_x is known to meet:

$$a_x = \omega^2 R = \omega^2 \sqrt{R_x^2 + R_y^2} \quad (8)$$

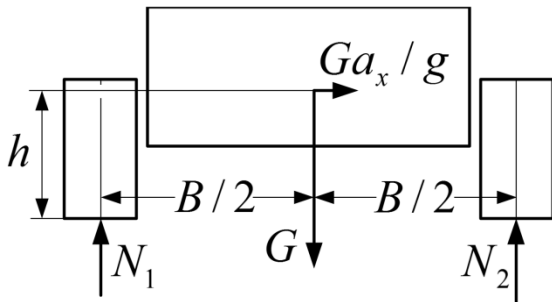


Fig.3 Vertical load transverse distribution of UGV

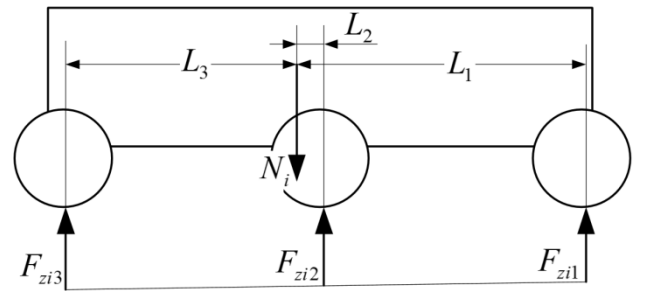


Fig.4 Vertical load longitudinal distribution of UGV

As for one side of the vehicle, the vertical load of the tire ij is marked as F_{zij} . As shown in Fig.4, there are equations (9):

$$\left\{ \begin{array}{l} \sum_{j=1}^3 F_{zij} = N_i \\ \sum_{j=1}^3 F_{zij} L_j = 0 \\ \frac{F_{zi3} - F_{zi2}}{L_3 - L_2} = \frac{F_{zi2} - F_{zi1}}{L_2 - L_1} \end{array} \right. \quad (9) \quad \left\{ \begin{array}{l} F_{zi1} = \frac{D_1}{D} \left[\frac{1}{2} + \frac{(-1)^i \omega^2 R h}{g B} \right] G \\ F_{zi2} = -\frac{D_2}{D} \left[\frac{1}{2} + \frac{(-1)^i \omega^2 R h}{g B} \right] G \\ F_{zi3} = \frac{D_3}{D} \left[\frac{1}{2} + \frac{(-1)^i \omega^2 R h}{g B} \right] G \end{array} \right. \quad (10)$$

By solving equations (7)(8)(9), we can get the computational formulas (10), that can calculate the vertical load forces. In equations (10), the D , D_1 , D_2 , D_3 are calculated by:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ L_1 & L_2 & L_3 \\ L_3 - L_2 & L_1 - L_3 & L_2 - L_1 \end{vmatrix}, D_1 = \begin{vmatrix} L_2 & L_3 \\ L_1 - L_3 & L_2 - L_1 \end{vmatrix}, D_2 = \begin{vmatrix} L_1 & L_3 \\ L_3 - L_2 & L_2 - L_1 \end{vmatrix}, D_3 = \begin{vmatrix} L_1 & L_3 \\ L_3 - L_2 & L_1 - L_3 \end{vmatrix}.$$

4. Calculation of Longitudinal and Lateral Forces Based on Magic Formula

Based on the physical prototype of tire, the Magic Formula is a set of mathematical expressions, which can be used to accurately describe the forces of tire under steady state[7]. The Magic Formula's inputs are longitudinal slip ratio, slip angle, roll angle and vertical load force of the tire, while the outputs are longitudinal braking force, lateral force and the gyroscopic moment. Generally, the gyroscopic moment is very small, and it is usually ignored when analyzing the vehicle dynamics[8], so here we only consider the longitudinal and lateral forces. The basic form of the Magic Formula is [9]:

$$\left\{ \begin{array}{l} y(x) = D \sin\{C \arctan[Bx - E(Bx - \arctan(Bx))]\} \\ Y(x) = y(x) + S_v \\ x = X + S_h \end{array} \right. \quad (11)$$

Where Y is the outputs, the longitudinal and lateral forces, and X is inputs, $\tan \beta$ or K . Parameters D , C , B , E , S_h and S_v are crest factor, shape factor, stiffness coefficient, curvature factor, horizontal shift and vertical shift, and their numerical values are functions of F_z and γ .

To describe the tire mechanical property under combined conditions, a weight function G is lead up to correct the model. Multiply the formula under pure conditions, equations (11), by G , we can get the lateral and longitudinal forces' computational formulas under combined conditions[9]. The expression of weight function G is:

$$G = D \cos[C \arctan(Bx)] \quad (12)$$

According to the basic structure of the Magic Formula, this paper matches a part of parameters in limited extent. Then we get the the Magic Formula under combined conditions with the zero roll angle:

$$[F_x, F_y]^T = MF(F_z, K, \beta) \quad (13)$$

According to the model (13), we can calculate the longitudinal forces with different tire slip angles, as well as the lateral forces with different longitudinal slip ratios, as shown in Fig.5 and Fig.6.

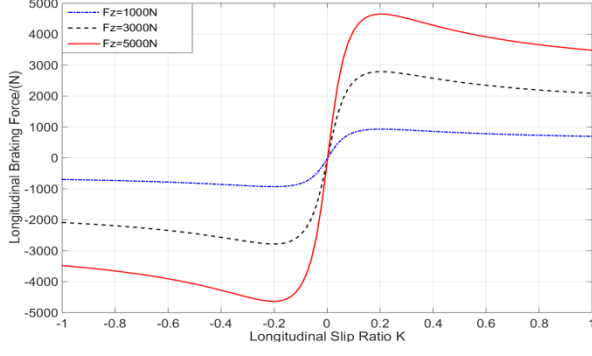


Fig.5 Longitudinal braking force changing with the longitudinal slip ratio curves
($\beta = 5^\circ$, $-1 \leq K \leq 1$)

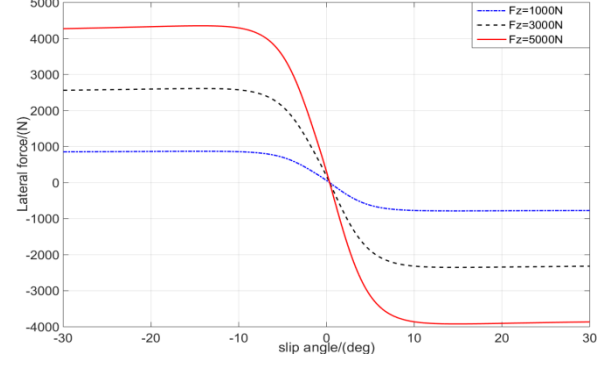


Fig.6 Lateral force changing with the slip angle curves
($K = 0.1$, $-30^\circ \leq \beta \leq 30^\circ$)

From Fig.4 and Fig.5, we can find that the longitudinal and lateral forces all become steady when the slip angle or longitudinal slip ratio increase to a certain level. This trend is difficult to reflect for the simple friction model.

5. Turning Model under Steady State and Numerical Analysis

5.1 Turning Model under Steady State.

When the UGV is turning under steady state, the longitudinal acceleration is zero, and the lateral acceleration, as shown in figure 6, is a_x . According to the dynamic equilibrium equation, we have:

$$\left\{ \begin{array}{l} \sum_{j=1}^3 (F_{x1j} + F_{x2j}) = 0 \\ \sum_{j=1}^3 (F_{y1j} + F_{y2j}) - \frac{G}{g} a_x = 0 \\ \sum_{j=1}^3 (F_{x2j} - F_{x1j}) \frac{B}{2} + \sum_{j=1}^3 (F_{y2j} + F_{y1j}) L_j = 0 \end{array} \right. \quad (14)$$

Combine equations (14) with (1)(3)(5)(6)(10)(13), eliminate the of intermediate variables, we can get a transcendental equation, with the independent variables ω , R_x , R_y , and the parameters ω_{ij} , L_j , B , r , h , G of the UGV:

$$\Psi(\omega, R_x, R_y, \omega_{ij}, L_j, B, r, h, G) = 0 \quad (15)$$

The equation (15) reflects the inner relationship between turning parameters (ω, R_x, R_y) and structure parameters (L_j, B, r, h) , as well as the motion parameters ω_{ij} . And it is the turning model of the six-wheeled skid-steering UGV under steady state.

5.2 Numerical Analysis of Structure Parameter.

When the parameters of UGV is confirmed, the equation (15) has a determined solution. But the equation is a transcendental equation, there is no analytical but numerical solution. Change a parameter, then we can know the changes of the turning states through equation (15), so we can study the parameter's effect to the UGVs' turning. Here we study the effect of the structural parameter L_2 , which reflects the position of the middle axle.

Table 1 Simulation Parameters of UGV

$B(m)$	$r(m)$	$G(N)$	h
1.5	0.313	9150	0.505

The simulation parameters of the UGV are given in Table 1, and we set $\omega_{11} = \omega_{12} = \omega_{13} = 1 \text{ rad/s}$, $\omega_{21} = \omega_{22} = \omega_{23} = 2 \text{ rad/s}$. To study the middle axle only, we have to confirm position of the first and last axle, here we make $L_1 = 1.25$ and $L_3 = -1.25 \text{ m}$. Then changes L_2 from $L_3 + 2r = -0.625 \text{ m}$ to $L_1 - 2r = 0.625 \text{ m}$, we can calculate ω and R by method of numerical analysis according to equation (15). The results are presented by Fig.7 and Fig.8.

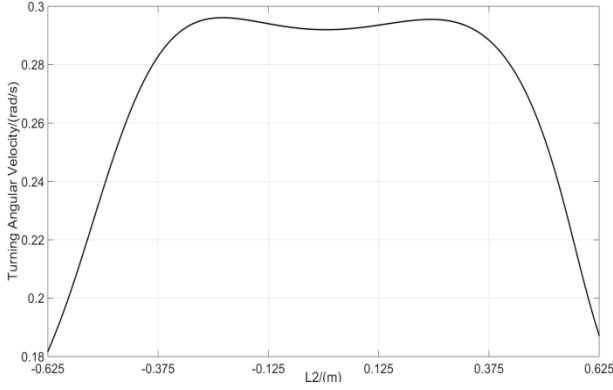


Fig.7 Curve graph of $\omega - L_2$ under steady turning state

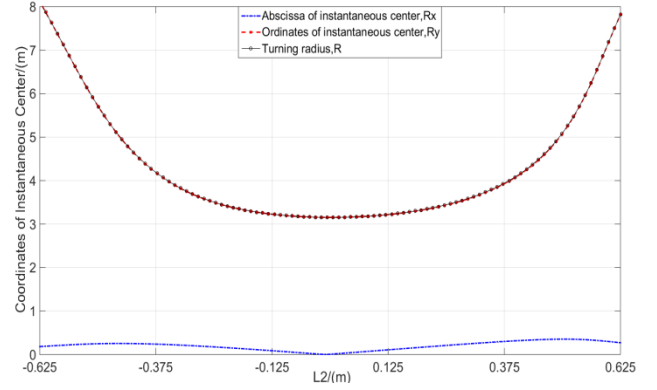


Fig.8 Curve graph of $R_x - L_2$, $R_y - L_2$ and $R - L_2$ under steady turning state

From Fig.7, we can see that when L_2 is in the range of $(-0.25, 0.25)$, the value of ω is stable relatively in a higher level. When the absolute value of L_2 continues increase, decays rapidly. This tendency of ω indicates that the middle axle position has a great influence on the turning angular velocity ω under steady state.

From Fig.8, we know that R_x is much smaller to R_y and R , so that the value of R is very approximate to R_y . Besides, when L_2 is in the range of $(-0.25, 0.25)$, the value of R is stable relatively in a lower level. When the absolute value of L_2 continues increase, R expands rapidly. This tendency of R also indicates that the middle axle position has a great influence on the turning radius R under steady state.

Generally speaking, when a vehicle is turning, we all hope the vehicle turns rapidly with a bigger turning angular velocity and a smaller turning radius. Besides, when the vehicle moves, many factors like vibration can lead to the variation of L_2 in a certain extent. So in the structure design, a befitting position of middle axle can make the turning parameters insensitive to L_2 . In a word, set the value of L_2 in the interval $(-0.25, 0.25)$ is the most reasonable in the structure design of a six-wheeled skid-steering UGV. It is a remarkable fact that $-0.25 \leq L_2 \leq 0.25$ is to say $-(L_1 - L_3)/5 \leq L_2 \leq (L_1 - L_3)/5$, so we can conclude that when the middle axle's position eccentricity is under one fifth of the distance between the first and the last axles, the steering performance is steadily, and the design is most reasonable.

6. Summary

This paper aims to establish the turning model of a six-wheeled skid-steering UGV under steady state conditions. Based on the Magic Formula, the longitudinal forces and lateral forces of each tires are calculated to build the dynamic equilibrium equations under steady state conditions, then through the equations, the paper finally establishes the turning model as a transcendental equation. By the numerical analysis of the turning model, the paper analyzes the effect of middle axis' position to the vehicle steering performance. In conclusion, the related results can be summarized as follow:

(1) The longitudinal and lateral forces of tires all become steady when the slip angle or longitudinal slip ratio increase to a certain level;

(2) L_2 is in the range of $(-0.25, 0.25)$, the value of ω is stable relatively in a higher level, while the value of R is stable relatively in a lower level;

(3) When the middle axle's position eccentricity is under one fifth of the distance between the first and the last axles, the steering performance is steadily, and the design is most reasonable.

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