

A Method of Attitude Control for Quadrotor Helicopter Based on Disturbance Observer

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Abstract. In this paper, an alternative control strategy based on the proportional integral (PI) observer is developed for attitude stabilization of a quadrotor subject to external disturbances. The PI observer has a simple structure with only two parameters that are needed to tune. Moreover, unlike high gain observers, gains of the PI observer can be controlled in a proper level because of applies of the integral action. This control strategy leads to a composite controller consisting of a backstepping controller and a PI disturbance observer. The design procedure for this composite controller consists of two stages. In the first stage, a PI disturbance observer is designed to estimate the external disturbances. In the second stage, a backstepping controller is designed to stabilize the attitude of a quadrotor under the assumption that the disturbance is measurable. And then the controller is integrated with the previous designed observer by replacing the disturbance in control law with its estimation. The resulting observer based controller can stabilize the attitude of a quadrotor subject to external disturbances and model inaccuracy. These results are confirmed by numerical simulations.

Introduction

A wide variety of attitude controller for quadrotor helicopters already exists in the literature. In [1], independent single-input-single-output (SISO) linear controllers are designed to regulate quadrotor attitude. The work in [2] shows that the classic model-independent PD controller can stabilize asymptotically the attitude of a quadrotor by considering the gyroscopic effects. Moreover, based on compensating Coriolis and gyroscopic torques and the use of PD2 feedback structure, an exponentially stabilizing controller is proposed. In [3], the rotor dynamics are considered in the model. Backstepping and sliding mode techniques are used to control the helicopter. Besides, Backstepping approaches applied to the quadrotor helicopter can also be found in [4]-[6].

However, few of the control strategies consider external disturbances and parametric uncertainties of the model. To achieve good performance in a hostile environment, a natural way is to estimate the disturbance by designing an observer, and then use the estimation to compensate for the disturbance. In [7], a high gain observer and sliding mode controller for a quadrotor is designed. This observer based controller is proved of good performance in simulations that consider a class of time-varying disturbances. But the high gain feedback solutions have drawbacks that they may saturate actuators, excite high frequency modes etc. The work in [8] designs a robust controller using sliding mode control driven by sliding mode disturbance observer (SMC-SMDO) approach for a small quadrotor helicopter. The SMC-SMDO technique allows for a continuous control robust to external disturbance and model uncertainties. However, as the parametric structure is more complex than other traditional observers, the parameter tuning is a difficult issue for sliding mode observer.

Problem Statement

The quadrotor has four rotors to generate thrust forces T_i and rolling moments M_i , $i = 1, 2, 3, 4$. Let $B = \{O_b, x_b, y_b, z_b\}$ denotes the body fixed frame, whose origin is at the center of mass of the vehicle, and $E = \{O_e, x_e, y_e, z_e\}$ denotes an inertial frame fixed with respect to the earth. The attitude of a

quadrotor is represented by three Euler angles $\Theta = (\phi, \theta, \psi)^T$ and the body angular velocities by $\omega = (p, q, r)^T$.

The quadrotor is considered to be a rigid body vehicle, whose dynamic equation of attitude motion

$$\dot{\omega} = -J^{-1}\omega \times J\omega + J^{-1}\tau + d \quad (1)$$

where $\tau \in R^3$ is the input torques applied to the center of mass of the vehicle, $J \in R^{3 \times 3}$ is the known inertia matrix, and $d = (d_x, d_y, d_z)^T$ is the unknown external disturbances.

The designed τ is generated from aerodynamic forces and rolling moments produced by four propellers. Aerodynamic force of a propeller can be shown to be proportional to square of its rotational speed using momentum theory. It is modeled as [1]

$$T = C_T \rho A (\Omega R)^2 \quad (2)$$

where C_T is the thrust coefficient, ρ is the air density, A is the propeller disk area, Ω is the propeller rotational speed (in radian) and R is rotor radius.

Similarly, aerodynamic rolling moment of a propeller can be shown to be proportional to square of its rotational speed using momentum theory. It is modeled as [1]

$$M = C_M \rho A (\Omega R)^2 R \quad (3)$$

where C_M is the rolling moment coefficient.

Then the input torques $\tau = (\tau_x, \tau_y, \tau_z)^T$ and the total thrust F produced by all four propellers can be expressed by

$$\begin{pmatrix} \tau_x \\ \tau_y \\ \tau_z \\ F \end{pmatrix} = \begin{bmatrix} 0 & b_T & 0 & -b_T \\ -b_T & 0 & b_T & 0 \\ b_M & -b_M & b_M & -b_M \\ b_T & b_T & b_T & b_T \end{bmatrix} \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{pmatrix} \quad (4)$$

where $b_T = C_T \rho A l R^2$, $b_M = C_M \rho A l R^3$ and l is the horizontal distance from the propeller centre to centre of gravity. Given desired input torques τ and total thrust F, the desired rotor speeds can be obtained by solving (5).

$$G(s) = \frac{0.8669}{1 - 0.6573s} \quad (5)$$

This transfer function is identified from measured input-speed data.

Design of Disturbance Observer

It is well known that the PI observer is useful for linear systems with constant disturbance [5]. However, disturbances acted on a flying quadrotor depend on the state of aircraft and time. Since, in general, there is no prior information about the disturbances, it is reasonable to make following assumption.

Assumption 1: The disturbance d is bounded and satisfies

$$\dot{d} = \zeta(t) \quad (6)$$

where $\zeta(t)$ is an unknown function whose upper bound $\sup_{t \in [0, \infty)} \|\zeta(t)\| \leq b_\zeta$ and b_ζ is a positive constant. This assumption implies that the disturbance varies in a bounded speed relative to the observer dynamics.

As shown in Fig. 2, the dynamics of a PI observer for system (1) can be described by [13]

$$\begin{cases} \dot{\hat{\omega}} = -J^{-1}\omega \times J\omega + J^{-1}\tau + L_1\eta + \hat{d} \\ \dot{\hat{d}} = L_2\eta \end{cases} \quad (7)$$

where $L_1 \in R^{3 \times 3}$ and $L_2 \in R^{3 \times 3}$ are matrices designed latter, $\hat{\omega} = (\hat{p}, \hat{q}, \hat{r})^T$ is the estimate of angular velocities ω by observer and $\hat{d} = (\hat{d}_x, \hat{d}_y, \hat{d}_z)^T$ is the estimate of the disturbance d , η is the estimation error of angular velocities ω , i.e. $\eta = \omega - \hat{\omega}$. Then the problem is how to design the parametric matrices L_1 and L_2 , such that the observer can approximately estimate the states and disturbances.

Define the estimation error of disturbance as $e = d - \hat{d}$. Then from (1), (2) and (7), we have

$$\begin{pmatrix} \dot{\eta} \\ \dot{e} \end{pmatrix} = \begin{bmatrix} -L_1 & I_3 \\ -L_2 & 0 \end{bmatrix} \begin{pmatrix} \eta \\ e \end{pmatrix} + \begin{pmatrix} 0 \\ I_3 \end{pmatrix} \zeta = A \begin{pmatrix} \eta \\ e \end{pmatrix} + B\zeta \quad (8)$$

where I_n denotes the identity matrix of dimension n . From (8) the following result can be obtained.

Theorem 1: Under Assumption 1, for system (1), if the parametric matrices of PI observer (7) for system (1) are

$$\begin{aligned} L_1 &= k_1 I_3, \quad k_1 > 0 \\ L_2 &= k_2 I_3, \quad k_2 > 0 \end{aligned} \quad (9)$$

Then we have

$$\lim_{k_1, k_2 \rightarrow \infty} \lim_{t \rightarrow \infty} \begin{pmatrix} \omega(t) - \hat{\omega}(t) \\ d(t) - \hat{d}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (10)$$

For any initial states $\hat{\omega}(0)$ and $\hat{d}(0)$.

Design of Composite Controller

Original Controller Design. order to design a backstepping controller for a quadrotor, we will rewrite its dynamic model (1) in state-space form $\dot{\mathbf{X}} = f(\mathbf{X}, \mathbf{U})$ with inputs vector \mathbf{U} and states vector \mathbf{X} chosen as

$$\mathbf{X} = (x_1, x_2, x_3, x_4, x_5, x_6)^T = (\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi})^T \quad (11)$$

$$\mathbf{U} = (u_1, u_2, u_3)^T = (\tau_x, \tau_y, \tau_z)^T \quad (12)$$

As the perturbations in stability flight are small, the transformation matrix between the rates of change of attitude Θ and the body angular velocities ω can be considered as unity matrix. This implies that $(\dot{\phi}, \dot{\theta}, \dot{\psi}) \approx (p, q, r)$ is hold. By assuming that the structure of quadrotor is symmetrical, the inertia matrix is

$$J = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (13)$$

Given desired roll angular x_1^d and its measurement x_1 , the tracking error is

$$z_1 = x_1^d - x_1 \quad (14)$$

Consider the positive definite Lyapunov function as follow

$$V(z_1) = \frac{1}{2} z_1^2 \quad (15)$$

Then we use the Lyapunov theorem by considering the time derivative of (16) negative semi-definite

$$\dot{V}(z_1) = z_1(\dot{x}_1^d - x_2) \quad (16)$$

The stabilization of z_1 can be guaranteed by introducing a virtual control input x_2^d

$$x_2^d = \alpha_1 z_1 + \dot{x}_1^d \quad \alpha_1 > 0 \quad (17)$$

Then the time derivative of the Lyapunov function (15) is $\dot{V}(z_1) = -\alpha_1 z_1^2 \leq 0$.

Now our goal is design input u_1 to make the roll angular velocity tracking the virtual control x_2^d . The tracking error between measured x_2 and the virtual control is

$$z_2 = \alpha_1 z_1 + \dot{x}_1^d - x_2 \quad (18)$$

Then we consider the augmented Lyapunov function

$$V(z_1, z_2) = \frac{1}{2} (z_1^2 + z_2^2) \quad (19)$$

and its time derivative

$$\dot{V}(z_1, z_2) = z_1 \dot{z}_1 + z_2 (\alpha_1 \dot{z}_1 + \ddot{x}_1^d - \dot{x}_2) = z_2 (-a_1 x_4 x_6 - b_1 u_1 - d_x) - \alpha_1^2 z_1 z_2 - \alpha_1 z_1^2 \quad (20)$$

Note that during the attitude stabilization control, we always expect the angular accelerate to be zero, i.e. $\ddot{x}_i^d = 0, i = 1, 3, 5$. From (21), the input u_1 is then designed to make $\dot{V}(z_1, z_2) < 0$:

$$u_1 = (I_{zz} - I_{yy})x_4 x_6 + I_{xx} (\alpha_2 z_2 - \alpha_1^2 z_1 - d_x) \quad (21)$$

with $\alpha_1 > 0$.

The same steps are followed to obtain u_2 and u_3 .

$$u_2 = (I_{xx} - I_{zz})x_2 x_6 + I_{yy} (\alpha_4 z_4 - \alpha_3^2 z_3 - d_y) \quad (22)$$

$$u_3 = (I_{yy} - I_{xx})x_2 x_4 + I_{zz} (\alpha_6 z_6 - \alpha_5^2 z_5 - d_z) \quad (23)$$

Composite Controller Design. The observer based composite controller is obtained by replacing the disturbances in (21), (22) and (23) with its estimation yielded by the PI disturbance observer.

To guarantee the stabilization of roll angular control subsystem, we design the parameter α_2 in (21) as follow

$$\alpha_2 = \begin{cases} \frac{\beta_1}{|z_2|} & z_2 \neq 0 \\ 0 & z_2 = 0 \end{cases} \quad (24)$$

Similarly we can also design α_4 and α_6 for u_2 and u_3 to make the pitch and yaw angular control subsystem stable.

$$\alpha_4 = \begin{cases} \frac{\beta_2}{|z_4|} & z_4 \neq 0 \\ 0 & z_4 = 0 \end{cases} \quad \alpha_6 = \begin{cases} \frac{\beta_3}{|z_6|} & z_6 \neq 0 \\ 0 & z_6 = 0 \end{cases} \quad (25)$$

Composite Controller Simulation

In order to evaluate the proposed composite controller, simulations have been performed on the complete closed loop system in Matlab environment. External disturbances are assumed to be

$$d = \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \begin{pmatrix} 0.5 + \sin(\phi + \theta) + 0.2 \sin(t) \\ 0.5 + \sin((\phi + \theta)/2) + 0.2 \sin(2t) \\ 0.25 \end{pmatrix} \quad (26)$$

From (26) it can be seen that d is time varying and non-vanishing. Especially, the disturbance torque acted on b_z axis is constant and small due to the yaw angular of quadrotor aircraft is relatively hard to disturb.

The initial states of the quadrotor and the PI observer are $X = (0.5, 0.5, 0.5, 0.5, 0, 0)^T$ and $\hat{\omega} = (0, 0, 0)^T$, $\hat{d} = (0, 0, 0)^T$. The desired attitude is set to be $x_1^d = x_3^d = x_5^d = 0$. Parameters of the composite controller are chosen to be $\alpha_1 = \alpha_3 = 5$, $\alpha_2 = \alpha_4 = 8$, $\alpha_5 = 4$ and $\alpha_6 = 6$. From Theorem 1, the gain parameters of PI observer is tuned at $k_1 = 10$ and $k_2 = 100$.

It should be noted that, in practice, the parameters of dynamic model in observer (7) is obtained by identification or empiricism, which implies that there must be some difference between the parameters in observer (7) and the flight dynamic model of the quadrotor. To take into account such inaccurate of parameters, the dynamic parameters in observer (7) are obtained as follow

$$J_{\text{observer}} = J + J \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \quad (27)$$

where $\varepsilon_{1,2,3}$ is uniformly distributed random variable on $[0.2, 0.5]$

Therefore, our simulations are carried out in two cases. In case 1 we use the true parameters for the PI observer (7) and in case 2, inaccurate parameters obtained from (38) are used. The proposed composite controller is able to achieve good performance in both case 1 and case 2 (see Fig. 1). This proves the efficiency of the proposed control strategy. Furthermore, as shown in Fig. 2, the control inputs are continuous and smooth without the phenomenon of chattering. This implies that the proposed composite controller can overcome the drawbacks of conventional high gain observer, which may lead to the actuator saturate and high frequency modes.

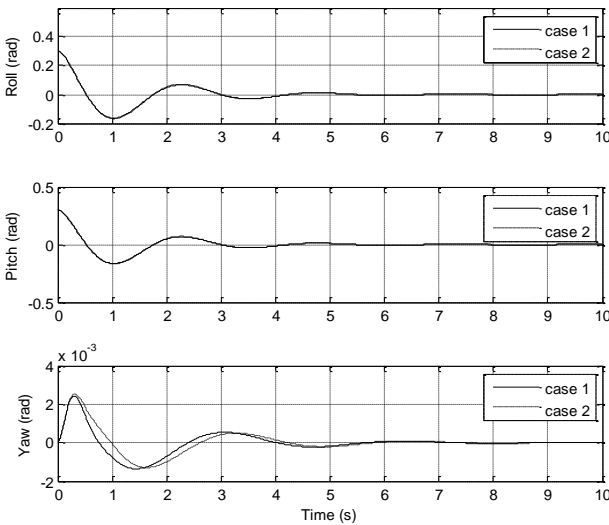


Figure 1. Attitude controlled in case 1 and case 2

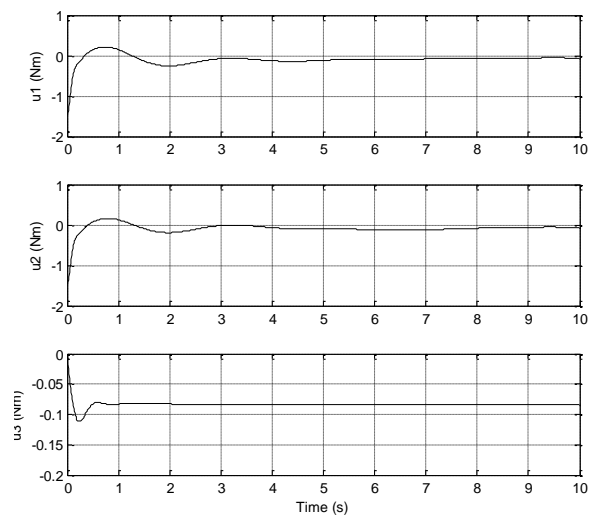


Figure 2. Control inputs in case 2

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