

An Equivalent Model of Corrugated Structures

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Keywords: Corrugated Structures, Orthotropic Plate, Finite Element Method, Modal Analysis.

Abstract. Corrugated structures have wide application in engineering. The corrugated structure is replaced by an orthotropic plate with equivalent stiffness properties in this paper. After equivalent stiffnesses are obtained, deflections and modals of simply supported orthotropic plates can be analyzed theoretically, which are compared with finite element solutions modeled with corrugated structures. Finally, it is clearly that corrugated structures can be replaced by an orthotropic plate with equivalent stiffness in deflection and modal analysis.

Introduction

Over last decades, corrugated structures have wide application in civil, aerospace, naval and automotive engineering, which is used as sensing elements, fiberboards, folded roofs, container walls, sandwich plate cores, bridge decks, ship panels, etc. The main mechanical property of corrugated structures is high specific stiffness, especially under bending.

When the dimensions of corrugated structure are much larger than the period of the corrugations, a homogenisation-based analytical model could be used for any corrugation shape, in which the corrugated structure is replaced by an orthotropic plate with equivalent stiffness properties. Briassoulis^[1] and McFarland^[2] investigated the equivalent bending stiffness with sinusoidal and rectangular corrugations, respectively. Samanta and Mukhopadhyay performed the static and dynamic analyses of trapezoidal corrugated structures by considering both extensional and bending stiffness^[3]. Yokoze et al. analyzed the mechanical properties of corrugated laminates made from carbon epoxy composites theoretically and experimentally^[4]. Peng et al. investigated the equivalent elastic properties of sinusoidal and trapezoidal corrugated plates by means of a mesh-free Galerkin method^[5]. Liew et al. used this method for the geometrically nonlinear analysis of corrugated plates. Both the equivalent extensional and bending properties were employed in the analyses^[6, 7]. Winkler and Kress derived accurate analytical expressions of equivalent orthotropic plate for circular corrugations^[8-10]. Xia et al. replaced the corrugated structure by an orthotropic plate, based on the equivalent energy and force properties^[11]. Giorgio Bartolozzi et al. investigated corrugated structures with any shapes by curved beam theories, and obtained equivalent Elasticity modulus and Poisson's ratio^[12]. Ye et al. summarized the equivalent model of corrugated structures in the past fifty years, and present their own equivalent models with poor theoretical derivation^[13].

This paper replaces the corrugated structure by an orthotropic plate with equivalent stiffness properties. Deflections and modals of simply supported orthotropic plates can be analyzed orthotropic plates theoretically, which are compared with finite element solutions modeled with corrugated structures.

Equivalent Stiffness

The corrugated structures are generated from a periodic shape in the XZ plane that is extruded in the Y direction to produce a structure, as seen in Fig.1.

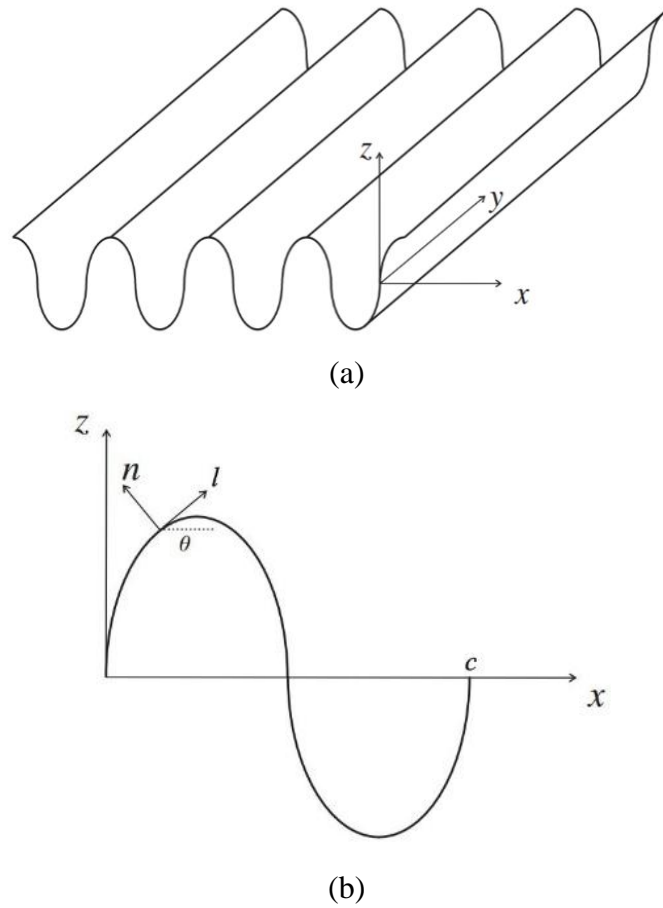
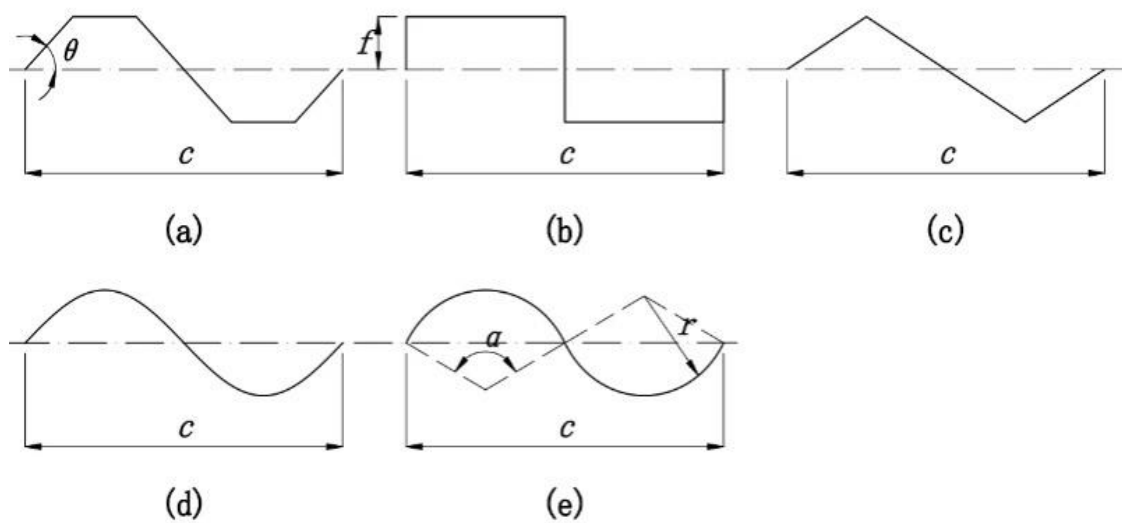


Fig.1 Definition of the Coordinate Systems

The five most common corrugated structures were discussed included trapezoidal, rectangular, toothed, sinusoidal and circular corrugations, as shown in Fig.2.



(a) Trapezoidal; (b) Rectangular; (c) Toothed; (d) Sinusoidal; (e) Circular

Fig.2 The Most Common Corrugated Structures

In order to have a better understanding of extension stiffness and bending stiffness of corrugated structures, which replaced by orthotropic plates, ANSYS FEA simulations are implemented to simulate the mechanical behaviors of the five most corrugated structures, which modeled with elastic shell element SHELL63.

Orthotropic structures are replaced by orthotropic plates. If coupling stiffnesses \mathbf{B} is ignored, constitutive equations of equivalent orthotropic plates can be written as

$$\begin{Bmatrix} \overline{N_x} \\ \overline{N_y} \\ \overline{N_{xy}} \\ \overline{M_x} \\ \overline{M_y} \\ \overline{M_{xy}} \end{Bmatrix} = \begin{bmatrix} \overline{A_{11}} & \overline{A_{12}} & 0 & 0 & 0 & 0 \\ \overline{A_{12}} & \overline{A_{22}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{A_{66}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{D_{11}} & \overline{D_{12}} & 0 \\ 0 & 0 & 0 & \overline{D_{12}} & \overline{D_{22}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{D_{66}} \end{bmatrix} \begin{Bmatrix} \overline{\varepsilon_x} \\ \overline{\varepsilon_y} \\ \overline{\gamma_{xy}} \\ \overline{\kappa_x} \\ \overline{\kappa_y} \\ \overline{\kappa_{xy}} \end{Bmatrix} \quad (1)$$

From Eq. (1), if one of strains is maintained 1, and the rest are 0, the value of the force equals the value of responding stiffness. The equivalent stiffnesses are shown in Table 1, where I_{sy} is inertia moment relative to the neutral axis (X axis), and s and t are the corrugation length and the thickness of corrugated structure, respectively.

Tab.1 Equivalent Extension Stiffness and Bending Stiffness of Corrugated Structures

	Extension stiffness		Bending stiffness
$\overline{A_{11}}$	$\frac{Ect^3}{48 \int_0^{\frac{c}{2}} z^2(x) \sqrt{1+z'(x)^2} dx}$	$\overline{D_{11}}$	$\frac{c}{s} \frac{Et^3}{12}$
$\overline{A_{12}}$	$\nu \overline{A_{11}}$	$\overline{D_{12}}$	$\nu \overline{D_{11}}$
$\overline{A_{22}}$	$Et \frac{s}{c}$	$\overline{D_{22}}$	$\frac{EI_{sy}}{c}$
$\overline{A_{66}}$	$\frac{c}{s} \frac{Et}{2(1+\nu)}$	$\overline{D_{66}}$	$\frac{s}{c} \frac{Et^3}{24(1+\nu)}$

In the numerical examples, the material properties and the geometry parameters are $E=21\text{GPa}$, $\nu=0.3$, $b=c=0.1\text{m}$, $f=0.015\text{m}$, $t=0.004\text{m}$. Trapezoidal corrugated structure with $\theta=45^\circ$, $s=0.12485\text{m}$; Rectangular corrugated structure is a particular case of trapezoidal corrugated structures when $\theta=90^\circ$, with $s=0.16\text{m}$; Toothed corrugated structure is also a particular case of trapezoidal corrugated structures when bottom side, with $\theta=30.964^\circ$, $s=0.11662\text{m}$; Sinusoidal corrugated structure with the curve $z(x)=0.015\sin(20\pi x)\text{m}$, $s=0.11945\text{m}$; Circular corrugated structure with $r=0.028333\text{m}$, $\alpha=123.855^\circ$, $s=0.1225\text{m}$. Theoretical value and FEA value equivalent stiffnesses of the five most common corrugated structures are listed in Table 2.

Based on Table 2, bending stiffness along the corrugations direction $\overline{D_{11}}$ is much smaller than that vertical the corrugations direction $\overline{D_{22}}$. Additionally, both theoretical results and FEA results $\overline{A_{22}}$, $\overline{A_{66}}$, $\overline{D_{22}}$ and $\overline{D_{66}}$ are fairly close; the error $\overline{A_{11}}$, $\overline{A_{12}}$, $\overline{D_{11}}$ and $\overline{D_{12}}$ is about 7%-9%.

Tab.2 Equivalent Stiffness of the Five Most Common Corrugated Structures

		Trapezoidal	Rectangular	Toothed	Sinusoidal	Circular
\overline{A}_{11} (MN/m)	Theoretical	0.729	0.415	1.28	0.904	0.7844
	FEM	0.784	0.446	1.37	0.971	0.8438
	Error (%)	7.02	6.95	6.46	6.89	7.04
\overline{A}_{12} (MN/m)	Theoretical	0.219	0.125	0.38	0.2712	0.2353
	FEM	0.235	0.134	0.41	0.2915	0.2534
	Error (%)	5.96	6.72	6.76	6.96	7.14
\overline{A}_{22} (MN/m)	Theoretical	104.88	134.4	98	100.338	102.9
	FEM	105.63	135.32	98.7	101.09	103.62
	Error (%)	0.71	0.74	0.79	0.744	0.695
\overline{A}_{66} (MN/m)	Theoretical	25.88	20.192	27.7	27.047	26.37
	FEM	25.893	20.195	27.7	27.063	26.412
	Error (%)	0.05	0.015	0	0.06	0.145
		Trapezoidal	Rectangular	Toothed	Sinusoidal	Circular
\overline{D}_{11} (N m)	Theoretical	89.71	70	96	93.763	91.43
	FEM	96.88	75.691	103	101.15	98.804
	Error (%)	7.4	7.52	7.08	7.3	7.46
\overline{D}_{12} (N m)	Theoretical	26.91	21	28.8	28.13	27.43
	FEM	29.573	23.077	31.7	30.914	30.149
	Error (%)	8.99	9.00	9.00	9.00	9.00
\overline{D}_{22} (kN m)	Theoretical	12.95	22.792	7.35	10.548	11.82
	FEM	13.16	23.132	7.53	10.631	12.227
	Error (%)	1.6	1.47	2.48	0.16	3.32
\overline{D}_{66} (N m)	Theoretical	53.78	68.923	50.2	51.455	52.769
	FEM	54.51	69	51.6	53.265	54.43
	Error (%)	1.34	0.11	2.58	3.4	3.05

Displacement Analysis

Displacement of simply supported orthotropic plate under uniformly distributed load in Ref. [14]

$$w = \frac{16q}{\pi^6} \frac{\sum_{m=1,3,5}^{\infty} \sum_{n=1,3,5}^{\infty} \frac{1}{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{D_{11} \left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{m}{b}\right)^2 \left(\frac{n}{b}\right)^2 + D_{22} \left(\frac{n}{b}\right)^4} \quad (2)$$

The parametric study was conducted by ANSYS finite element software. Corrugated structures are modeled with elastic shell element SHELL63. 3-D structural surface effect element SURF154 is used for uniformly distributed load applications in corrugated structure analysis. The length of both sides of the orthotropic plate is 1m, and the geometry parameter of corrugation is the same as mentioned before; the uniformly distributed load is 69kN/m². Displacement of simply supported orthotropic plate under uniformly distributed load can be calculated from Eq.(2), which is compared with the corrugated structure FEA results, as shown in Figs.3-4.

From Figs.3-4, it is obvious that corrugated structures can be replaced by orthotropic plates with equivalent stiffness properties, while analysis of the displacement.

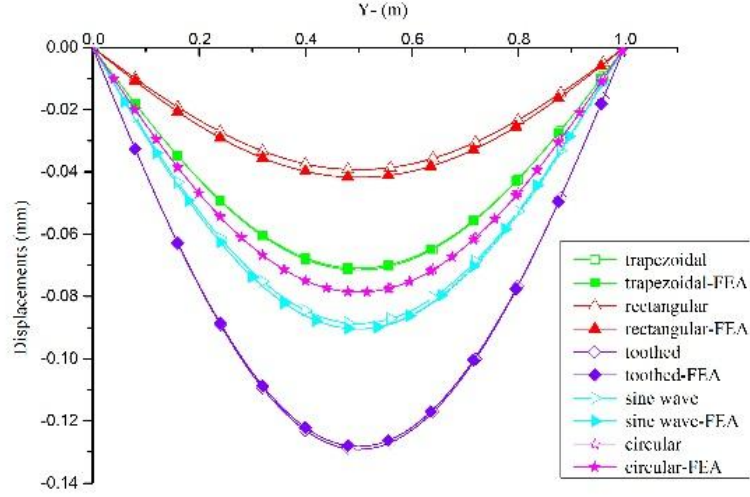


Fig.3 Displacements along Centre Line in X Direction

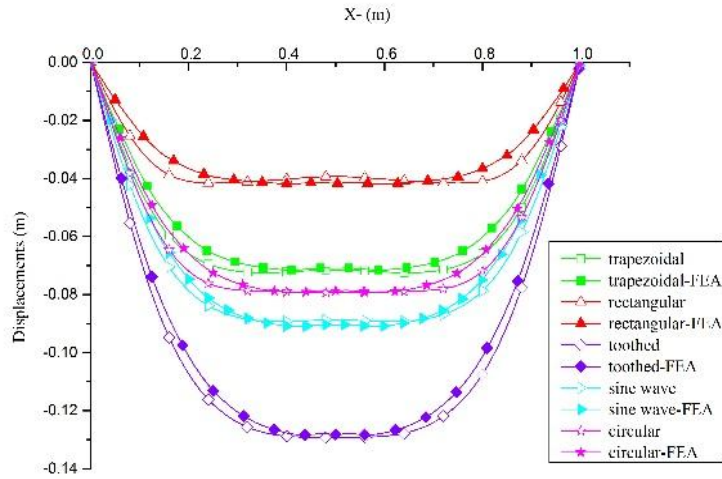


Fig.4 Displacements along Centre Line in Y Direction

Modal Analysis

Natural frequencies of simply supported orthotropic plate under uniformly distributed load in Ref.^[14]

$$\omega^2 = \frac{\pi^4}{\rho} \left[D_{11} \left(\frac{m}{a} \right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{m}{b} \right)^2 \left(\frac{n}{b} \right)^2 + D_{22} \left(\frac{n}{b} \right)^4 \right] \quad (3)$$

Natural frequencies of simply supported orthotropic plate under uniformly distributed load can be calculated from Eq. (3), which are compared with the corrugated structure FEA results, as shown in Table 3.

As shown in Table 3, natural frequencies of equivalent orthotropic plates are close to that of corrugated structures; n maintains 1 in the first five natural frequencies, as bending stiffness D_{11} is much smaller than D_{22} .

Tab.3 Natural Frequencies for Simply Supported Corrugated Structures

	(m,n)	1(1,1)	2(2,1)	3(3,1)	4(4,1)	5(5,1)
Trapezoidal	Theoretical	28.9419	31.1947	37.7459	50.3124	69.0446
	FEM	29.299	32.824	41.144	55.397	75.432
	Error (%)	1.2188%	4.9637%	8.2590%	9.1785%	8.4678%
Rectangular	Theoretical	33.7403	35.1679	39.2206	47.4247	60.4822
	FEM	34.034	37.083	43.491	54.104	69.039
	Error (%)	0.8630%	5.1644%	9.8190%	12.3453%	12.3942%
Toothed	Theoretical	22.0167	25.8511	34.2932	49.2996	70.5347
	FEM	23.177	27.207	36.936	53.192	75.544
	Error (%)	5.0063%	4.9836%	7.1551%	7.3176%	6.6310%
Sinusoidal	Theoretical	26.7825	29.3522	36.7244	50.4856	70.5692
	FEM	26.996	30.758	39.764	55.075	76.405
	Error (%)	0.7909%	4.5705%	7.6441%	8.3330%	7.6380%
Circular	Theoretical	27.9488	30.3362	37.2398	50.3247	69.6439
	FEM	28.49	32.147	40.788	55.51	76.107
	Error (%)	1.8996%	5.6329%	8.6991%	9.3412%	8.4921%

Conclusions

This paper uses equivalent stiffness method to replace corrugated structures by orthotropic plates. After equivalent stiffnesses are obtained, deflections and modals of simply supported orthotropic plates can be analyzed theoretically, which are compared with finite element solutions modeled with corrugated structures. Finally, it is clearly that corrugated structures can be replaced by an orthotropic plate with equivalent stiffness in deflection and modal analysis.

Acknowledgement

The authors gratefully acknowledge the financial support provided by the Science and Technology Scheme of Guangzhou City (No. 201510010013), the Science and Technology Scheme of Guangdong Province (No. 2012A030200003) and the National Natural Science Foundation of China (No.11032005).

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