

Study of Shale Gas Debris' Fractal Theory and Rock Drillability

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ABSTRACT: With the developing of shale gas industry, in order to predict rock drillability in drilling time, we should set up the rock drillability profile, which has significant meaning. The article introduces the fractal theory briefly and gives detailed theoretical derivation on fractal dimension D . It has many disadvantages in actual production like short of core and high-cost. In this article, it uses upward shale gas debris to study rock drillability. In view of the FuLing-shale-gas-blocks, this article establishes a relationship between fractal dimension and drillability of extremum, the model has high fitting degree, which has considerable meaning for the actual production.

KEYWORD: Fu-Ling area; Debris; Fractal; Drillability; Fractal dimension

1 THEORETICAL DERIVATION OF FRACTAL DIMENSION

In nature, many irregular things can't be described by Euclidean geometry, while this problem can be solved by Fractal theory (Botsis J et al, 1987). For example the Menger Sponge, the appearance of a cube is three-dimensional, but actually sponge's structure is based on many chaotic micropore. When under a certain pressure, the sponge will become a flat surface, and then it became two-dimensional. This says that the appearance of a cube is actually part of 3-D structure, the real dimension is greater than 2 but less than 3. So we can give a logical conclusion, Euclidean geometry just only reflects the apparent phenomenon of object, however fractal dimension describes the internal characteristics of the object and reveals the inherent law.

According to the fractal theory, a relatively independent part of object is the image of the whole and microcosm in a way. Bit breaks rock constantly in drilling time, debris morphology show irregular shape, and the last it becomes smaller and smaller. Different sizes of debris has self-similarity in the overall form and partial form, so we can use fractal theory to study upward debris and establish the relationship with rock drillability (Li Shibin et al, 2007).

Theoretical model of fragments fractal distribution:

$$N = Cr^{-D} \quad (1)$$

In the formula, r represents the debris' diameter, N represents the number of debris corresponding to r , C is a constant, D represents fractal dimension.

Assumptions that a cube of side length is a , and broken base is B (B is greater than or equal to 2), so the cube of side length is a/B when broken in first time, and so on, the cube of side length is a/B^m when broken m time (Ma Hai et al, 2008) (Mandelbrot B B, 1982). According to theoretical fractal distribution model, we give following formula.

$$N_m = Cr_m^{-D} = Ca^{-D} B^{mD} \quad (2)$$

In the formula, N_m represents the number of debris corresponding to r_m . N_{rm} represents the adding up debris corresponding from r_1 to r_m .

$$\begin{aligned} N_{rm} &= N_1 + N_2 + N_3 + \dots + N_m \\ &= Ca^{-D} (B^D + B^{2D} + B^{3D} + \dots + B^{mD}) \\ &= Ca^{-D} \frac{B^D (B^{mD} - 1)}{B^D - 1} \end{aligned} \quad (3)$$

Combined with fractal dimension, D is greater than or equal to 1.73 given in the document (Li Shibin et al, 2007), when m is greater than or equal to 4, then B^{mD} is greater than or equal to 1. So (3) can be changed to the following formula.

$$N_{rm} = \frac{CB^D}{B^D - 1} a^{-D} B^{mD} = \frac{CB^D}{B^D - 1} r_m^{-D} = C_1 r_m^{-D} \quad (4)$$

In the formula, C_1 is a constant. Due to a large quantity of debris, it's inconvenient in statistics. The article turn to research relationship between part of debris' quality and debris' diameter. We can get de-

bris' quality from electronic scales and scales from sieve mesh.

$$N_x = C_1 r_{\min}^{-D} - C_1 x^{-D} \quad (5)$$

In the formula, N_x represents the number of debris corresponding to less than diameter x , r_{\min} represents debris' minimum diameter.

$$\begin{aligned} M_r &= \sum_{x=0}^r \frac{4}{3} \pi x^3 \rho \Delta N_x \\ &= \sum_{x=0}^r \frac{4}{3} \pi \rho (-C) (-D) x^{2-D} \Delta x \end{aligned} \quad (6)$$

In the formula, M_r represents the quality of debris corresponding to less than sieve mesh diameter r , x represents any debris' diameter from 0 to r , ρ represents density. We can change (6) to (7) through differential and integral fundamental theorem.

$$M_r = \frac{4}{3} \pi \rho C D \int_0^r x^{2-D} dx = \frac{4}{3} \pi \rho C D \frac{r^{3-D}}{3-D} \quad (7)$$

then (8) can be obtained

$$\frac{M_r}{M_0} = \frac{\frac{1}{3-D} \cdot r^{3-D}}{\frac{1}{3-D} \cdot r_{\max}^{3-D}} = \left(\frac{r}{r_{\max}} \right)^{3-D} \quad (8)$$

In the formula, M_r/M_0 represents the quality percentage of debris corresponding to less than sieve mesh diameter r , M_0 represents the total debris' quality, r_{\max} represents the debris' maximum diameter.

Based on theoretical model of fragments fractal distribution, the article gives actual fractal distribution regularity about fragments. If condition is satisfied, the expression of both expression forms are

consistent. The article gives detailed theoretical derivation on fractal dimension D , at the same time, we establishes monadic linear regression relationship between $\ln(M_r/M_0)$ and $\ln(r/r_{\max})$. The slope of regression straight line is 3 minus D in rectangular coordinate and thus we can get fractal dimension D .

2 DETERMINE THE DEBRIS' DIMENSION D

This paper selects the stratigraphic position including Longtan, Maokou, Qixia, Hanjiadian, Xiaoheba and Longmaxi. The lithology mainly contains mud and shale. During the experiment, the quality of debris is controlled at 100g. We should dry off debris, so as to enhance the accuracy of experimental date.

In this experiment, the sieve mesh diameter are 4.75mm, 2.00mm, 1.00mm, 0.5mm, 0.35mm, respectively. The vibration time of the separator is set to 5 minutes. Debris is weighted (shown in table one) and recorded for analysis (shown in figure one) on each level.



Fig1 debris weighing

Tab.1 Upward debris quality percentage

stratigraphic position	Depth(m)	sieve mesh diameter(mm)				
		0.35	0.5	1	2	4.75
		Cumulative percentage after screening (%)				
Longtan	1061-1062	6.12	11.19	38.27	78.56	91.21
Maokou	1245-1248	5.05	13.12	39.11	71.07	86.25
Qixia	1290-1294	6.81	15.12	32.15	76.62	91.13
Hanjiadian	1630-1631	6.11	17.23	36.47	47.25	77.68
Xiaoheba	2082-2086	7.83	21.21	33.25	43.54	76.17
Longmaxi	2210-2212	8.15	23.57	31.25	55.15	72.17

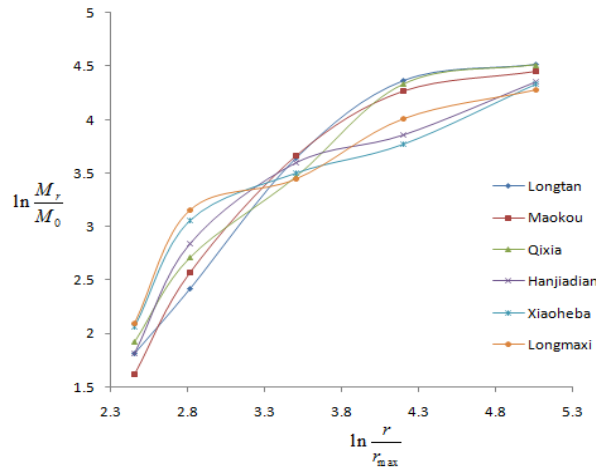


Fig2 Fractal debris of different layer

We have analyzed the six layers and drawn a diagram of the relationship between $\ln(M_r/M_0)$ and $\ln(r/r_{max})$.

The statistics and analysis for the results of each sample debris screening have been carried out. From the result, we calculate the fractal dimension D and relative coefficient.

Tab2 Fractal dimension of different layers

Stratigraphic position	Fractal dimension D	Relative coefficient
Longtan	1.924	0.901
Maokou	1.935	0.876
Qixia	2.005	0.918
Hanjiadian	2.120	0.875
Xiaoheba	2.237	0.884
Longmaxi	2.251	0.860

The relative coefficients are all above 0.86, so we can see that fractal dimension D and relative coefficient have good correlation. This conforms to the fractal theory. Fractal dimension increases with depth, which also conforms to the compaction law.

3 THE RELATIONSHIP BETWEEN FRACTAL DIMENSION AND ROCK DRILLABILITY

The research of rock drillability is based on laboratory experimental data. Automatic tester is used to get the drillability extreme value (Wang Qianyan et al, 1994) (Xie Heping, 1997) (Yang Minghe et al, 2008) (Yan Tie et al, 2007), so as to establish the relationship between rock drillability and fractal dimension in Fu-Ling area. Curve of least square method has been used to enhance fitting accuracy. Setting a function $S(x)$, so the value in the node is $S(x_i)$ ($i=1,2,\dots,n$). And then we can get an approximate one $S(x) = \sum_{i=0}^n a_j \varphi_j(x)$. The power function $w(x)$ is not used here because of the consistency of the data

of different points (Yan Tie et al, 2014) (Zhang Hui et al, 2006).

$$P = \sum_{i=0}^m [S(x_i) - y_i]^2 = \sum_{i=0}^m [\sum_{j=0}^n a_j \varphi_j(x_i) - y_i]^2 \quad (9)$$

(9) formula is intended to find the minimum of P. According to the essential conditions of function extreme value, we can get (10) formula.

$$\frac{\partial P}{\partial a_k} = 2 \sum_{i=0}^m [S(x_i) - y_i] \varphi_k(x_i) = 0 \quad (10)$$

(10) simplifies (11)

$$\sum_{j=0}^n [\sum_{i=0}^m \varphi_k(x_i) \varphi_j(x_i)] a_j = \sum_{i=0}^m \varphi_k(x_i) y_i \quad (11)$$

The mathematical solution of (11) is established and the matrix equation is established. (if we choose linear equation then $\varphi_2(x) = 0$, $a_2 = 0$)

$$\begin{pmatrix} \sum_{i=0}^m \varphi_0(x_i) \varphi_0(x_i) & \sum_{i=0}^m \varphi_0(x_i) \varphi_1(x_i) & \sum_{i=0}^m \varphi_0(x_i) \varphi_2(x_i) \\ \sum_{i=0}^m \varphi_1(x_i) \varphi_0(x_i) & \sum_{i=0}^m \varphi_1(x_i) \varphi_1(x_i) & \sum_{i=0}^m \varphi_1(x_i) \varphi_2(x_i) \\ \sum_{i=0}^m \varphi_2(x_i) \varphi_0(x_i) & \sum_{i=0}^m \varphi_2(x_i) \varphi_1(x_i) & \sum_{i=0}^m \varphi_2(x_i) \varphi_2(x_i) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^m \varphi_0(x_i) y_i \\ \sum_{i=0}^m \varphi_1(x_i) y_i \\ \sum_{i=0}^m \varphi_2(x_i) y_i \end{pmatrix} \quad (12)$$

The fractal dimension is independent variable and the drillability extreme value is the function of the variable. Though the data in table 3, we find that scatter points conform to linear or quadric curve. So we assume linear equation is $y=a_0+a_1x$ and quadric curve equation is $y=a_0+a_1x+a_2x^2$.

Tab3 Fractal dimension of different layers and drillability

Stratigraphic position	Fractal dimension D	Rock drillability Kd
Longtan	1.924	4.89
Maokou	1.935	5.38
Qixia	2.005	5.84
Hanjiadian	2.120	6.18
Xiaoheba	2.237	6.45
Longmaxi	2.251	6.61

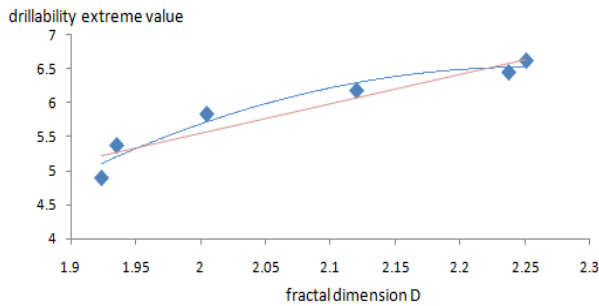


Fig3 Fractal dimension and drillability scatter plot

According to the assumptive linear equation, the matrix equation is established.

$$\begin{pmatrix} 6 & 12.472 \\ 12.472 & 26.0316 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 35.35 \\ 73.9372 \end{pmatrix} \quad (13)$$

We calculate the equation $y=4.286x-3.018$ and relative coefficient $R^2=0.8978$.

According to the assumptive quadric curve equation, the matrix equation is established.

$$\begin{pmatrix} 6 & 12.472 & 26.0316 \\ 12.472 & 26.0316 & 54.5557 \\ 26.0316 & 54.5557 & 114.7988 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 35.35 \\ 73.9372 \\ 155.2677 \end{pmatrix} \quad (14)$$

We calculate the equation $y=-9.184x^2+42.645x-42.907$ and relative coefficient $R^2=0.9412$. Compared the relative coefficient of two cases, we decide to use quadric curve equation to establish the relationship between fractal dimension and rock drillability.

4 CONCLUSION

(1) After the study of upward debris on the distribution, it indicates that the rock fragment distribution has good fractal characteristics and the relative coefficient is all above 0.86. So we can use the fractal dimension to analysis the rock distribution.

(2) In the experiment, a part of debris will be remained on the sieve and it will affect the accuracy of experimental data due to its difficulty to be removed.

(3) We have established the relationship between the fractal dimension and rock drillability. Curve of least square method has been used to reduce the error between the actual and the theoretical data. We have chosen the method of quadratic curves which has highly fitting degrees.

(4) This method of using upward debris is more simple and cheaper. It could take sample of debris constantly in drilling time. This article has certain theoretical and application value.

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