

# High Resolution RD Reconstruction for Two-dimension Sparse HF Radar

Guodong Jin and Libin Lu

Xi'an research institute of high technology Xian, China

jinguodong\_army@163.com lulibin@126.com

**Keywords:** Range-Doppler (RD); Compressed sensing (CS); High resolution range profiles (HRRPs)

**Abstract.** A novel high resolution range-Doppler (RD) reconstruction method for high frequency (HF) radar via compressed sensing (CS) is proposed. In this framework, discontinuous frequency sub-bands and incomplete pulse bursts are used to reconstruct high resolution range profiles (HRRPs) and Doppler spectrum simultaneously. The method is capable of effective suppression of sidelobes introduced by bandwidth discontinuity and preservation of temporal coherence of the signal in Doppler analysis despite the time samples deficiency. Simulation and experiment results have demonstrated the effectiveness of the proposed method.

## Introduction

HF radars are restricted to operating within narrow frequency bands due to serious user congestion. As a result, the achievable range resolution is poor. In order to overcome this problem, sparse frequency waveform (SFW) is adopted [1]. SFW is created by transmitting interference-free HF sub-bands selected by a frequency monitor to synthesize HRRPs. Simultaneously, a number of pulse bursts are employed for Doppler analysis. The HF radar signal is usually contaminated by strong transient interference, such as impulsive lightning and meteor echoes. Filtering out the interference in the time domain is possible [2], but it results in missing pulse bursts. Transmitting the SFW and excising the transient interference results in signal deficiency in 2D, simultaneously in the frequency domain and time domain. The spectrum discontinuity induces high range sidelobes which decrease the signal dynamic range. The time samples absence may destroy the temporal coherence of the signal and affect the Doppler accumulation. Prediction and interpolation are useful methods in solving these problems [3]. However, the application of these methods encounters difficulties from both noise and model errors. In this paper, the target scene is represented as a sparse signal. Simultaneous reconstruction of HRRPs and Doppler frequency with low sidelobes is accomplished via sparse signal estimation by using CS.

## Problem Formulation

Within the coherent integral time (CIT),  $N_c$  pulse bursts are transmitted for Doppler processing. Each burst contains  $N_{rs}$  sub-bands selected randomly from a certain large frequency band to synthesize a large bandwidth. Suppose that the bandwidth of a sub-band and the synthetic bandwidth is  $B_r$  and  $B$ , respectively, and  $B/B_r = N_r$ ,  $N_{rs} < N_r$ . The coarse range cell is defined as  $\Delta R_r = C/2B_r$ , where  $C$  is the velocity of light. After sub-pulse stretch processing [4], the signal within CIT of a certain coarse range cell containing  $K$  moving targets with different velocities can be expressed as a  $N_{rs} \times N_c$  matrix as follows:

$$\mathbf{x} = [\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_j, \dots, \mathbf{x}_{N_c-1}] \quad , \quad \mathbf{x}_j = \sum_{k=1}^K A_k \exp[-j2\pi \mathbf{f}_j \boldsymbol{\tau}_{k,j}] \quad , \quad 0 \leq j \leq N_c - 1 \quad (1)$$

where  $A_k$  is the  $k$ th target's backward scattering amplitude,  $\boldsymbol{\tau}_{k,j}$  is the delay vector in the  $j$ th burst of  $k$ th target, and  $\tau_{k,j}(p) = \frac{2(R_0 + v_k(N_{rs} \times j + p)T_r)}{C}$ ,  $0 \leq p \leq N_{rs} - 1$ , where  $R_0$  is the range of

the coarse range cell,  $v_k$  is the velocity of the  $k$ th target, and  $T_r$  is the pulse repetition interval (PRI). The carrier frequency is denoted by  $f_j = f_0 + \mathbf{k}_j B_r$ , where  $f_0$  is the initial frequency, and  $\mathbf{k}_j$  is a subset of  $N_{rs}$  elements selected from  $[0:N_r-1]$ . Here  $\mathbf{k}_j$  represents the available time-varying clean frequency sub-bands of HF radar. By removing the polluted sub-bands and corrupted pulse bursts contaminated by transient interference [2], the 2D deficient signal becomes:

$$\mathbf{x}' = [\mathbf{x}'_0, \mathbf{x}'_1, \dots, \mathbf{x}'_q, \dots, \mathbf{x}'_{N_{cs}-1}] \quad , \quad 0 \leq q \leq N_{cs}-1 \quad , \quad N_{cs} < N_c \quad (2)$$

where  $\mathbf{x}'$  is a submatrix of  $\mathbf{x}$ . Signal (2) contains only a subset of the frequency sub-bands and an incomplete number of the pulse bursts, so it is capable of avoiding polluted frequency bands in the frequency domain and transient interference in the time domain. Inspired by stepped frequency waveform [5], the HRRPs can be generated by applying the inverse fast Fourier transform (IFFT) to a burst of sub-pulses of a coarse range cell, with missed frequency bands filled by zeros. Subsequently, the Doppler frequency of targets can be obtained by applying fast Fourier transform (FFT) to the bursts of high resolution range cells with missed bursts filled by zeros. Unfortunately, the bandwidth discontinuity introduces high range sidelobes which may degrade the detection performance. Meanwhile, the deficiency of time samples may cause difficulties in conventional Doppler analysis.

### High Resolution 2D Reconstruction Via CS

Consider a  $K$ -sparse discrete signal  $\mathbf{s}$  of length  $N$  under an appropriate basis  $\Phi$ ,  $K \ll N$ . It makes sense that we should only have to measure a signal  $M \geq O(K \log N)$  times instead of  $N$  [6-7]. Applying a linear measurement process to  $\mathbf{s}$ , we have  $\mathbf{y} = \mathbf{A}\Phi\mathbf{s} + \mathbf{n} = \Psi\mathbf{s} + \mathbf{n}$ , where  $\mathbf{A}$  is a random measurement matrix,  $\Psi$  is defined as the dictionary of size  $M \times N$ , and  $\mathbf{n}$  is additive noise. The CS theory indicates that accurate recovery of  $\mathbf{s}$  is possible if the matrix  $\Psi$  has optimal restricted isometry property (RIP) [8].

Unlike the traditional method of obtaining the HRRPs and Doppler frequency of targets separately, we attempt to obtain them simultaneously via CS. Assuming that the number of targets is small, the signal can be regarded as sparse in the RD plane. Let  $M = N_{rs}N_{cs}$ ,  $N = N_rN_c$ ; by reshaping the signal (2) into a vector and taking the noise into account, we express the measurements as follows:

$$y(m) = \sum_{k=1}^K A_k \exp[-j4\pi f(m) \frac{R_0 + v_k t_m}{C}] + n(m) \quad , \quad 0 \leq m \leq M-1 \quad (3)$$

where  $f(m)$  is randomly distributed over  $[f_0, f_0 + B]$ ,  $n(m)$  is the noise, and  $t_m = G(m)T_r$  represents the sampling instant of the  $m$ th sub-pulse we used,  $G(m)$  is selected from  $[0:N_{rs}N_c]$ . The synthetic bandwidth is  $B$ , and the high resolution range cell is defined as  $\Delta R = C/2B$ . The Doppler and velocity resolution are defined as  $\Delta f_d = 1/N_{rs}T_rN_c$  and  $\Delta v = \Delta f_d \lambda / 2$ , respectively, where

$\lambda$  is the wavelength. Then the dictionary can be constructed as:

$$\Psi = \{\Psi_0, \Psi_1, \dots, \Psi_{i \times N_c + j}, \dots, \Psi_{N-1}\} \quad , \quad 0 \leq i \leq N_r-1, \quad 0 \leq j \leq N_c-1, \quad N = N_rN_c \quad (4)$$

$$\Psi_{i \times N_c + j}(m) = \exp[-j2\pi f(m)\tau'_{i,j}(m)] \quad , \quad 0 \leq m \leq M-1 \quad (5)$$

where  $\tau'_{i,j}(m) = \frac{2(\Delta R \times i + \Delta v \times j \times t_m)}{C}$ , which represents a target's delay of the sampling instant

$t_m$  with velocity  $\Delta v \times j$  within the  $i$ th high resolution range cell. The dimension of dictionary  $\Psi$  is  $M \times N$  and  $M < N$ . As indicated in [7-8], the RIP holds for  $\Psi$  generally. The target scene can be reshaped as an  $N \times 1$  sparse vector  $\mathbf{s}$  which represents the target distribution in the RD plane. This way the modified optimization problem with constraint to estimate the HRRPs and Doppler

frequency becomes:

$$\min(\|\mathbf{s}'\|_1), \text{ subject to } \|\Psi\mathbf{s}' - \mathbf{y}\|_2 \leq \varepsilon \quad (6)$$

where  $\|\cdot\|_p$  denotes  $l_p$  norm and  $\min(\cdot)$  denotes minimization,  $\mathbf{s}'$  represents the estimated vector of  $\mathbf{s}$ , and  $\varepsilon$  is the noise level which should be estimated precisely for good reconstruction [8]. Both convex optimization and greedy algorithms can be applied to solve the norm optimization. After reshaping  $\mathbf{s}'$  into a 2D grid, the HRRPs and Doppler frequency are obtained.

## Results and Discussion

In this section, the proposed algorithm was evaluated using simulated and experimental data.

Table 1 Parameters of a 2D sparse signal for simulation

$T_r$	$f_0$	$B_r$	$B$	$N_r$	$N_{rs}$	$N_c$	$N_{cs}$
10m s	10MH z	10KHz	320K Hz	32	16	32	16

In our simulation, the parameters of a 2D sparse signal are listed in Table I. A target scene consisting of five targets with  $N_r \times N_c = 32 \times 32$  grids is presented in Fig. 1(a). Only half of the sub-pulses and half of the bursts are randomly selected for reconstructing the target scene. The signal-to-noise ratio (SNR) is set at 10dB after sub-pulse stretch processing. Fig. 1(b) shows the 2D recovery by constructing the dictionary  $\Psi$  according to (4) and solving the optimization of (6). Five targets are clearly resolved in the RD plane with noise and sidelobes suppressed to a low level.

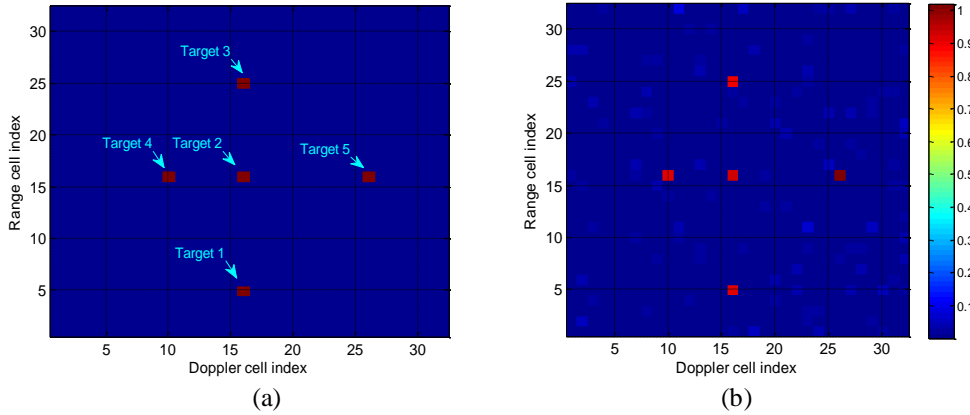


Figure 1. Target scene reconstruction  
(a) original targetscene (b) recovered target scene with SNR=10

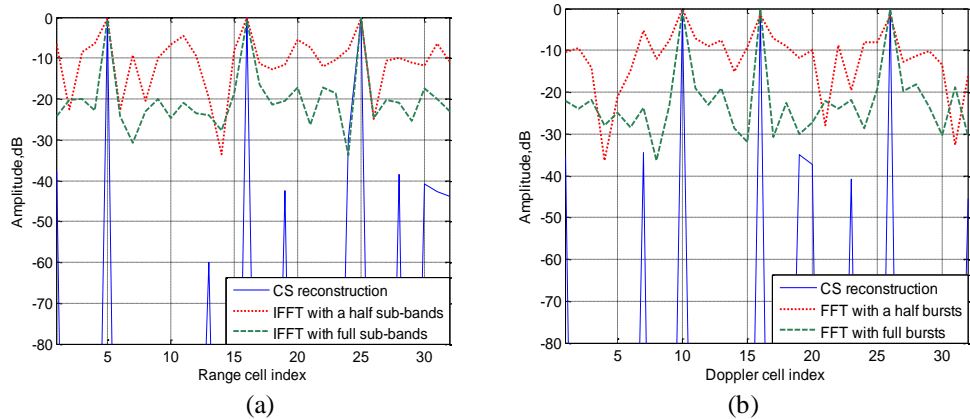


Figure 2. Comparison of the results of different methods  
(a) HRRPs for target 1,2 and 3 (b) Doppler fortarget 2,4 and 5

Fig. 2(a) demonstrates the reconstructed HRRPs of target 1, 2 and 3 compared with the results of IFFT with the full number of sub-bands and half of the sub-bands, respectively. It is obvious that the traditional method suffers from high sidelobes, whereas the proposed method can effectively suppress the sidelobes to -38dB. Fig. 2(b) depicts the recovered Doppler frequency of target 2, 4 and 5 compared with the traditional FFT Doppler analysis with the full number of bursts and half of the bursts, respectively. It can be shown that the targets are resolved with the peak sidelobe at -34.5dB, whereas with -5dB and -18dB for FFT with half of the bursts and all the bursts, respectively. The proposed approach exhibits good performance in sparse bandwidth synthesis and temporal coherence preservation. An additional benefit from sparse signal estimation by using CS is that the noise and sidelobes are effectively suppressed.

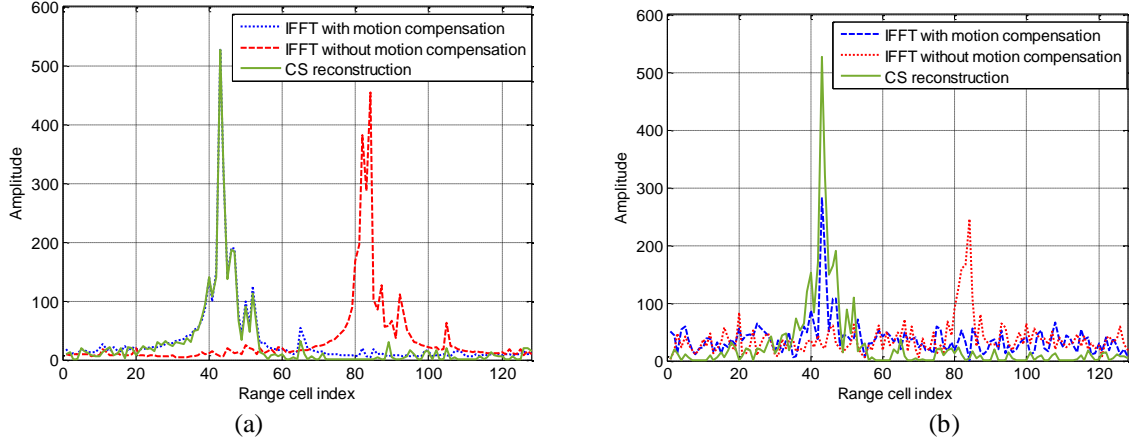


Figure 3. Real data results (a) CS result compared with IFFT with full data (b) CS result compared with IFFT with partial data

The proposed algorithm was also evaluated using the experimental data from an S-band step frequency radar.  $N_c = 32$  pulse bursts are transmitted. And each burst contains  $N_r = 128$  pulses. The PRF  $f_r = 1\text{KHz}$ , the carry frequency  $f_0 = 2.4\text{GHz}$ , and the step frequency  $\Delta f = 1\text{MHz}$ . And the synthetic bandwidth  $B = 128\text{MHz}$ . To validate the proposed algorithm,  $N_{cs} = 16$  pulse bursts are randomly selected, and in each burst  $N_{rs} = 64$  pulses are randomly adopted. There is a target in the testing range cell with the velocity of  $21\text{m/s}$ . The dictionary  $\Psi$  is constructed according to (3)-(5). The length of Doppler sequence is set as  $N_c = 32$ . And  $M = N_{rs}N_{cs}$ ,  $N = N_rN_c$ . Thus the dimension of dictionary is  $M \times N = N_{rs}N_{cs} \times N_rN_c = 1024 \times 4096$ . In order to improve the computational efficiency, we use Orthogonal matching pursuit (OMP) algorithm to solve the optimization problem in (6). In each iteration of the OMP, we get a group of indexes of fine range bin and velocity cell simultaneously. Then we can obtain the high range profile and Doppler frequency exactly. Figure 3(a) depicts the comparison of the high range profile by IFFT with full data ( $N_r \times N_c$  pulses) and OMP optimization. The blackish green line demonstrates the CS reconstruction of range profile. While the red line and blue line show the high range profile by IFFT without or with motion compensation when the velocity of target is preset to  $21\text{m/s}$ . Figure 3(b) depicts the comparison of the high range profile by FFT with partial data ( $N_{rs} \times N_{cs}$  pulses) and OMP optimization. The blackish green line demonstrates the CS reconstruction of range profile. While the red line and blue line show the high range profile by IFFT without or with motion compensation, with the missing data zero-padded. Compared with Figure 3(a), the distortion of the spectrum of target is more serious because of target's motion. Meanwhile the level of noise and sidelobe is higher than figure 3(a) due to data missing of some pulses. CS reconstruction can significantly focus the target's spectrum and reduce the granting sidelobes, which is helpful for further analysis of detection of small moving targets. More importantly, using CS reconstruction, we can estimate the Doppler frequency and high range profile simultaneously.

## Conclusions

A general high resolution RD reconstruction method for 2D sparse signal of HF radar is presented. HRRPs and the Doppler frequency are simultaneously reconstructed precisely with noise and sidelobes suppressed. The effectiveness of proposed method is validated by simulation and experimental results.

## References

- [1] Liu, W. X., Lesturgie, M., Lu, Y. L.: ‘Real-time sparse frequency waveform design for HFSWR system’, *Electron. Lett.*, 2007, 43, (24), 1387–1389.
- [2] Barnum, J. R., Simpson, E. E.: ‘Over-the-Horizon Radar sensitivity Enhancement by Impulsive Noise excision’. *IEEE National Radar Conference*, Syracuse, NY, USA, 1997, pp. 252–256.
- [3] Borison, S. L., Bowling, S. B., and Cuomo, K. M.: ‘Super-resolution methods for wideband radar’, *Lincoln Lab. J.*, 1992, 5, (3), pp. 441–461.
- [4] Richards, M.: ‘Fundamentals of Radar Signal Processing’. McGraw-Hill, 2005.
- [5] Wehner, D. R., ‘High resolution radar’. Artech House, Norwood, MA, 1995.
- [6] Herman, M. A., Strohmer, T.: ‘High-Resolution Radar via Compressed Sensing’. *IEEE Trans. Signal Processing*, 2009, 57, (6), pp. 2275-2284.
- [7] Wang, H. X., Quan, Y. H., Xing, M. D., Zhang, S. H.: ‘ISAR Imaging via Sparse Probing Frequencies’. *IEEE Lett. Geosci. Remote Sens.*, 2011, 8, (3), pp.451-455.
- [8] [Candès, E., Romberg, J., and Tao, T.: ‘Near-optimal signal recovery from random projections: universal encoding strategies?’ *IEEE Trans. Information Theory*, 2006, 52, (2), pp. 489–509.