

Euler-Maruyama Approximation for Mean-Reverting Regime Switching CEV Process

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Abstract—The mean-reverting constant elasticity of variance (CEV) process with regime switching is one of the most successful continuous-time models of the short term rate, volatility, and other financial quantities. However, most SDEs with Markovian switching do not have explicit solutions. This paper obtains the Euler-Maruyama approximate solution for mean-reverting Regime Switching CEV processes and provides a detailed proof of the convergence of the EM approximate solution to the exact solution.

Keywords—CEV process; mean-reverting; regime switching; Euler-Maruyama; Lipschitz condition

I. INTRODUCTION

Option pricing is one of the most important research fields in financial economics from both practical and theoretical point of view. The work of Black and Scholes [1] and Merton [2] laid the foundations of the research field and motivated important research in option pricing theory, its mathematical models and its computational techniques. The Black-Scholes-Merton formula is one of the important products of economic research of the last century and it has been widely adopted by traders, analysts, investors and other finance researcher.

Despite its popularity, the Black-Scholes-Merton formula is not without flaws. It has been documented in many studies in empirical finance that the Geometric Brownian Motion (GBM) assumed in the Black-Scholes-Merton model does not provide a realistic description for the behavior of asset price dynamics. One of substitutes is the CEV model, which is originally introduced by Cox [3] and Cox and Ross [4]. Many empirical studies have been conducted in the literature to justify the use of the CEV model, for instance, Mendoza-Arriaga and Linetsky [5], Ruas, Dias and Nunes [6], Larguinho, Dias and Braumann [7], Thakoor, Tangman and Bhuruth [8].

Markovian regime-switching models have drawn a significant amount of attention in recent years due to their ability to include the influence of macroeconomic factors on individual asset price dynamics^[9-13]. There are substantial empirical evidences in support of the existence of regime switching effects on stock market returns and default probabilities. Using the CRSP stock market returns over the period 1929-1989, Schaller and Norden [14] demonstrate that there is compelling evidence of regime switching in US stock market returns and the evidence for switching is robust to

different specifications such as switching in means, switching in variances, and switching in both means and variances. Ang and Timmermann [15] also show that regime-switching models can capture the stylized behavior of many asset returns.

In this paper, we investigate the Euler-Maruyama approximate solution of a stochastic differential equation, where we generalize the mean-reverting CEV process by replacing the constant parameters with the corresponding parameters modulated by a continuous-time, finite-state, Markov chain. This paper obtain the Euler-Maruyama approximate solution for mean-reverting Regime Switching CEV processes and provides a detailed proof of the strong convergence of the EM approximate solution to the exact solution.

This paper is organized as follows. In Section II, we develop a mean-reverting CEV process with regime switching. The Euler-Maruyama approximate solution is provided in Section III. In Section IV, we provide a detailed proof of the strong convergence of the EM approximate solution to the exact solution. Conclusion is given in Section V.

II. MEAN-REVERTING REGIME SWITCHING CEV PROCESS

We let $(\Omega, F, \{F_t\}_{t \geq 0}, P)$ be a complete probability space with a filtration $F_{t \geq 0}$ satisfying the usual conditions, upon which all stochastic processes are defined. Let $X(t)$ be a finite-state continuous-time Markov chain taking values among G different states, where G is the total number of states considered in the economy. Each state represents a particular regime and is labeled by an integer i between 1 and G . Hence the state space of $X(t)$ is given by $\mathfrak{S} := \{1, 2, \dots, G\}$ which can be used to model factors of the economy. Here the Markov chain is assumed to be observable and serve as a proxy for some exogenous economic factors such as GDP and stock price indices. One might interpret the states of $X(t)$ as different stages of a business cycle. By interpreting the states of the Markov chain $X(t)$ as different stages of a business cycle, one could suppose that $G = 5$ and that state 1, state 2, \dots , and state 5 represent expansion, peak, \dots , and recovery, respectively.

To obtain the transition probabilities of the Markov chain $X(t)$, we need to specify its generator matrix Q . For easy

exposition, we assume that a constant generator $Q = (q_{ij})_{G \times G}$ is given. Clearly it is straightforward to extend the framework to the case of time varying generator. From Markov chain theory, the elements $(q_{ij})_{G \times G}$ in the matrix Q satisfy:

1. $q_{ij} \geq 0$ if $i \neq j$;
2. $q_{ii} \leq 0$ and $q_{ii} = -\sum_{j \neq i} q_{ij}$ for each $i = 1, \dots, G$.

Assume that the Markov chain $X(t)$ at any time $t > 0$ is in a regime $i \in \mathfrak{S}$. Then after a period of time Δt , the Markov chain $Y_{t+\Delta t}$ may stay in regime i with probability $P^Y(i, i)$ or jump to any other regime $j \in \mathfrak{S}$ with probability $P^X(i, j)$, where the one-step transition probabilities $P^X(i, j)$ of the Markov chain $X(t)$ are given by

$$p^x(i, j) = \begin{cases} e^{q_{ii}\Delta t}, & j = i \\ (1 - e^{q_{ii}\Delta t}) \frac{q_{ij}}{-q_{ii}}, & j \neq i \end{cases}.$$

Let $W(t)$ be a standard Brownian motion defined on the probability space $(\Omega, F, \{F\}_{t \geq 0}, P)$. We consider the following regime-switching mean-reverting CEV process

$$dY(t) = a_{X(t)}(b_{X(t)} - Y(t))dt + \sigma_{X(t)}Y(t)^\beta dW(t), t \geq 0, \quad (1)$$

with initial values $Y_0 = y_0$ and $X_0 = x_0$. $Y(t)$ represents an underlying variable (for example, the stochastic interest rate or default intensity), $a_{Y(t)}$ denotes the speed of mean reversion, $b_{Y(t)}$ denotes the long term mean of the variable, and $\sigma_{Y(t)}$ is the volatility coefficient. $a_{Y(t)}$, $b_{Y(t)}$ and $\sigma_{Y(t)}$ are positive real number dependent on the Markov chain $Y(t)$, indicating that they can take different values in different regimes.

Since the underlying variable $Y(t)$ is mainly used to model stochastic volatility or interest rate or an asset price, it is critical that $Y(t)$ will never become negative. Mao et al. [10] discuss its analytical properties when $\frac{1}{2} \leq \beta \leq 1$ and show that for given any initial data $Y_0 = y_0 > 0$ and $X_0 = x_0 \in \mathfrak{S}$, the solution $Y(t)$ of (1) will remain positive with probability 1, namely $Y(t) > 0$ for all $t \geq 0$ almost surely, if one of the following two conditions holds:

1. $\frac{1}{2} < \beta \leq 1$;
2. $\beta = \frac{1}{2}$ and $\sigma_i^2 \leq 2a_i b_i$ for all $i \in \mathfrak{S}$.

A. Lemma 1.

The coefficients of (1) satisfy the local Lipschitz condition for given initial value $Y_0 = y_0 > 0$, i.e., for every integer $k > 1$, there exists a positive constant L_k such that for all

$i \in \mathfrak{S}$, and those x, y with $x \in [0, k]$ and $y \in [0, k]$, and

$$|a_i(b_i - x) - a_i(b_i - y)| \leq L_k |x - y|, |\sigma_i x^\beta - \sigma_i y^\beta| \leq L_k |x - y| \quad (2)$$

and thus there exists a unique local solution to equation (1).

The following theorem reveals the existence of the positive solution.

B. Lemma 2

For any given initial value $Y_0 = y_0 > 0$, a_i , b_i and $\sigma_i > 0$ for all $i \in \mathfrak{S}$, there exists a unique positive global solution $Y(t)$ to Eq. (1) on $t \geq 0$.

III. THE EULER-MARUYAMA APPROXIMATE SOLUTION

To define the Euler-Maruyama approximate solution, we will need the following lemma.

C. Lemma 3.

Given a step size $\Delta t > 0$, let $x_k = X_{k\Delta t}$ for $k = 0, 1, 2, \dots$. Then x_k is a discrete-time Markov chain with the one-step transition probability matrix

$$p^x(i, j) = \begin{cases} e^{q_{ii}\Delta t}, & j = i \\ (1 - e^{q_{ii}\Delta t}) \frac{q_{ij}}{-q_{ii}}, & j \neq i \end{cases}.$$

Then the discrete-time Markov chain x_k can be simulated as follows:

1. Compute the transition-probability matrix $P^x(i, j)$;
2. Let $\widetilde{X}_0 = i_0$ and generate a random number ξ_1 which is uniformly distributed in $[0, 1]$. Let $i_1 \neq G$ and define

$$\widetilde{X}_1 = \begin{cases} i_1, & \sum_{j=1}^{i_1-1} p^{\widetilde{X}}(i_0, j) \leq \xi_1 < \sum_{j=1}^{i_1} p^{\widetilde{X}}(i_0, j) \\ G, & \sum_{j=1}^{G-1} p^{\widetilde{X}}(i_0, j) \leq \xi_1 \end{cases}$$

3. Generate independently a new random number ξ_2 which is again uniformly distributed in $[0, 1]$ and then Let $i_2 \neq G$ and define

$$\widetilde{X}_2 = \begin{cases} i_2, & \sum_{j=1}^{i_2-1} p^{\widetilde{X}}(i_1, j) \leq \xi_2 < \sum_{j=1}^{i_2} p^{\widetilde{X}}(i_1, j) \\ G, & \sum_{j=1}^{G-1} p^{\widetilde{X}}(i_1, j) \leq \xi_2 \end{cases}$$

4. Repeating this procedure, a trajectory of \widetilde{X}_k , $k = 0, 1, 2, \dots$ can be generated. This procedure can be carried out independently to obtain more trajectories.

After explaining how to simulate the discrete-time Markov chain \widetilde{X}_k , we can now define the EM approximate solution for (1). Given a step size $\Delta t > 0$. Compute the discrete approximations $y_k \approx Y_{k\Delta t}$ by setting $y_0 = Y_0$, $\widetilde{X}_0 = X_0$ and

$$y_{k+1} = y_k + a_{\widetilde{X}_k}(b_{\widetilde{X}_k} - y_k)\Delta t + \sigma_{\widetilde{X}_k} y_k^\beta \Delta W_k, \quad (3)$$

where $\Delta W_k = W_{(k+1)\Delta t} - W_{k\Delta t}$. Let

$$\bar{y}(t) = y_k, \bar{x}(t) = \widetilde{X}_k, t \in [k\Delta t, (k+1)\Delta t], k = 0, 1, 2, \dots, \quad (4)$$

and define the continuous EM approximate solution by

$$y(t) = y_0 + \int_0^t a_{\bar{x}(s)}(b_{\bar{x}(s)} - \bar{y}(s))ds + \int_0^t \sigma_{\bar{x}(s)} \bar{y}(s)^\beta dW(s), \quad (5)$$

Note that $Y_{k\Delta t} = y_k = \bar{y}_{k\Delta t}$, that is, $Y(t)$ and $\bar{y}(t)$ coincide with the discrete approximate solution at the grid points.

IV. CONVERGENCE OF THE EM APPROXIMATE SOLUTION

The following theorem describes the convergence in probability of the EM solution to the exact solution under the local Lipschitz condition.

A. Theorem 1

For $Y(t)$ in (1) and $y(t)$ in (5),

$$\lim_{\Delta t \rightarrow 0} (\sup_{0 \leq t \leq T} |Y(t) - y(t)|^2) = 0, \text{ in probability.}$$

B. Proof.

We divide the whole proof into three steps.

C. Step 1

For sufficiently large R , define the similar stopping time

$$\tau_R = T \wedge \inf\{t \geq 0 \mid Y(t) \geq R\}.$$

Applying the generalized Ito formula to a C^2 function $V_{X_t}(y(t))$ yields

$$EV_{X(t \wedge \tau_R)}(Y(t \wedge \tau_R)) = V_{x_0}(y_0) + E \int_0^{t \wedge \tau_R} L V_{X(s)}(Y(s)) ds$$

Using the Gronwall inequality, we obtain

$$EV_{X(t \wedge \tau_R)}(Y(t \wedge \tau_R)) \leq e^{KT} [V_{x_0}(y_0) + KT],$$

for some positive number K .

Let

$$V_R = \inf\{V_i(Y(t)), Y(t) \geq R, i \in \mathfrak{I}\}.$$

We can derive that

$$P\{\tau_R \leq T\} \leq \frac{e^{KT}}{V_R} [V_{x_0}(y_0) + KT]. \quad (6)$$

D. Step 2

For sufficiently large R , define the similar stopping time

$$\eta_R = T \wedge \inf\{t \geq 0 \mid y(t) \geq R\}.$$

Using (5) and applying the generalized Ito's formula to $V_{X(t)}(y(t))$ defined in (2) yields

$$\begin{aligned} EV_{X(t \wedge \eta_R)}(y(t \wedge \eta_R)) &= V_{x_0}(y_0) \\ &+ E \int_0^{t \wedge \eta_R} [a_{\bar{x}(s)}(b_{\bar{x}(s)} - \bar{y}(s))(\frac{1}{2}\theta_{X(s)}y(s)^{-\frac{1}{2}} - 2\gamma_{X(s)}y(s)^{-3}) \\ &+ \frac{1}{2}\sigma_{\bar{x}(s)}^2\bar{y}(s)^{2\beta}(-\frac{1}{4}\theta_{X(s)}y(s)^{-\frac{2}{3}} + 6\gamma_{X(s)}y(s)^{-4}) \\ &+ \sum_{j=1}^G q_{ij}V_j(y(s))]ds. \end{aligned}$$

Since $LV_{\bar{x}(s)}(\bar{y}(s)) \leq K[1 + V_{\bar{x}(s)}(\bar{y}(s))]$, Rearranging the terms on the right-hand side by plus-and-minus technique we obtain that

$$\begin{aligned} &EV_{X(t \wedge \eta_R)}(y(t \wedge \eta_R)) \\ &\leq V_{x_0}(y_0) + KE \int_0^{t \wedge \eta_R} [1 + V_{X(s)}(y(s))]ds \\ &+ KE \int_0^{t \wedge \eta_R} [\theta_{\bar{x}(s)}(\sqrt{\bar{y}(s)} - \sqrt{y(s)}) + \gamma_{\bar{x}(s)}(\bar{y}(s)^{-2} - y(s)^{-2})]ds \\ &+ KE \int_0^{t \wedge \eta_R} [(\theta_{\bar{x}(s)} - \theta_{X(s)})\sqrt{y(s)} + (\gamma_{\bar{x}(s)} - \gamma_{X(s)})y(s)^{-2}]ds \\ &+ E \int_0^{t \wedge \eta_R} a_{\bar{x}(s)}(b_{\bar{x}(s)} - \bar{y}(s))[\frac{1}{2}\theta_{X(s)}(y(s)^{-\frac{1}{2}} - \bar{y}(s)^{-\frac{1}{2}}) - 2\gamma_{X(s)}(y(s)^{-3} - \bar{y}(s)^{-3})]ds \\ &+ E \int_0^{t \wedge \eta_R} a_{\bar{x}(s)}(b_{\bar{x}(s)} - \bar{y}(s))[\frac{1}{2}(\theta_{X(s)} - \theta_{\bar{x}(s)})\bar{y}(s)^{-\frac{1}{2}} - 2(\gamma_{X(s)} - \gamma_{\bar{x}(s)})\bar{y}(s)^{-3}]ds \\ &+ \frac{1}{2}E \int_0^{t \wedge \eta_R} \sigma_{\bar{x}(s)}^2\bar{y}(s)^{2\beta}[-\frac{1}{4}\theta_{X(s)}(y(s)^{-\frac{2}{3}} - \bar{y}(s)^{-\frac{2}{3}}) + 6\gamma_{X(s)}(y(s)^{-4} - \bar{y}(s)^{-4})]ds \\ &+ \frac{1}{2}E \int_0^{t \wedge \eta_R} \sigma_{\bar{x}(s)}^2\bar{y}(s)^{2\beta}[-\frac{1}{4}(\theta_{X(s)} - \theta_{\bar{x}(s)})\bar{y}(s)^{-\frac{2}{3}} + 6(\gamma_{X(s)} - \gamma_{\bar{x}(s)})\bar{y}(s)^{-4}]ds \\ &+ E \int_0^{t \wedge \eta_R} \sum_{j=1}^G q_{X(s)j}[\theta_j(\sqrt{y(s)} - \sqrt{\bar{y}(s)}) + \gamma_j(y(s)^{-2} - \bar{y}(s)^{-2})]ds \\ &+ E \int_0^{t \wedge \eta_R} \sum_{j=1}^G (q_{X(s)j} - q_{\bar{x}(s)j})(\theta_j\sqrt{\bar{y}(s)} + \gamma_j\bar{y}(s)^{-2})ds. \end{aligned} \quad (7)$$

By condition (iii) we have

$$\begin{aligned}
& E \int_0^{t \wedge \eta_R} [\theta_{\bar{x}(s)}(\sqrt{\bar{y}(s)} - \sqrt{y(s)}) + \gamma_{\bar{x}(s)}(\bar{y}(s)^{-2} - y(s)^{-2})] ds \\
& \leq E \int_0^{t \wedge \eta_R} [\Theta |\sqrt{\bar{y}(s)} - \sqrt{y(s)}| + \Gamma |\bar{y}(s)^{-2} - y(s)^{-2}|] ds \\
& \leq C_1(R) E \int_0^{t \wedge \eta_R} |\bar{y}(s) - y(s)| ds \\
& \leq C_1(R) E \int_0^T |\bar{y}(s \wedge \eta_R) - y(s \wedge \eta_R)| ds \\
& \leq C_1(R) \int_0^T (E |\bar{y}(s \wedge \eta_R) - y(s \wedge \eta_R)|^2)^{\frac{1}{2}} ds
\end{aligned}$$

Similarly, we can get

$$\begin{aligned}
& E \int_0^{t \wedge \eta_R} a_{\bar{x}(s)}(b_{\bar{x}(s)} - \bar{y}(s)) [\frac{1}{2} \theta_{X(s)}(y(s)^{-\frac{1}{2}} - \bar{y}(s)^{-\frac{1}{2}}) - 2\gamma_{X(s)}(y(s)^{-3} - \bar{y}(s)^{-3})] ds \\
& \leq ABE \int_0^{t \wedge \eta_R} [\frac{1}{2} \Theta |y(s)^{-\frac{1}{2}} - \bar{y}(s)^{-\frac{1}{2}}| - 2\Gamma |y(s)^{-3} - \bar{y}(s)^{-3}|] ds \\
& \leq C_2(R) E \int_0^{t \wedge \eta_R} |\bar{y}(s) - y(s)| ds \\
& \leq C_2(R) \int_0^T (E |\bar{y}(s \wedge \eta_R) - y(s \wedge \eta_R)|^2)^{\frac{1}{2}} ds.
\end{aligned}$$

We can show, in the same way as above, that

$$\begin{aligned}
& E \int_0^{t \wedge \eta_R} [(\theta_{\bar{x}(s)} - \theta_{X(s)})\sqrt{y(s)} + (\gamma_{\bar{x}(s)} - \gamma_{X(s)})y(s)^{-2}] ds \\
& \leq E \int_0^{t \wedge \eta_R} [2\Theta\sqrt{y(s)} + 2\Gamma y(s)^{-2}] I_{\{\bar{x}(s) \neq X(s)\}} ds \quad (8) \\
& \leq C_2(R) \sum_{k=0}^{N-1} \int_{t_k}^{t_{k+1}} E[I_{\{X(t_k) \neq X(s)\}} | X(t_k)] ds.
\end{aligned}$$

Since

$$\begin{aligned}
E[I_{\{X(t_k) \neq X(s)\}} | X(t_k)] &= \sum_{i=1}^G I_{\{X(t_k)=i\}} P(X(s) \neq i | X(t_k) = i) \\
&= \sum_{i=1}^G I_{\{X(t_k)=i\}} \sum_{j \neq i} (q_{ij}(s - t_k) + o(s - t_k)) \\
&\leq (\max_{1 \leq i \leq G} (-q_{ii})) \Delta t + o(\Delta t).
\end{aligned}$$

Substituting this into (8) gives

$$\begin{aligned}
& E \int_0^{t \wedge \eta_R} [(\theta_{\bar{x}(s)} - \theta_{X(s)})\sqrt{y(s)} + (\gamma_{\bar{x}(s)} - \gamma_{X(s)})y(s)^{-2}] ds \\
& \leq C_2(R) T (\max_{1 \leq i \leq G} (-q_{ii}) \Delta t + o(\Delta t)).
\end{aligned}$$

We can similarly estimate the other terms on the right-hand side of (7) to get that

$$\begin{aligned}
& EV_{X(t \wedge \eta_R)}(y(t \wedge \eta_R)) \\
& \leq V_{x_0}(y_0) + KT + E \int_0^{t \wedge \eta_R} V_{X(s)}(y(s)) ds \\
& + C_1(R) \int_0^T (E |\bar{y}(s \wedge \eta_R) - y(s \wedge \eta_R)|^2)^{\frac{1}{2}} ds \\
& + C_1(R) T (\max_{1 \leq i \leq G} (-q_{ii}) \Delta t + o(\Delta t)), \quad (9)
\end{aligned}$$

where $C_1(R)$ is a constant dependent on R but independent of Δt . Similarly we can show that

$$E |\bar{y}(s \wedge \eta_R) - y(s \wedge \eta_R)|^2 \leq C_2(R) \Delta t,$$

Substituting this into (9) yields that

$$\begin{aligned}
& EV_{X(t \wedge \eta_R)}(y(t \wedge \eta_R)) \\
& \leq V_{x_0}(y_0) + KT + C_3(R)(\sqrt{\Delta t} + o(\Delta t)) + K \int_0^t EV_{X(s)}(y(s)) ds,
\end{aligned}$$

By the Gronwall inequality,

$$EV_{X(T \wedge \eta_R)}(y(T \wedge \eta_R)) \leq e^{KT} [V_{x_0}(y_0) + KT + C_3(R)(\sqrt{\Delta t} + o(\Delta t))].$$

E. Step 3

In the same way as (6) was obtained, we can then show that

$$P\{\eta_R \leq T\} \leq \frac{e^{KT}}{V_R} [V_{x_0}(y_0) + KT + C_3(R)(\sqrt{\Delta t} + o(\Delta t))].$$

Now, let $\varepsilon, \delta \in (0, 1)$ be arbitrarily small. Set

$$\Omega = \{\omega : \sup_{0 \leq t \leq T} |\bar{y}(t) - y(t)|^2 \geq \delta\}$$

We compute

$$\begin{aligned}
P(\Omega) &= P(\Omega \cap \{\tau \geq T\}) + P(\tau < T) \\
&\leq P(\Omega \cap \{\tau \geq T\}) + P(\theta < T) + P(\rho < T) \\
&\leq \frac{C_4(R)}{\delta} (\Delta t + o(\Delta t)) + \frac{2e^{KT}}{V_R} [V_{x_0}(y_0) + KT] \\
&\quad + \frac{e^{KT}}{V_R} C_3(R)(\sqrt{\Delta t} + o(\Delta t))
\end{aligned}$$

Recalling that $V_R \rightarrow \infty$ as $R \rightarrow \infty$, we can choose R sufficiently large for

$$\frac{2e^{KT}}{V_R} [V_{x_0}(y_0) + KT] \leq \frac{\varepsilon}{2}$$

and then choose Δt sufficiently small for

$$\frac{C_4(R)}{\delta} (\Delta t + o(\Delta t)) + \frac{e^{KT}}{V_R} C_3(R)(\sqrt{\Delta t} + o(\Delta t)) \leq \frac{\varepsilon}{2},$$

to obtain

$$P(\Omega) = P\{\omega : \sup_{0 \leq t \leq T} |\bar{y}(t) - y(t)|^2 \geq \delta\} \leq \varepsilon.$$

This proves the assertion (5).

V. CONCLUSION

In this paper we obtain the Euler-Maruyama approximate solution for mean-reverting Regime Switching CEV processes and provides a detailed proof of the strong convergence of the EM approximate solution to the exact solution.

ACKNOWLEDGMENT

The authors would like to thank the Youth Project of National Social Sciences Foundation (No.12CGL021), the Youth Project in Humanities and Social Science Research of the Ministry of Education of China (No.11YJC790224), and the National Natural Science Foundation of China (No.71173203) for financial support.

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