Free trade networks based on R&D perspective
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ABSTRACT: Based on Goyal and Joshi, we study a setting with many countries, in which there are firms that can sell in the domestic as well as foreign markets, and assumed that each firm compete to take the form of non-price, and analysis the incentive of country to sign FTA with other countries via Classical Competition Model. Our principal finding is that 1) two isolated countries have no incentive to sign an FTA, and the empty network is stable. 2) The complete trading network is stable if \( N \geq 3 \); 3) There is no improving path from the empty network to the complete network.

Introduction
FTA as bilateral trade agreements between countries has played a huge advantage, and become an important means to achieve the globalization of trade.

In order to investigate technical in the role of the FTA partner countries market makers occupation, the authors introduced the classic competition model. Each firm only concerns the market share in their own country and the FTA partner countries, and have complete information of other firms in technology spending. In the course of repeated games, technology spending of each firm finally reached equilibrium. In previous papers, Goyal and Joshi (2006) assumed that in each country there is a single firm producing a homogeneous good and competing as a Cournot oligopolist in all countries. A lot of follow-up research expand Goyal and Joshi (2006). Such as, Chen and Joshi (2010) considered the decision of the country to enter into an FTA is also crucially dependent on the participating countries' existing FTA relationships with third countries. In another example, Mauleon, Song and Vannetelbosch (2010) studied whether trade unions in different countries influence on global FTA network formation. In another example, Daisuke and Furusawa (2014) and Zhang, Cui and Zu (2014) studied dynamic evolution of FTA networks by introducing the dynamic analysis. Different from the existing literature, We assume that firms adopt technology investment strategy to compete in the same market.

In their lay articles Goyal and Joshi find that if countries are symmetric, a complete network is consistent with the incentives of individual countries. In other words, the complete network is stable. Furusawa and Konishi based on Goyal and Joshi lay articles also find that the complete network is stable. Zhang, Cui and Zu analysis the dynamic FTA networks, and find that the complete network is stable, if \( N \geq 3 \). Cui and Gao studied a free trade agreement network formation game where bilateral free trade agreements imply unrestricted FDI, and finds that star FTA network is pairwise stable. In this article, we find that (1) any two independent states has no incentive to sign FTA and (2) the complete networks is stable, if \( N \geq 4 \).

The basic model
We consider a setting with \( N \) countries, each of which has one firm producing a homogeneous good, \( N = \{1, 2, 3, \ldots, N\}, N \geq 3 \). Each firm can sell in the domestic market as well as in each of the foreign markets. We assume that any country offers a tariff-free access to its domestic market, if any two countries have bilaterally negotiated an FTA; otherwise, each country imposes a nonzero tariff on the imports from the other. Bilateral relations between any two countries is represented as follows: for any \( i, j \in N \), the pairwise relationship between the two countries is captured by a binary variable, \( g_{ij} \in \{0, 1\} \); \( g_{ij} = 1 \) means that FTA is established between countries \( i \) and \( j \), whereas \( g_{ij} = 0 \) means that no FTA is in effect. Additional, \( g_{ii} = 1 \forall i \in N \). Let \( T_i(g) \) be the tariff faced by
firm $i$ in country $j$ in the network $g$. Note that $T'_j(g) = T'_i(g) = 0$, if $g_y = 1$. However, in general, impose prohibitive tariffs between countries $i$ and country $j$, if $g_y = 0$.

Let $N_i(g) = \{ j \in N : g_y = 1 \}$, Each country in $N_i(g)$ is involved in an FTA with country $i$ in the network $g$. Let $i \in N_i(g), g_i = |N_i(g)|$. $g^e$ represent the empty network; $g^c$ represent the complete network. Therefore, $N(g^e) = \Phi$ and $N(g^c) = N$. Let $g + g_y$ denote the network obtained by replacing $g_y = 0$; Similarly, let $g - g_y$ denote the network obtained by replacing $g_y = 1$ in network $g$ by $g_y = 0$.

For each new FTA partner country, it compete for market share through R&D investment. Let $A'_i$ represent R&D investment of firm $i$ in country $j$. We let the output of firm $j$ in country $i$ be denoted by $Q'_i$. We assume that each firm only concerned with its market share in FTA partner countries and in their own country. In general, in each country $i \in N_i(g)$, each country faces an function of income given by

$$S_i(g) = \sum_{k \in N_i(g), j} [Q_i(g) - A'_k(g)] \quad (1)$$

The relationship between output of firm $j$ in country $i$ and the investment of R&D is given by

$$Q'_i(g) = Q'_i(A'_i(g), A'_{i^c}(g))$$

Among them, other countries’ R&D investment in country $i$ is expressed as $A'_{i^c}(g)$. Simplicity, $A'_i(g)$ will be denoted by $A'_i$. For further, we assume that the relationship between the market share of firm $j$ in country $i$ and the R&D investment is given by

$$Q'_i(g) = \frac{A'_i}{A'_i + \sum_{k \in N_i(g), j} A'_k} \quad (2)$$

On the relationship between the market share of firm and the R&D investment, we can obtain the following properties.

**PROPOSITION 1.** $Q'_i(g)$ is a monotonically increasing function with respect to $A'_i$. Moreover, $Q'_i(g)$ is a monotonically decreasing function with respect to $A'_i$ ($k \neq j$).

**Proof.** For (2) is carried out first derivation on $A'_i$:

$$\frac{\partial Q'_i(g)}{\partial A'_i} = \sum_{k \in N_i(g), j} A'_k \left( A'_i + \sum_{k \in N_i(g), j} A'_k \right) > 0$$

Therefore, $Q'_i(g)$ is a monotonically increasing function with respect to $A'_i$. The same can be certified, $Q'_i(g)$ is a monotonically decreasing function with respect to $A'_i$ ($k \neq j$). Thus, the R & D expenditure of country $i$, $i \in N$ increases in country $j$, it can increase its market share in the country $j$. Moreover, the R & D expenditure of other country $k, k \in N_i(g) \setminus j$ increases in country $j$, it can decrease the market share of country $i$ in the country $j$.

The income function of $i$ in country $j$ can be obtained from (1)and (2):

$$S'_i(g) = \frac{A'_i}{A'_i + \sum_{k \in N_i(g), j} A'_k} - A'_i \quad (3)$$

To obtain the best technology investment, income function $S'_i(g)$ is carried out first derivation on $A'_i$: 

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\[
\frac{\partial S_i'(g)}{\partial A_i'} = \frac{\sum_{k \in N_i(g) \setminus j} A_i^k}{(A_i' + \sum_{k \in N_i(g) \setminus j} A_i^k)^2} - 1 = 0
\]

Therefore, we can get the optimal R&D investment of each firm
\[
A_i' = \frac{\eta_i(g) - 1}{[\eta_i(g)]^2} \quad (4)
\]

Known by the above formula, if \( \eta_i(g) = 1 \), the optimal R&D investment of firm is \( 0 \). However, each firm’s R&D expenditure will reach the maximum, if \( \eta_i(g) = 2 \). The income function of \( i \) in country \( j \) can be obtained from (3) and (4):
\[
S_i'(g) = \frac{1}{[\eta_i(g)]^2} \quad (5)
\]

For future,
\[
\frac{\partial S_i'(g)}{\partial \eta_i(g)} = \frac{-2}{[\eta_i(g)]^3}
\]

So \( S_i' \) is a decreasing function with respect to \( \eta_i(g) \) which means that as a country forms more trade agreements and more foreign firms become active in its home market, the social welfare of firms in this market is decreasing.

From (5) we can obtain the social welfare of country \( i \).
\[
S_i(g) = \sum_{j \in N_i(g)} \frac{1}{[\eta_j(g)]^2} \quad (6)
\]

The stable and efficient networks

We employ a relatively weak notion of stability that is based on the idea that although FTAs are formed bilaterally, they can be severed unilaterally. We have borrowed this definition of stable networks from Goyal and Joshi (2006):

Definition 1 The network \( g \) is stable if for all \( i, j \in N \)

(i) \( S_i(g + g_j) - S_i(g) > 0 \)

(ii) \( S_j(g) - S_j(g - g_j) \geq 0 \); \( S_j(g) - S_j(g - g_j) \geq 0 \)

In words, a stable networks means that any country \( i, j \in N \) in a stable network FTA has no incentive to sever an existing FTA with another, and any countries \( i, j \in N \) that are not involved in an FTA have no incentive to form an agreement.

(1) When the country \( i \) and country \( j \) to sign an new FTA, it will form new FTA network \( g + g_j \). Therefore, the social welfare of country \( i \) is given by
\[
S_i(g + g_j) = \frac{1}{[\eta_i(g)]^2} + \sum_{k \in N_i(g) \setminus \{i, j\}} \frac{1}{[\eta_k(g)]^2} + \frac{1}{[\eta_k(g) + 1]^2}
\]

The marginal returns to country \( i \) from forming a link with country \( j \) are now given by
\[
S_i(g + g_j) - S_i(g) = \frac{-2\eta_i(g) - 1}{\eta_i(g)(1 + \eta_i(g))^2} + \frac{1}{(\eta_i(g) + 1)^2}
\]

The country \( i \) will sign a new FTA with country \( j \) if \( S_i(g + g_j) - S_i(g) > 0 \).

(2) If country \( i \) sever an existing FTA with country \( j \), it will form a new FTA network \( g - g_j \). Therefore, the social welfare of country \( i \) is given by
The marginal returns to country $i$ from severing a link with country $j$ are now given by
\[ S_i(g - g_j) = \frac{1}{|\eta_i(g) - 1|^2} + \sum_{k \in \eta_i(g)} \frac{1}{|\eta_k(g)|^2} \]

The country $i$ will serve FTA with country $j$, if $S_i(g - g_j) - S_j(g) \geq 0$.

**The main result**

**PROPOSITION 2.** The empty trading network is stable.

**PROOF.** For an empty FTA network $g^e$, the market is monopolized. Thus when $\eta_i(g) = 1$, the social welfare of country $i$ is given by
\[ S_i(g) = 1 \]

When the country $i$ and country $j$ to sign an new FTA, the marginal returns to country $i$ from forming a link with country $j$ are now given by
\[ S_i(g^e + g_j) - S_i(g^e) = -\frac{1}{2} < 0 \]

Thus, the social welfare of country $i$ will reduce, when the country $i$ and country $j$ to sign a new FTA. The same reason, when the country $j$ and country $i$ to sign a new FTA, the social welfare of country $j$ will reduce. Therefore, the empty trading network is stable.

For any country in the empty network, it will occupy all of the market. However, when the country $i$ and country $j$ to sign an new FTA, the firm of country $i$ will lose the entire market as long as the firm of country $j$ have a little technology investment. Thus, in order to compete for the market share, they will continue to increase R&D investment. When the two firms achieve the optimal R&D investment, market share will be split. In this case the R&D investment of each firm is greater than 0, so each country has no incentive to sever an existing FTA with another.

**PROPOSITION 3.** The complete trading network is stable if $N \geq 4$.

The social welfare of country $i$ in the complete network is given by
\[ S_i(g) = \sum_{k \in \eta_i(g)} \frac{1}{N} \text{=} \frac{1}{N} \]

For further, if country $i$ sever an existing FTA with country $j$, the social welfare of country $i$ is given by
\[ S_i(g - g_j) = \frac{1}{(N-1)^2} + (N-2) \frac{1}{N^2} \]

The marginal returns to country $i$ are now given by
\[ S_i(g^e) - S_i(g^e - g_j) = 2 \frac{1}{N^2} - \frac{1}{(N-1)^2} \]

Thus, $S_i(g^e) - S_i(g^e - g_j) > 0$, if $N \geq 4$. Thus, country $i$ has no incentive to serve FTA with country $j$. So The complete trading network is stable if $N \geq 4$.

**PROPOSITION 4.** There is no improving path from the empty network $g^e$ to the complete network $g^c$.

Suppose that there are there countries in the world. An isolated country $a$ wishes to remain autarkic, but country $b$ has a certain number of FTAs, it has strict incentives to form FTAs with all other countries.

**PROOF.** If country $a$ and country $b$ to sign an new FTA, it will form a new FTA network.
We can rewrite the marginal payoff to country $a$ from a bilateral free-trade agreement as follows:

$$S_a(g + g_{ab}) - S_a(g) = \frac{-6 - 17}{36} < 0$$

The social welfare of country $a$ will reduce, when the country $a$ and country $b$ to sign an new FTA. Thus, The country $a$ will have no incentive to serve FTA with country $b$.

The marginal returns to country $a$ from forming a link with country $b$ are now given by

$$S_b(g + g_{ab}) - S_b(g) = \frac{12 - 8}{36} = \frac{4}{9} > 0$$

Thus, The country $b$ will have incentive to serve FTA with country $a$.

Thus, there is no improving path from the empty network $g^e$ to the complete network $g^c$, if $N \geq 3$.

**Conclusion**

Our interest has been in the following question: What structure of FTA is consistent with the incentives of individual countries? We have developed a simple model of network formation to analyze this question. Based on Goyal and Joshi, we assumed that each firm competes for market share only by R&D investment via Classical Competition Model and find that no country has incentive to sign FTA with other country if the country is isolated. Therefore, the empty FTA network is stable. However, the complete network is still stable. Thus, the best choice for each country is that they doesn’t sign ant FTA with other country. Otherwise, it has to try to sign FTA with other country to reduce the social welfare losses. However, we find that there is no improving path from the empty network to the complete network.

**References**


