A Graph Clustering Algorithm Based on the Intra Vertex Adjacent Ratio

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Abstract—This paper proposes a new clustering coefficient based on the number of connecting vertices within a cluster (called inter vertex adjacent ratio, IVAR) and proposes a new clustering algorithm based on IVAR. Finally, the analysis of algorithm applicability shows that the algorithm is more applicative and effective for regular graphs than the vertex-clustering algorithm based on intra connection ratio [L. Moussiades and A. Vakali, Clustering dense graph: A web site graph paradigm. Information Processing and Management, vol. 46, pp.247-267, 2010].

Keywords—component; graph; clustering algorithm; intra vertex connection ratio

I. INTRODUCTION

Clustering is an important tool for data analysis and processing, has been widely used in many fields. Graph-clustering is an important variant of data clustering. According to some given index, by Graph-clustering, the graph can be divided into several subgraphs which contain relatively closely connected vertices and their associated edge.

Graph-clustering algorithm plays an important role in many fields: monitoring on computer network executive purpose, visualization knowledge based on support of understanding of complex data structure, measurement data cluster, detection source code plagiarism, network data cluster and online community identification [3,5,6,7]. Clustering based on graph theory is one of the clustering method developed rapidly recently, which is based on graph theory and computer graphics [1,2,4,8]. Graph-clustering is a very widely applied research topic, especially in the network research, including e-mail network, social network, gene networks and so on.

Generally, graph-clustering clusters are groups with a high density of within-group edges and a lower density of between-group edges. Moussiades and Vakali proposed a vertex-clustering algorithm based on intra connection ratio (we call the algorithm as MV-ICR algorithm) in Ref. [7]. A graph can be clustered according to the degree of internal vertex by MV-ICR. For the internal vertices of a class, classification index is not only related with the degree of a vertex, but also the number of the external vertices connected to the class. This paper proposes a new clustering algorithm based on the ratio of the number of connecting vertices, and the algorithm is more reasonable and effective than MV-ICR algorithm for some especial graphs, such as regular graph.

II. DEFINITIONS AND NOTATIONS

Let $G=(V, E)$ be a graph, a clustering of G vertices is denoted as $\mathcal{C}(G)$, a clustering of G vertices, the element of $\mathcal{C}(G)$ is called a cluster. $\forall v \in V(G)$, the degree of v is denoted as $d(v)$. The following definitions and notations are mainly from Ref. [7].

Definition 1. Let $C \in \mathcal{C}(G)$ be a clustering of G , for any $C \in \mathcal{C}(G)$, $|C|/(|C|+|N(C)|)$ is called the inter vertex adjacent ratio, denoted as $ivar(C)$, where

$$N(C) = \{v | (u, v) \in E(G), u \in C, v \notin C\}$$

Definition 2. Let $\mathcal{C}(G)$ be a clustering of G , for any $C, S \in \mathcal{C}(G)$,

$$\frac{|CUS|}{|CUS|+|N_G(CUS)|} - \frac{|C|}{|C|+|N_G(C)|} - \frac{|S|}{|S|+|N_G(S)|}$$

is called inter vertex adjacent ratio of merging C and S (CUS), denoted as $\Delta_{IVAR}(C, S)$ (Δ_{IVAR} for short).

For $C = \{u\}, S = \{v\}$, the inter vertex adjacent ratio of CUS is

$$\Delta_{IVAR}(u, v) = \frac{2}{2+|N(\{u, v\})|} - \frac{1}{1+d(u)} - \frac{1}{1+d(v)}.$$

III. CLUSTERING ALGORITHM BASED ON IVAR

A. A Graph-Clustering Algorithm Based on IVAR

Algorithm 1: A graph-clustering algorithm based on IVAR

S1: INPUT graph $G=(V, E)$

S2: Let $\mathcal{C}(G) = \{C_1, C_2, \dots, C_{|V|}\}, C_i = \{v_i\}$

S3: $\forall C_i, C_j \in \mathcal{C}(G)$, if $d(C_i, C_j) > 0$, then

$$\Delta_{IVAR}(C_i, C_j) = \frac{|C_i \cup C_j|}{|C_i \cup C_j| + |N(C_i \cup C_j)|} - \frac{|C_i|}{|C_i| + |N(C_i)|} - \frac{|C_j|}{|C_j| + |N(C_j)|};$$

S4: IF $|\mathcal{C}(G)| > 1$ THEN

- finding the cluster subset with the maximum Δ_{IVAR} .
- if it does not exist, then exit.
- if there is a cluster subset $\{C_{i_1}, C_{i_2}, \dots, C_{i_k}\} (k \geq 3)$ with

the maximum Δ_{IVAR} and equal Δ_{IVAR} among each other, then merge those clusters into S :

$$S = \bigcup_{j=1}^k C_{i_j},$$

$$\mathcal{C}(G) = (\mathcal{C}(G) - \{C_{i_1}, C_{i_2}, \dots, C_{i_k}\}) \cup S;$$

- else merge C_i, C_j with the maximum Δ_{IVAR} into S :

$$S = C_i \cup C_j, \mathcal{C}(G) = (\mathcal{C}(G) - \{C_i, C_j\}) \cup S;$$

$$\varepsilon. \forall C_i, C_j \in \mathcal{C}(G), \text{ if } d(C_i, C_j) > 0, \chi\alpha\lambda\chi\upsilon\lambda\alpha\tau\varepsilon \Delta_{IVAR}(C_i, C_j).$$

S5: END

B. Example Based on IVAR

We cluster the vertices of the graph G in Figure I with Algorithm 1 as follows:

FIGURE I. GRAPH G

(1) Every vertex in the graph is a cluster $\mathcal{C} = \{\{0\}, \{1\}, \{2\}, \dots, \{19\}\}$, then calculate Δ_{IVAR} . Because $\Delta_{IVAR}(5, 7) = \Delta_{IVAR}(18, 19)$ is the maximum value, we choose one pair of $\{18\}$ and $\{19\}$, merging into one cluster $\{18, 19\}$ (see Figure II) and get the clustering as follows:

$$S = \{18\} \cup \{19\} = \{18, 19\},$$

$$\mathcal{C} = (\mathcal{C} - \{\{18\}, \{19\}\}) \cup S = \{\{1\}, \{2\}, \dots, \{17\}, \{18, 19\}\}.$$

FIGURE II. MERGING $\{18\}$ AND $\{19\}$ TOGETHER

(2) Calculating Δ_{IVAR} of $\mathcal{C}$, and finding the maximum value $\Delta_{IVAR}(5, 7)$, then we merge $\{5\}$ and $\{7\}$ (see Figure III), get the clustering as follows:

$$\mathcal{C} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5, 7\}, \{6\}, \{8\}, \{9\}, \dots, \{17\}, \{18, 19\}\}.$$

FIGURE III. MERGING $\{5\}$ AND $\{7\}$ TOGETHER

(3) Calculating Δ_{IVAR} of $\mathcal{C}$, and finding the maximum value $\Delta_{IVAR}(\{5, 7\}, \{6\})$, then we merge $\{5, 7\}$ and $\{6\}$ (see Figure IV), and get the clustering as follows:

$$\mathcal{C} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5, 6, 7\}, \{8\}, \{9\}, \dots, \{17\}, \{18, 19\}\}.$$

FIGURE IV. MERGING $\{5, 7\}$ AND $\{6\}$ TOGETHER

(4) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between each pair of $\{8\}$, $\{9\}$, $\{10\}$, $\{11\}$ and $\{12\}$ is maximum, then we merge them into one cluster of $\{8, 9, 10, 11, 12\}$ (see Figure V), and get the cluster as follows:

$$\mathcal{C} = \{\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5, 6, 7\}, \{8, 9, 10, 11, 12\}, \{13\}, \{14\}, \dots, \{17\}, \{18, 19\}\}.$$

FIGURE V. MERGING $\{8\}$, $\{9\}$, $\{10\}$, $\{11\}$ AND $\{12\}$ TOGETHER

(5) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between each pair of $\{0\}$, $\{1\}$, $\{2\}$, $\{3\}$ is maximum, then we merge them into one cluster of $\{0, 1, 2, 3\}$ (see Figure VI), and get the cluster as follows:

$$\mathcal{C} = \{\{0, 1, 2, 3\}, \{4\}, \{5, 6, 7\}, \{8, 9, 10, 11, 12\}, \{13\}, \{14\}, \dots, \{17\}, \{18, 19\}\}.$$

FIGURE VI. MERGING $\{0\}$, $\{1\}$, $\{2\}$ AND $\{3\}$ TOGETHER

(6) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between each pair of $\{13\}$, $\{14\}$, $\{15\}$, $\{16\}$ is maximum, then we merge them into one cluster of $\{13, 14, 15, 16\}$ (see Figure VII), and get the cluster as follows:

$$\mathcal{C} = \{\{0, 1, 2, 3\}, \{4\}, \{5, 6, 7\}, \{8, 9, 10, 11, 12\}, \{13, 14, 15, 16\}, \{17\}, \{18, 19\}\}.$$

FIGURE VII. MERGING $\{13\}$, $\{14\}$, $\{15\}$ AND $\{16\}$ TOGETHER

(7) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between $\{4\}$ and $\{5, 6, 7\}$ is maximum, then we merge them into one clustering of $\{4, 5, 6, 7\}$, and get the cluster as follows:

$$\mathcal{C} = \{\{0, 1, 2, 3\}, \{4, 5, 6, 7\}, \{8, 9, 10, 11, 12\}, \{13, 14, 15, 16\}, \{17\}, \{18, 19\}\}.$$

(8) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between $\{17\}$ and $\{18, 19\}$ is maximum, then we merge them into one cluster of $\{17, 18, 19\}$, and get the cluster as follows:

$$\mathcal{C} = \{\{0, 1, 2, 3\}, \{4, 5, 6, 7\}, \{8, 9, 10, 11, 12\}, \{13, 14, 15, 16\}, \{17, 18, 19\}\}.$$

(9) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between $\{0, 1, 2, 3\}$ and $\{8, 9, 10, 11, 12\}$ is maximum, then we merge them into one cluster of $\{0, 1, 2, 3, 8, 9, 10, 11, 12\}$, and get the cluster as follows:

$$\mathcal{C} = \{\{0, 1, 2, 3, 8, 9, 10, 11, 12\}, \{4, 5, 6, 7\}, \{13, 14, 15, 16\}, \{17, 18, 19\}\}.$$

(10) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between $\{0, 1, 2, 3, 8, 9, 10, 11, 12\}$ and $\{4, 5, 6, 7\}$ is maximum, then we merge them into one cluster and get the cluster as follows:

$$\mathcal{C} = \{\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}, \{13, 14, 15, 16\}, \{17, 18, 19\}\}.$$

(11) Calculating Δ_{IVAR} of $\mathcal{C}$, the Δ_{IVAR} between $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ and $\{13, 14, 15, 16\}$ is maximum, then we merge them into one cluster and get the cluster as follows:

$$\mathcal{C} = \{\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}, \{17, 18, 19\}\}.$$

(12) Finally, all vertices will be merged into one cluster:

$$\mathcal{C} = \{\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19\}\}.$$

IV. ANALYSIS OF ALGORITHM APPLICABILITY

(1) IVAR is better than MV-ICR for those graphs containing the complete subgraph.

We can cluster the K_5 according to MV-ICR (Ref.[7]), and get the clustering approach shown in Figure VIII.

FIGURE VIII. CLUSTERING PROCESS OF K_5 WITH MV-ICR.

We can cluster the K_5 according to IVAR: the initial clustering is $\{0\},\{1\},\{2\},\{3\},\{4\}$; Calculating Δ_{IVAR} between all clusters, they are equal between each pair, then $\{0\},\{1\},\{2\},\{3\},\{4\}$ will be merged into one cluster, and all vertices will be merged into one cluster. This algorithm can not only improve the clustering efficiency, but also make clustering more reasonable.

(2) IVAR is better than MV-ICR for regular graph.

IVAR is an algorithm based on the number of vertices, while MV-ICR is an algorithm based on the degree of a vertex. For some especial graphs, such as regular graph, an algorithm based on the degree of a vertex is inapplicable. The graph in Figure IX(g) is a 3 regular graph, that is, the degree of all vertices in graph is $d(v)=3$. Clustering according to MV-ICR, the Δ_{IVAR} is equal between each pair of vertices ($\Delta_{ICR}(\{v_i\},\{v_j\}) = \frac{2}{d(v_i)+d(v_j)}$), that make the choice more difficult.

If clustering the graph vertices according to IVAR, we can get the clustering process illustrated in Figure IX.

FIGURE IX. CLUSTERING PROCESS

V. CONCLUSION

In the clustering coefficient field, this paper studies merely on graph-clustering related to degree of vertex and the number of vertex, without considering the other parameters. We will discuss the clustering algorithm based on or combined with other parameters in the future research. This paper only discusses the applicability of MV-ICR and IVAR clustering algorithm for simple graphs, we will probe into the applicability for general graphs to find a new graph-clustering algorithm with wild adaptability.

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