Nodal gap energy in high-\(T_c\) cuprate superconductors: A new paradigm

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\textbf{Abstract.} In high-\(T_c\) cuprate superconductors, the \(d\)-wave-like superconducting gap appears to depart from the standard proportionality with critical temperature \(T_c\), and shows different doping dependences between the nodal and antinodal regions of momentum space. The failure of the attempts so far to formulate these momentum-resolved gaps with \(T_c\) has posed questions to their roles in electron pairing and their mutual relationship. Here, we report a simple proportional relation between nodal gap and critical temperature in superconducting \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8\) and \(\text{Bi}_2\text{Sr}_{1.6}\text{La}_{0.4}\text{CuO}_6\). Using low-energy synchrotron radiation on angle-resolved photoemission spectroscopy, we observed the strongly curved gap profile in the underdoped cuprates, and found that a nodal-gap component \(2\Delta_N\) at the nodal limit of the gap slope closely follows \(8.5k_BT_c\) and concomitantly related to the antinodal-gap energy \(\Delta^*\) through a reduction factor of square-root superfluid density \(\rho_s\). These empirical formulae provide a new gap-based paradigm on the principle governing \(T_c\) in strong-coupling superconductors.

\textbf{Introduction}

In classical Bardeen-Cooper-Schrieffer (BCS) theory, a superconducting critical temperature \(T_c\) is proportional to an energy gap in the electronic excitation spectrum \([1]\). However, this is not the case in underdoped high-\(T_c\) cuprates: as hole concentration \(p\) decreases from the optimum, the amplitude of \(d\)-wave-like gap \(\Delta^*\) increases whereas \(T_c\) decreases \([2,3]\). Instead, superfluid density \(\rho_s\) decreases along with such underdoping, and thus is considered as a candidate of the superconducting order parameter in the cuprates \([4-10]\). Recently, a variety of experimental gap energies in the superconducting state have been classified into two groups \([11-13]\). One increases like \(\Delta^*\) and the other decreases like \(T_c\) with underdoping. The former energy \(\Delta^*\) remains as a pseudogap above \(T_c\) for the underdoped cuprates, as typically seen in the electronic excitation spectra around an antinode \([14]\). In the close vicinity of a node, by contrast, the gap closes above \(T_c\) \([15,16]\). The further experimental evidences suggest that the low-energy near-nodal excitations are more relevant to \(T_c\) than the antinodal ones \([16,17]\). Nevertheless, the gap energies defined under a dichotomy between near-nodal and antinodal regions have been difficult to formulate along with the doping dependences of \(T_c\) and \(\rho_s\) \([17-19]\). The unformulated behaviors of the nodal and antinodal gaps posed severe challenges not only to their mutual relationship but also to their standard role in electron pairing. The identities of these gaps have thus been at the center of controversy over a unified picture of the unconventional pairing state in the high-\(T_c\) cuprates, and intensively examined for clues to the principles underlying the peculiar behavior of \(T_c\).
Angle-resolved photoemission spectroscopy (ARPES) is an excellent tool for studying the anisotropic energy gap as it provides direct information of the electronic structure in energy-momentum space. In the previous ARPES studies, two phenomenological fitting models have been suggested to quantify the nodal-gap component from the momentum-resolved gaps $\Delta(0)$. One is linear-fit analysis and the other is high-harmonic-fit analysis. The former is based on the presumption that the character of excitation gap discontinuously changes on a certain region of Fermi surface [15,17,18,20,21]. The latter is that the gap variation is described by $d$-wave gap function including the higher harmonic term [22-25], indicating that there is no definable boundary from the node to the antinode. It is debatable whether the near-nodal region is well defined or not. The problem is that it is difficult to determine the nodal gap from only a few $k$-points in the vicinity of the node. Here, we have utilized low photon energies with synchrotron radiation in ARPES measurement. The low-energy photons facilitate improvement in energy and momentum resolution, because in-plane momenta are magnified into large emission angles under a given instrumental angular resolution [15,18,19,21-23,26,27]. Furthermore, the tunability of the photon energy enables us to select the photoelectron final state suitable for the observation of superconducting gap from the node to the antinode [22,23,26,27].

In this paper, we show simple empirical forms,

$$8.5 k_B T_c = 2\Delta_n \propto \sqrt{\rho_s}$$

through a refined ARPES data of bilayer and single-layer cuprate superconductors, Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi2212) and Bi$_2$Sr$_{1.6}$La$_{0.4}$CuO$_{6+\delta}$ (Bi2201). Low-energy ARPES provided clear-cut gap images over the entire Fermi surface, and revealed that the gap slope decreases asymptotically toward the node. This observation led us to determine the pure nodal-gap component $\Delta_n$ from the limit of the gap slope. As a result, the nodal-gap energy $\Delta_n$ is proportional to $T_c$, indicating its relevance to the condensation of the electron pairs. A high plateau value of $\Delta_n/T_c$ ratio exhibits sufficiently strong electron coupling both for Bi2212 and Bi2201. Furthermore, the nodal and antinodal gaps are quantitatively related by a factor of the square-root superfluid density, $\Delta_n/\rho_s \propto \sqrt{\rho_s}$, in a wide hole-concentration range. This reduction factor is identical to the theoretical prediction for the effect of incoherent pair excitations [28,29], implying that the antinodal gap energy is relevant to the formation of the electron pairs. Thus, we argue that the strong coupling makes a critical difference to the pairing state, and that a substantial number of electrons remain paired, despite dropping out of the coherent superfluid in the underdoped superconducting cuprates. These gap-based formulae reasonably describe a crucial part of the mechanism governing $T_c$.

Experiment

High-quality single crystals of Bi2212 and Bi2201 were grown by traveling-solvent floating-zone method. Details of sample preparation are described elsewhere [30,31]. Hole concentrations $p$ for Bi2212 have been deduced from the samples’ $T_c$ using a phenomenological relation, $T_c/T_c^{\text{max}} = 82.6(p-0.16)^2$, with $T_c^{\text{max}} = 91$ K from Ref. 32. Heavily-underdoped and heavily-overdoped samples were made of Dy-substituted and Pb-substituted crystals, respectively. The value of $p$ for Bi2201 was calculated by the form, $p = 0.21 - 0.15x$, where $x$ is the La concentration in this system [33]. ARPES experiments were performed on a helical undulator beamline, BL-9A of the Hiroshima Synchrotron Radiation Center, using a SCIENTA R4000 analyzer. Total instrumental energy resolution was set at 5 meV. Clean surfaces were obtained under ultrahigh vacuum better than $5 \times 10^{-11}$ Torr. Energies were calibrated with reference to the intermittently monitored Fermi edge of polycrystalline gold.
Results

Figure 1(a) shows intensity plots of ARPES spectra around $|\omega| = 36$ meV for the optimally-doped Bi2212 ($T_c = 91$ K). We have studied photon-energy dependence of low-energy ARPES, and found that switching the photon energy from $h\nu \leq 7$ eV used in laser-based ARPES studies [15,18,19,21] to 8.5 eV results in a dramatic improvement in the spectral intensity in the off-nodal region. This enabled our high-resolution gap study covering from the node through to the antinode for both the bonding band (BB) and antibonding band (AB) of the CuO$_2$ bilayer. As typically seen in ARPES spectra of Figs. 1(b)-(d), the superconducting gap gradually opens away from the node, indicating a $d$-wave-like gap structure. The Fermi surfaces were determined rigorously from the minimum-gap loci, because both inaccuracy and broadening of the cutting path result in an overestimate of the gap energy. The spectra along the Fermi surface of BB are displayed in Fig. 1(e) as a function of off-node angle $\theta$ (Fig. 1(a)). The seamless evolution of the gap image is obtained from the node to the antinode. Similarly, the gap image for the optimally-doped Bi2201 ($T_c = 33$ K) has been extracted from the low-energy ARPES data, as shown in Fig. 1(j). The overall reduction of gap energies for Bi2201 in Figs. 1(g)-1(j) explain its lower $T_c$. The gap size at $\theta = 22^\circ$ for Bi2201 is approximately 42% with respect to that for Bi2212, which is consistent with previous results [21].

![Figure 1](image.png)

Figure 1  Low-energy synchrotron-radiation ARPES data of optimally-doped Bi2212 and Bi2201. The spectra were measured at $T = 10$ K with $h\nu = 8.5$ eV for Bi2212 (upper panels) and $h\nu = 7.8$ eV for Bi2201 (lower panels). (a),(f) Intensity plot of the spectral weight, which is symmetrized with respect to the diagonal of the Brillouin zone. The off-node angle $\theta$ is defined about the $(\pi,\pi)$ point. (b)-(d) and (g)-(i) ARPES spectra along three typical momentum cuts as indicated white curves in panel (a) and (f), respectively. (e),(f) Energy-versus-$\theta$ plots of the spectral images along the Fermi surfaces, which is symmetrized with respect to $\omega = 0$. BB and AB indicate the bonding and antibonding band of CuO$_2$ bilayer for Bi2212, respectively.

Deviation from the $d$-wave gap is directly imaged in the energy-versus-$\sin^2\theta$ plots of the ARPES spectra. Figures 2(a)-(c) reveal that the curvature of the gap profile in $\sin^2\theta < 0.6$ increases as hole concentration $p$ decreases from the optimum, making clear a departure from the standard $d$-wave form, $\Delta(\theta) \propto \sin^2\theta$, for the underdoped superconducting Bi2212. The superconducting gap energies $\Delta(\theta)$ were determined by fitting energy-distribution curves (EDCs). For the spectral function $A(\omega)$ at
a minimum gap locus, we adopted a widely-used phenomenological form introduced by Norman et al. [34],

\[
A(\omega) = \frac{1}{\pi} \text{Im} \left[ \frac{1}{\omega + i \Delta / \omega} \right]
\]

where the peak position and width are given by the superconducting-gap energy \( \Delta \) and the single-particle scattering rate \( \Gamma \), respectively. In accordance with the incoherent spectral weight increasing towards higher energies, we added a background linear in energy and then applied the multiplication by a Fermi-Dirac distribution function. As shown in Figs. 2(d)-(c), the EDCs are well captured by this phenomenological function. Here the artifact of the antinodal peak smearing is ruled out of the causes for this deviation [35], because the curvature in \( \sin 2\theta \) is established within the region, \( \sin 2\theta \leq 0.75 \), where the well-defined sharp and intense peak is observed for an underdoped sample with \( T_c = 66 \) K (UD66), as shown in Figs. 2(b) and (e). We found that the gap slope in \( \sin 2\theta \) decreases continuously and asymptotically toward the nodal limit \( \theta \to 0 \) for the underdoped Bi2212. This result indicates that the deviation from the \( d \)-wave gap penetrates into the close vicinity of the node. So far, it has often been assumed that there is some discontinuity between the nodal and antinodal spectra or a certain region with pure \( d \)-wave gap [15,17,18,20,21]. However, it is difficult to define them with the all-round seamless images in Fig. 2(a)-(b), because all the spectral features, such as the energy gap, the peak width, the peak intensity, and their derivatives along the Fermi surface, smoothly evolves in going from the node to the antinode. The continuous spectral evolution in momenta has also been reported by Vishik et al. [36]. These imply that the nodal region is indefinable from ARPES data for Bi2212. In other words, even near the node, a certain percentage of the antinodal character likely mixes in the spectral behavior. Indeed, the near-nodal gap does not close exactly at \( T_c \) [18,19], and the gap-closing temperature gradually decreases to \( T_c \) on approaching the nodal limit, \( \theta \to 0 \), in a way similar to the deviation from the \( d \)-wave gap [15,37].

Therefore, we have adopted the high-harmonic fit over the unbounded region for parameterization of the anisotropic superconducting gap [24,25]. The nodal and antinodal gap energies are defined as \( \Lambda_\theta = 0.5(d\Delta/d\theta)_{\theta=0} \) and \( \Lambda' = \Lambda(\theta)_{\theta=0} \), respectively, so that \( \Delta_n / \Delta' = 1 \) is satisfied for the ideal \( d \)-wave gap as depicted in Fig. 2(c). The fitting function is expressed as

\[
\Lambda(\theta) = \Lambda_\theta \sin 2\theta + \left( \Lambda' - \Lambda_\theta \right) \left( 3\sin 2\theta - \sin 6\theta \right) / 4
\]

where the first term is solely responsible for the nodal-gap slope, the second term models the gap deviation without adjustable angle parameter and its asymptotic behavior is consistent with the empirical indistinctness of the pure \( d \)-wave region. Note that the onset angle, where the gap deviates from the linearity in \( \sin 2\theta \), is indefinable under this fitting model. Consequently, this function well captures the curved gap profiles \( \Lambda(0) \) (solid curves) and their nodal tangents \( \Delta_n \sin 2\theta \) (blue lines) for all the doping levels of Bi2212 and for the optimally-doped Bi2201, as overlaid in Figs. 2(a)-(c) and 3(a). One can find the fact that the nodal limit of the gap slope \( \Delta_n \) decreases with underdoping from OP91 to UD66, and then to UD42, and seems to follow a similar doping dependence as \( T_c \).
Figure 2  Doping dependence of the gap profiles in Bi2212.  Samples are labeled by a doping-level prefix, namely underdoped (UD) and optimum (OP), and a trailing number denoting $T_c$.  (a)-(c) Energy-versus-$\sin 2\theta$ plots of the spectral images for BB, taken in the superconducting state at $T = 10$ K and symmetrized with respect to $\omega = 0$.  Circles, curves and blue lines denote the gap energies, the next-higher-harmonic fits and the nodal tangents, respectively.  (d)-(f) EDCs at the momenta marked by black triangles in panels a-c.  Black curves and vertical bars denote the fits and the gap energies, respectively.  The original data have been reported in Ref. 22.

The gap energies scaled by $T_c$ are plotted in Fig. 3(d), which reveals that the nodal limit of $\Delta(0)/T_c$ remains unchanged with underdoping and in different materials in contrast to the antinodal limit.  We have determined the two gap parameters, $\Delta_N$ and $\Delta^*$, from the high-harmonic fit and plotted as a function of hole concentration in Fig. 3(b).  As hole concentration $p$ decreases from the overdoped limit, both the nodal and antinodal ones increase with keeping a constant proportion, $\Delta_N = 0.87\Delta^*$.  This is the canonical behavior expected when the coupling is getting stronger.  With further decreasing $p$, the nodal gap $2\Delta_N$ closely follows the decrease in $T_c$ in contrast to the monotonic increases in the antinodal gap $\Delta^*$.  The gap-to-$T_c$ ratio is quantified as $2\Delta_N/k_B T_c = 8.5$, which is about twice the mean-field prediction 4.3 for $d$-wave weak-coupling superconductors.  Figure 3(c) shows that our data for Bi2212 and Bi2201 finely converge on the line of $2\Delta_N/k_B T_c = 8.5$ in particular from the optimum to a heavily underdoped level, and that similar values of $2\Delta_N/k_B T_c$ have been reported for other single-layer cuprates [20,21].  It is reasonable that $T_c$ is primarily determined by $\Delta_N$ rather than $\Delta^*$, because thermal quasiparticle excitations concentrate in the vicinity of the node and hardly occur around the antinode in particular for the strong-coupling case, $2\Delta^*/k_B T_c >> 4.3$.  The association between the nodal excitations and $T_c$ has also been suggested in various experiments [12,13,15-17].  In particular, the decrease in $\Delta_N$ with underdoping has been deduced from the low-energy slope of B$_{2g}$-Raman spectra [12] and the quasiparticle interference in scanning-tunneling images [25].  A new insight arises from the analogy between the doping $p$ and temperature $T$ dependences of $\Delta_N$.  With both increasing $p$ and decreasing $T$, the superconducting gap approaches the ideal $d$-wave form.
As $p$ decreases conversely, $\Delta_N$ is suppressed relative to $\Delta^*$, and expected to decrease towards zero on approaching the disappearance of the superconductivity along $2\Delta_N/k_B T_c = 8.5$. This behavior is quite analogous to what is seen with increasing $T$ (ref. 15). Noting that the superfluid density $\rho_s$ decreases to zero on approaching the critical temperature, $T = T_c$, irrespective of the pseudogap temperature [4,5,7,10], one may associate the nodal-gap suppression, $\Delta_N/\Delta^*$, with the low superfluid density $\rho_s$, which has widely been argued to play a critical role in the underdoped cuprates as the Uemura relation [4,5]. From this viewpoint, we found another simple relation, $\Delta_N/\Delta^* \propto \sqrt{\rho_s}$. Figures 3(e) and 3(f) compare the square of nodal-to-antinodal gap ratio, $(\Delta_N/\Delta^*)^2$, with the superfluid density deduced from the magnetic penetration depth [7,9,10]. The $\rho_s$ for the Bi2212 follows a single doping dependence, which is known as an universal curve for the superfluid density. It increases linearly in hole concentration $p$ with an onset at $p \sim 0.07$, and then shows a saturation at $p \sim 0.19$ (refs 7,10). The intimate relationship between the nodal and antinodal gaps is in accord with the continuity between the nodal and antinodal spectra (Fig. 2(a)-(c)).

Discussion

A general form of $\Lambda_{SC}/\Lambda = \sqrt{\rho_s'/\rho_s}$ has been theoretically deduced for the superfluid reduction due to the incoherent pair excitations inherent in the strong-coupling superconductors, where the degeneracy between the order parameter $\Delta_{SC}$ and the pair-formation energy $\Delta$ is split. The former is manifested in the energy of the near-nodal spectral peak as $\Delta$, whereas the latter dominates the

Figure 3  Doping dependence of gap parameters. (a) Gap energies as functions of $\sin^2 \theta$ in the superconducting state of optimally-doped Bi2212 (purple circles) and Bi2201 (open circles). (b) Doping dependences of the energies of nodal gap $2\Delta_N$ (circles), antinodal gap $\Delta^*$ (squares), and $8.5k_B T_c$ (black curve). Colored and opened symbols are results for Bi2212 and Bi2201, respectively. (c) Correlation between $T_c$ and $\Delta_N$ for Bi2212 (colored circles) and Bi2201 (open circle) from the present study. Also plotted are the data for optimally-doped Bi2201 (open square) [21] and La$_{2-x}$Sr$_x$CuO$_4$ (open diamond) [20]. (d) Superconducting-gap energies scaled by $T_c$, plotted as functions of $\sin^2 \theta$, and vertically offset by 8 for each sample. Solid curves and dotted lines denote the high-harmonic fits and the nodal tangents, respectively. (e) Doping dependence of square of nodal-to-antinodal gap ratio $(\Delta_N/\Delta^*)^2$. (f) Superfluid density, $\rho_s \equiv \lambda^2$, determined from magnetic penetration depth $\lambda$ with alternating-current susceptibility (triangles) [10], from heat capacity (crosses) [7] and isothermal reversible magnetizations (circles) [9].
antinodal peak energy $\Delta^*$, as shown by taking into account the lifetime effect [28,29]. In the weak-coupling BCS model, $\rho_s^{BCS}$, the superfluid density is approximately constant, because the Fermi velocity and Fermi-surface perimeter in the normal state are hardly dependent on the hole concentration $p$ [38]. Therefore, the strong-coupling scenario well explains our empirical form, $\Lambda_N/\Lambda^* \propto \rho_s^{1/2}$, with $\rho_s^{BCS} \approx 31 \, \mu m^{-2}$ and $14 \, \mu m^{-2}$ for Bi2212 and Bi2201, respectively, and is also consistent with the observations of the high gap-to-$T_c$ ratios and the strong renormalization of dispersion upon the superconducting transition [38]. Under this scenario for the nodal-gap suppression, the phase fluctuation persisting down to a temperature below $T_c$ is responsible for the decrease in superfluid density. However, the intrinsic phase fluctuation arising solely from the strong coupling diminishes at temperatures much lower than $T_c$ [28,29]. Hence, in conjunction with the strong coupling, some extra source generating the incoherent pair excitations at low temperatures is assumed to be present, on the basis of the phenomenology among $\Delta_N$, $\Delta^*$, and $\rho_s$. The possible candidates for this scattering source are the orders competing with the superconductivity [29]. In particular, it has indeed been observed by scanning-tunneling experiments that the nanoscale spatial domains of density-wave-like modulation spread over the underdoped Bi2212 [40]. Perhaps, the limit of $2\Delta_N/k_BT_c \leq 8.5$ may imply that such competing orders are practically inevitable for the strong-coupling superconductivity.

We combine two relations and finally obtain $8.5k_BT_c = 2\Lambda_N \propto \Lambda^*/\rho_s^{1/2}$. This formulation associates $T_c$ with the superfluid density $\rho_s$, as well as with the pair-formation energy, $\Delta \approx \Delta^*$ [4,5]. Notably, recent investigations have revealed that the rising exponent of $T_c$, as a power of $\rho_s$, is generally less than 1, and that specifically $T_c \propto \sqrt{\rho_s}$ is satisfied near $\rho_s = 0$ [8,9]. This critical scaling can be understood as a narrow-range limit of our form, $T_c \propto \Lambda^*/\rho_s^{1/2}$. Empirically, high phase stiffness appears to favor the lowest-order $d$-wave term over the higher-order-harmonics term. Thus, as another possible implication of $\Lambda_N/\Lambda^* \propto \rho_s^{1/2}$, the pairing of near-nodal electrons may be dominated more by the requirement of global phase coherence than by the attraction between themselves.

Summary

In conclusion, the ARPES using low-energy synchrotron radiation has revealed the empirical relation among the nodal, antinodal, and critical temperature in Bi2212 and Bi2201. The nodal-gap energy $2\Delta_N$ closely follows with $8.5k_BT_c$ in a wide hole-concentration range and is also proportional to the product of an antinodal gap energy $\Delta^*$ and a square-root superfluid density $\rho_s^{1/2}$, by the form of $8.5k_BT_c = 2\Lambda_N \propto \Lambda^*/\rho_s^{1/2}$. These relations reveal the nodal and antinodal gaps as representing the condensation and formation energies of electron pairs, respectively, being identical to the formula predicted by a strong-coupling phase-incoherent scenario. Regardless of interpretation, the simple phenomenological formulae put strong constraints on existing theories for the high-$T_c$ superconductivity, and will stimulate broad activities with their potential universality.

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