Optimal Train-speed Curve Based on Cooperative Train

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Abstract. This paper focuses on studying the problem of train-speed curve in railway transportation system. Different from the previous study, the characteristic of this paper is to optimize the train-speed curve by selecting the gear sequence and the switching locations. In the case of multi-vehicle cooperation, considering the energy efficiency and service quality, this paper formulates a train-speed curve optimization model to minimize the energy consumption and travel time. Then the genetic algorithm is used to optimize the train-speed curve and the obtain optimal control strategies of the train under different departure intervals.

Introduction

Compared with other modes of transport, the railway transport system has a good safety record and the highest energy efficiency. To evaluate the effectiveness and efficiency of a railway transport system, there are usually two kinds of the train operational performance metrics that need to be considered, i.e., the energy consumption and the travel time. More specifically, the energy consumption is related to the benefit of railway operating companies, and travel time is referred to evaluate the service quality for passengers. To the best of our knowledge, most of the existing studies focus on the cyclic process of traction and the coasting on a railway link. Such as Wong and Ho [1] presented several searching methods for the coasting control for mass rapid transit railways. From the perspective of mathematical justification, Howlet ([2], [3]) not only proved the existence of an optimal strategy within the framework of functional analysis theory, but also proposed the optimal driving strategy including maximum acceleration, cruising, coasting, and maximum braking phases. Effati and Roohparvar [4] used measure theory technique and iterative dynamic programming algorithm to solve the energy saving control problem.

In addition, traditional studies on energy-efficient techniques for railway transport systems mainly include mass reduction, resistance reduction, space utilization, and energy-efficient operation. For example, Bai et al. [5] investigated the energy-efficient driving strategy for freight trains based on the power consumption analysis. Ke et al. [6] presented a method for optimizing train driving strategies of the mass rapid transit systems with the aim of saving energy between successive stations. Recently, researchers have also paid more attention to the regenerative braking technique. Such as Pena et al. [7] is attempted to maximize the use of regenerative energy by optimizing the timetable, without considering a detailed energy consumption model during the trip.

The optimization of energy during the trip and maximum use of regenerative energy are two closely dependent parts of improving energy consumption, which should be combined to reduce the practical energy consumption. With the consideration of train operations under different gears, it is urgently needed to optimize the train-speed trajectory on the railway by proposing a cooperative train control model to both minimize the practical energy consumption and guarantee the service quality. Moreover, with a vector representation of solutions, we use the genetic algorithm (GA) to solve the objective problem for searching an approximate optimal train-speed trajectory.

Problem Description

Based on the background of high-speed railway, this research is intended to minimize the practical energy consumption for train operations with the consideration of regenerative braking technique.

To guarantee the economical operations and service quality, the dispatchers need to produce the detailed control guidance for each train's operations, which necessarily specifies the explicit speed-position relationship during the train's movement.

In the real-world operational environments, some physical conditions which can directly influence the finally produced speed trajectories should be taken into consideration, such as the speed limit, slope gradient, degree of curve and so on. Considering all of these physical limitations, the entire section will be divided into a series of speed limit intervals along the railway line, in which the minimum and maximum speeds are pre-determined by the railway companies to meet the requirements of both safety and efficiency. Thus, the considered problem turns out to be an optimization problem of speed control for the practical operations within a series of system constraints incurred by the line conditions. With the consideration of the speed limit of different interval, the optimal speed-shift sites and gears can be obtained by computing the speed-distance curves of different gears with known power outputs.

As shown in Fig. 1, an illustrative example is given to demonstrate the energy feedback process more clearly. Assuming that all trains run in the same direction sharing a common control strategy, the recovery energy that is generated by the braking train i can be fed back into the overhead contact line and immediately be used by the accelerating train $i+1$. Note that the transmission losses of electricity is ignored. In Fig. 1 we use a black arrow to indicate the direction of regenerative braking energy.

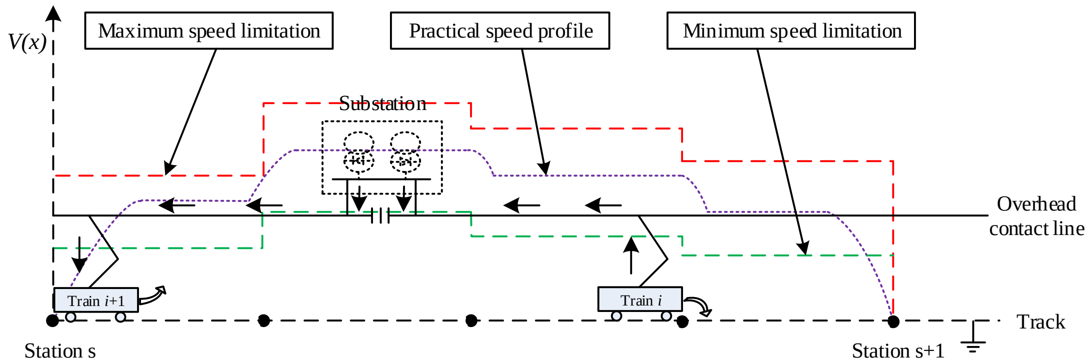

Figure 1. Energy feedback process

In the paper, we also need to consider the timeliness of the regenerative braking energy utilization. That is, the regenerative braking energy would be lose if it can’t be used in time. As shown in Fig. 2, the recovery energy which is generated by train i in time interval $[t_4^i, t_5^i]$ can be used by train $i+1$. However, the recovery energy that is generated by train $i+1$ in time interval $[t_6^{i+1}, t_7^{i+1}]$ will be lost because train i moved into the next station.

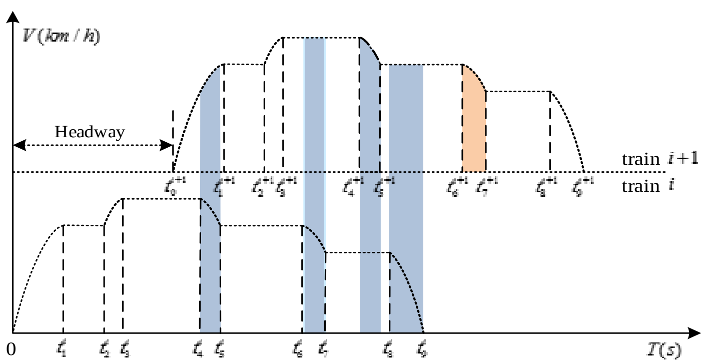

Figure 2. Overlapping time of the energy feedback

Multi-objective Optimization Model of Train Speed Curve

In this section we introduce a multi-objective train-speed trajectory optimization model to balance the energy consumption and the travel time.

Objective Functions. Energy Consumption. According to the train traction calculation rules, the total energy consumption can be calculated according to the following equation:

$$E = \int_0^T F(t)v(t)\mathrm{dt}, \tag{1}$$

Where T and E represent the total running time and the total energy consumption of the train, respectively. And $F(t)v(t)$ represents the rated power at time t.

In essence, computing the energy consumption on railway links is a complex formulation as it is difficult to deduce analytic functions $F(t)$ and $v(t)$ due to the complication of the realistic traffic environments. To simplify the computation, we adopt the Davis formula [8] to calculate the energy consumption. Specifically, the Davis formula is defined to calculate the acceleration of a train with respect to different velocities, which is given below.

$$R = R_0 + R_1 v + R_2 v^2, \tag{2}$$

Where R is the total resistance, R_0, R_1, R_2 are the resistance coefficients.

According to the actual situation, we can get the following equation:

$$F = ma + R, \tag{3}$$

and we can obtain the traction force and the braking force through positive a and negative a, respectively.

In addition, we should considering the energy feedback process in the actual operation of the train. Then, we can formulate the following objective to minimize the practical energy consumption:

$$E_p(V,G,X) = E(V,G,X) - \sum_{t\in T} \min\{E_f(V,G,X,t), E_r(V,G,X,t)\}, \tag{4}$$

And the V, G, X are decision vectors with respect to the cruising speed choic, the gear choice, speed-shift site choice, respectively. E is the total energy consumption， E_p is the practical energy consumption， E_f is the energy consumption during traction/cruise at time t, E_r is the feedback energy at time t.

Travel Time. The discrete distance method is used to calculate the total running time. Specifically, we divide the segment into $i\ (=1,2,...N)$ discretized intervals with the length δ. Then the travel time over different intervals can be calculated. Consider a distance interval denoted by $\left[x,\ x+\delta\right]$, and suppose that the velocities at two end points of this interval are $v(x)$ and $v(x+\delta)$, respectively. Then the travel time over this interval can be calculated in accordance to the following equation:

$$t_i = \begin{cases} \dfrac{\delta}{v(x)}, & v(x) = v(x+\delta) \\ \dfrac{v(x+\delta)-v(x)}{a}, & v(x) \neq v(x+\delta) \end{cases} \tag{5}$$

In this way, the total running time is：

$$T(V,G,X) = \sum_{i=1}^{N} t_i, \tag{6}$$

Train-speed Trajectory Optimization Model. As two objectives, namely E and T, are introduced to measure the performance, the discussed problem is essentially a multi-objective optimization problem. Fuzzy mathematical programming [9] is an efficient approach to solve multi-objective optimization problems, which models each objective as a fuzzy set whose membership function represents the degree of satisfaction of the objective. By using the fuzzy theory, we can get the membership function based on the energy consumption and running time, respectively. The formulation given below can be employed to represent the membership degree between the actual energy consumption and the target.

$$\bar{E}_p = \frac{(E_p - E_{p\min})}{(E_{p\max} - E_{p\min})}, \tag{7}$$

$$\bar{T} = \frac{(T - T_{\min})}{(T_{\max} - T_{\min})}, \tag{8}$$

In Eq. (7) and (8), $\bar{E}_p$, $E_{p\max}$, $E_{p\min}$ represent the membership function, the maximum value and the minimum value of the practical energy consumption E_P, respectively. $\bar{E}_p$ is closer to 1 as E_P is growing smaller. Similarly, $\bar{T}$, $T_{\max}$, $T_{\min}$ represent the membership function, the maximum value and the minimum value of travel time T, respectively.

Finally, combining the objective function and the membership function, we can construct the following formulation for the train-speed trajectory optimization problem:

$$\begin{aligned} &\max\min\{\bar{E}_p(V,G,X),\bar{T}(V,G,X)\} \\ &s.t. \quad v_k \in [\underline{v}_k,\bar{v}_k],\ k=1,2,\cdots,K \\ &\qquad T \in \left[\underline{t},\bar{t}\right], \\ &\qquad L=\sum_{k=1}^{K}(x_{k,k}-x_{k-1,k})+\sum_{k=1}^{K}(x_{k,k-1}-x_{k,k})+(x_{K+1,K+1}-x_{K,K+1}),\ k=1,2,\cdots,K \end{aligned} \tag{9}$$

In the Eq. (9), $\underline{v}_k$, $\bar{v}_k$ represent the minimum speed limit and the maximum speed limit within the k th speed limit block, respectively. $\underline{t}$, $\bar{t}$ represent the minimum running time and the maximum running time, $(x_{k,k}-x_{k-1,k})$ is the distance of traction/ deceleration, $(x_{k,k-1}-x_{k,k})$ is the cruise distance and $(x_{K+1,K+1}-x_{K,K+1})$ is the braking distance. In addition, the first constraint is respect to the speed of different speed limit, the second constraint is respect to the running time, and the last constraint is respect to the distance constraint.

Genetic Algorithm

Genetic algorithm (GA) introduced by Holland [10] is a stochastic searching method for seeking high-quality solutions for optimization problems, which simulates the process of natural evolution. In this section, we shall introduce detailed operations for seeking approximate optimal control strategies for the considered problem.

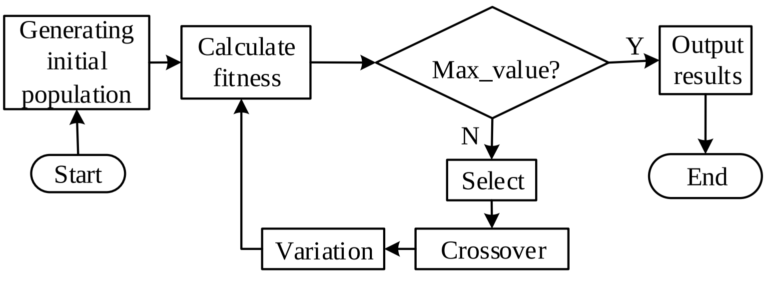

Figure 3. The genetic algorithm program flow

Coding Process. According to the problem description, the optimization of the train speed curve can be translated into the process of determining the speed-shift sites by selecting the speed and gear under pre-given speed limit interval. According to the idea of the genetic algorithm, the chromosome $X=(g_1,v_1,g_2,v_2,...,g_K,v_K)$ can be described by the selection of the gear and speed, where v_i is randomly generated speed in speed-limit interval i, $i=1,2,...,K$, g_i is the gear to be chosen by the operator from the $i-1$ interval to i interval.

Selection Process. The roulette-wheel based selection operation, which is designed on the basis of evaluation indices of different chromosomes, will be employed to select the chromosomes for the next generation. If the fitness of the individual j is F_j and the population size is pop_size, the probability that j is inherited to the next generation is: $p_j = F_j \Big/ \sum_{j=1}^{pop\_size} F_j$, $(j = 1, 2, ..., pop\_size)$. Therefore, the greater the degree of individual fitness, the individual will have a greater chance being selected if it has a greater fitness.

Cross Process. Crossover operations will be implemented among the specific chromosomes that are selected from the new population according to the crossover probability p_c. Specifically, randomly generate a number $r \in (0,1)$ for each selected individual, if $r \le p_c$, the chromosome will be selected for crossover operation. Then, the parent which has been selected to proceed with the crossover operation will produce the offspring by a linear combination.

Mutation Process. Denote the mutation probability by p_m. For each chromosome in the population, we firstly determine whether it can be selected for the mutation operation: randomly generate a number $r \in (0,1)$, then the chromosome will be selected for mutating if $r \le p_m$.

Numerical Examples

In this section, the genetic algorithm has been coded in C++ programming language. In the following, we adopt the resistance function in Yang et al. [11] as the experiment function (unit: kN), which is given as follows.

$$R = 11.4 + 0.101v + 0.001269v^2.$$

In this experiment, we assume that the distance between two stations is 40,000 meters. The maximum speed and the minimum speed of each interval is shown in Table 1.

Table 1 Cruising speed limit over different intervals

Distance interval (m)	Minimum speed limit (km/h)	Maximum speed limit (km/h)
(0，10,000)	160	220
(10,000，20,000)	200	260
(20,000，30,000)	240	300
(30,000，40,000)	190	250

In addition, preset the pop_size, the crossover probability, the mutation probability, the iteration number as 40, 0.7, 0.8, 500, respectively. In this case, we can get the optimal velocity curve and its objective function under different departure intervals. The simulation results show that if the train in each limit range all travel at a minimum speed, the running time is about 900s. However, based on the consideration of security and service levels, we set the headway between [420, 600]s. The corresponding train optimal speed curve (i.e. speed-position relationship) and the fitness curve are shown in Fig. 4:

As shown in Fig. 4, we can know that the optimal value is gradually reduced with the increasing of the headway. Here, based on the safety consideration, we select the result whose headway is 420s as the optimal control strategy. Accordingly, the speed-shift sites over different speed-limit intervals are 0m, 2885m, 9487m, 10000m, 16128m, 24299m, 24716m, 33414m, 37119m, 40000m. In this control strategy, the acceleration/deceleration rates and cruising speeds over each speed-limit interval are 0.6 m/s^2, 211.83 km/h, 0.2 m/s^2, 218.01 km/h, 0.2 m/s^2, 299.82 km/h, -0.2 m/s^2, 204.12 km/h, -0.6 m/s^2. This iteration process takes up to 14.34s in total. (Intel (R) Core(TM) i5-2450M CPU @2.50GHz).

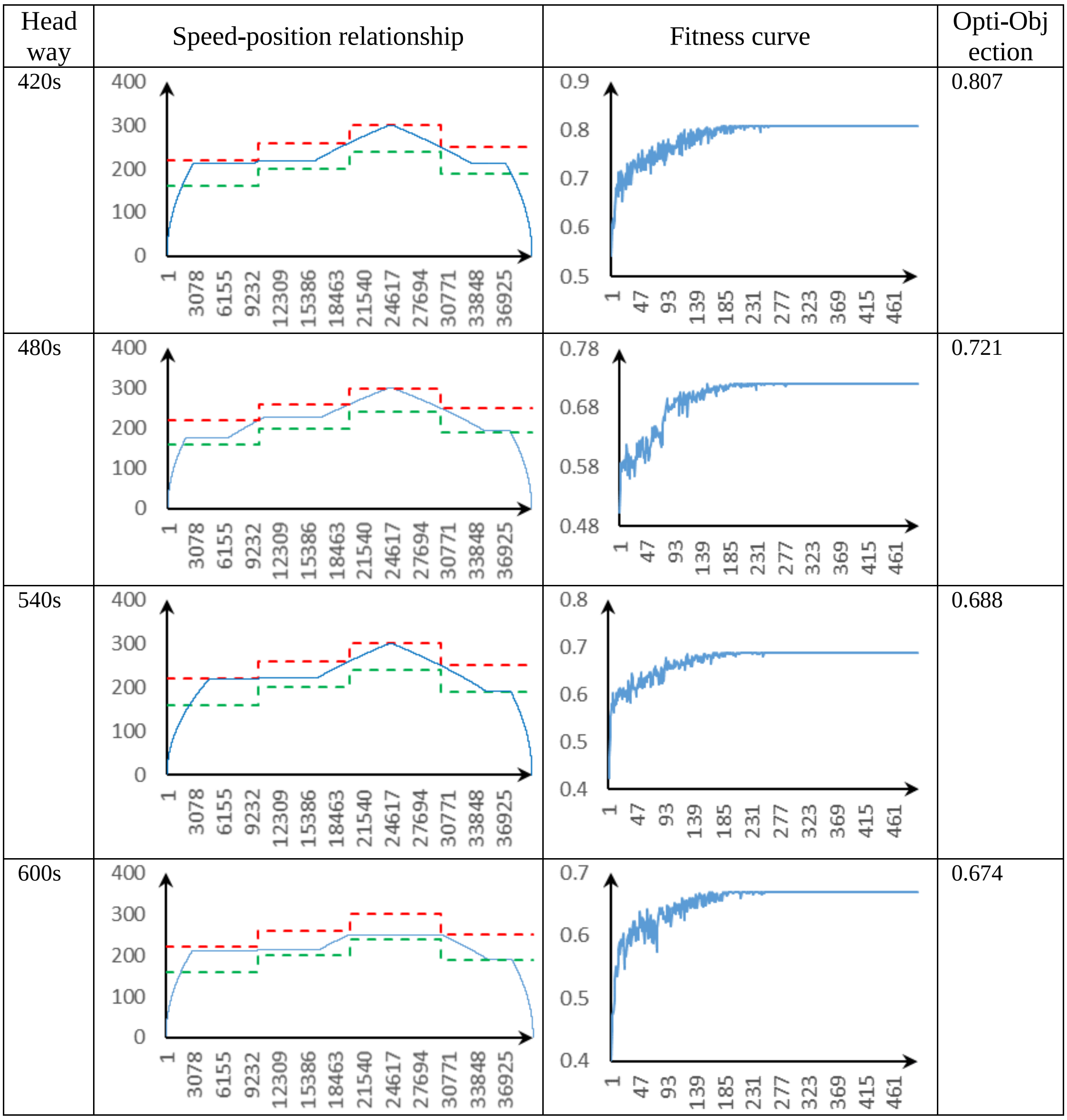

Figure 4. The results of the corresponding headway

In addition, according to the fitness function curve shown in Fig. 4, we can know that the optimal value has a significant increase with the increase of the number of iterations. When the headway is 420s, its fitness value is optimal in the 259 generation, and the optimal value is 0.807.

Conclusion

The optimization of train speed curve can significantly improve the performance of the railway transportation system. With consideration of the energy feedback, taking the speed, gear and speed-shift sites as decision variables, this paper used discrete time and discrete distance method to calculate the actual energy consumption and running time, respectively. In order to reduce the actual energy consumption and running time, corresponding mathematical optimization model was established by using the concept of membership function. After that, this paper used the genetic algorithm to solve the optimization model, and obtained the optimal control strategy of trains under different headways.

References

[1] Wong KK, Ho TK. Coast control for mass rapid transit railways with searching methods. IEE Proceedings Electric Power Applications 2004; 151(3): 365-376.

[2] Howlett P. Existence of an optimal strategy for the control of a train. School of Mathematics Report 3, University of South Australia, 1988.

[3] Howlett P. An optimal strategy for the control of a train. Journal of the Australian Mathematical Society Series B 1990; 31: 454-471.

[4] Effati S, Roohparvar H. The minimization of the fuel costs in the train transportation. Applied Mathematics and Computation 2006; 175: 1415-1431.

[5] Bai GZ, Guo J, Shi HG, Yang Y, Zhang TH. Optimization of Convergence based on the Coordination of Train Departure Time in the Subway Transfer Station. Journal of Transportation Systems Engineering and Information Technology 2013; 13(5): 134-139.

[6] Ke BR, Lin CL, Yang CC. Optimisation of train energy efficient operation for mass rapid transit systems. IET Intelligent Transport Systems 2012; 6(1): 58-66.

[7] Pena M, Fernandez A, Cucala A, Ramos A, Pecharroman RR. Optimal underground timetable design based on power flow for maximizing the use of regenerative-braking energy. Proceedings of the Institution of Mechanical Engineers Part F Journal of Rail & Rapid Transit 2012; 226(4): 397–408.

[8] Ghoseiri K, Szidarovszky F, Asgharpour MJ. A multi-objective train scheduling model and solution. Transportation Research Part B-Methodological 2004; 38(10): 927-952.

[9] Liu B. *Uncertainty Theory: an Introduction to its Axiomatic Foundations*. Springer-Verlag 2004.

[10] Holland JH. Adaptation in natural and artificial systems. Ann Arbor: University of Michigan Press; 1975.

[11] Yang L, Li K, Gao Z, Li X. Optimizing trains movement on a railway network. Omega The International Journal of Management Science 2012; 40(5): 619-633.