Distributed Control for Circumnavigating Multi-Target by a Group of Agents

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Abstract—In this study, a distributed cooperative control law is proposed to attain multi-agent uniform circumnavigation around moving targets. First, we construct a decentralized estimator for each agent to estimate the geometric center of the targets. Then, we develop the distributed control strategy by means of the estimator, which guarantees that the agents preserve the desired distance to the target center and rotate around the targets at uniformly spaced angles. In particular, the topology structure of the agents only needs to be a tree rather than the ring. Thus, the control scheme is more feasible and robust and requires less information. The convergence of the proposed algorithms is demonstrated based on Lasalle's invariance principle and algebraic graph theory. Several numerical simulations are provided to validate the results.

Keywords—multi-agent systems; coordination control; decentralized estimator; uniform circumnavigation

I. INTRODUCTION

With the rapid development of computation and communication technologies, achieving coordination control for multi-agent systems to perform a variety of challenging tasks has become possible. Coordination control has drawn more and more attention recently because of its significant benefits and broad applications. Various subjects in this field have been under intense investigation, including consensus, formation, and flocking. In particular, the circumnavigation problem for multi-agent systems has received increasing interest because of its numerous practical uses in science and engineering, such as security and surveillance, orbit maintenance, and source-seeking.

Considerable effort has been dedicated to solving this problem. For instance, a cyclic pursuit strategy was proposed in [1] to achieve uniform circumnavigation. With bearing-only measurements, [2] and [3] developed a distributed control law that steered the agents to encircle a static target. An adaptive scheme was designed in [4] to attain uniform distribution of the agents. Unlike situations where multiple agents circumnavigate the target along one orbit, the authors in [5] considered that the agents circled along different orbits. By contrast, less attention has been devoted to cases with multiple targets. In real applications, such as in cases where unmanned ground vehicles are protected and monitored by armed robotic vehicles, instructing agents to circumnavigate multiple targets is more practical and effective than instructing them to circumnavigate a single target. Therefore, analyzing scenarios with multiple

targets is required to improve the implementation of this strategy.

Furthermore, the circumnavigation problem is closely related to the target-enclosing problem, where various enclosing formations may be taken into account (e.g. [6,7]). The authors in [6] proposed a distributed control method that enabled the followers to surround the leaders with the polytope configuration. [7] provided an equidistant circular enclosing control law for moving targets. Although these articles investigated different enclosing configurations instead of the circumnavigation formation, their ideas may still be used to analyze the multi-target circumnavigation problem.

Motivated by the previously presented discussion, in this study, we investigate a case where a group of agents must circumnavigate a group of moving targets with only local information. Based on the estimation and control algorithms, we propose a distributed control method to settle the problem. First, a decentralized estimator is built to track the geometric center of the targets. Then, a cooperative control scheme is generated by means of the estimator, which stipulates the achievement of the desired global behavior; as the targets move, the agents uniformly circumnavigate the targets with the estimated center as its centroid. Unlike previous works [1-5], where only one target is involved, our work concerns multiple moving targets. Similarly, although the authors in [8] investigated the circumnavigation problem of multiple targets, only one agent was included. Compared with [7], where an equidistant circular enclosing control law was developed for multiple moving targets, our work also achieves rotation for multi-agent systems to enclose moving targets and the strategy we proposed is easy to implement. In addition, our work uses only the local information of the targets and the topology structure of the agents needn't be a ring, as in several other formation controls.

II. PRELIMINARIES

An undirected graph of order n is denoted as $G = (V, E)$, containing a vertex set $V = \{v_1, v_2, \dots, v_n\}$ and an edge set $E = \{(v_i, v_j)\} \subset V \times V$. The set of neighbors of vertex v_i is denoted by $N_i = \{v_j \mid (v_i, v_j) \in E\}$. Let $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ be the adjacency matrix, in which $a_{ij} > 0$ if $(v_i, v_j) \in E$ and $a_{ij} = 0$ otherwise. For simplicity, we assume $a_{ij} = 1$ when

$a_{ij} > 0$. Throughout the paper, $\|\cdot\|$ represents the Euclidean norm, $|S|$ denotes the number of elements in set S , and $\mathbf{0}$ represents the vector with all zeros.

III. PROBLEM STATEMENT

Consider a team of n agents moving in the plane, each of which is described by the following single-integrator dynamics:

$$\dot{x} = u_i, \quad i = 1, 2, \dots, n, \quad (1)$$

where $x_i, u_i \in \mathbb{R}^2$ denote the position and control input, respectively. Suppose in the plane there are m moving targets with positions $r_l \in \mathbb{R}^2$ for $l = 1, 2, \dots, m$. The set of the multi-agent system is denoted by $X = \{X_1, X_2, \dots, X_n\}$, whereas the set of the target group is denoted by $R = \{R_1, R_2, \dots, R_m\}$.

Let $G = (X, E)$ denote the topology structure of the multi-agent system. We assume that each agent is equipped with an onboard sensor that allows it to communicate with its neighbor agents and detect its neighbor targets, where the information flow can be described by $\bar{G} = (X \cup R, \bar{E})$. For agent X_i , let N_i^R denote the set of targets who can be detected by agent X_i . For target R_l , let N_l^X denote the set of agents who can access the information from target R_l . Furthermore, in this study, we assume that $\bar{G}$ is fixed. Moreover, if any agent has a detection relationship with the targets, then it will be informed of the number of agents correlated to the same target a priori.

Denote $\bar{r} = \frac{1}{m} \sum_{l=1}^m r_l$ as the geometric center of the target group and $p_i = x_i - \bar{r}$ as the relative position of agent X_i to the target center. The polar coordinate system centered at $\bar{r}$ is used to facilitate the analysis. Thus, the radius and angle of agent X_i can be defined as $\rho_i = \|p_i\|$, $\theta_i = \arctan2(p_i^y, p_i^x)$, where $p_i = [p_i^x, p_i^y]^T \in \mathbb{R}^2$ and $\arctan2$ represents a two-argument arctangent function returning the angle of point p_i as a numeric value ranging from 0 to 2π rad.

Now, we consider how to form a geometric pattern for a uniform circumnavigation task of multi-target by the group of n agents. The detailed control objectives are given as follows:

(i) $\lim_{t \rightarrow +\infty} \|x_i - \bar{r}\| = d$ for $i = 1, 2, \dots, n$, where $d \in \mathbb{R}$ is the radius of the desired enclosing circle.

(ii) $\lim_{t \rightarrow +\infty} (\dot{\theta}_i - \dot{\theta}_j) = 0$ for any $i, j = 1, 2, \dots, n$.

(iii) $\lim_{t \rightarrow +\infty} \psi_i = 2\pi / n$ for $i = 1, 2, \dots, n$, where $\psi_i = \theta_{i+1} - \theta_i$

θ_i when $\theta_{i+1} \geq \theta_i$, $\psi_i = \theta_{i+1} - \theta_i + 2\pi$ otherwise, and agent X_{n+1} is the same as X_1 .

Notably, $\bar{r}$ is unavailable to every agent because it only has local information about the targets. Thus, deriving an estimator of the target center for each agent is a necessary step toward ultimately solving the problem. We shall again assume that n estimators exist, denoted by $\hat{x}_i (i = 1, 2, \dots, n)$, each of which is embedded in an individual agent. Taken together, the circumnavigation problem investigated in this study can be stated formally as follows:

Concerning multi-agent system (1) and multiple moving targets, derive the estimator $\hat{x}_i$ and control input u_i such that the detailed control objectives (i)-(iii) are achieved.

Assumption 1: The topology structure $G = (X, E)$ of the multi-agent system is undirected and connected. One target can be detected at least by one agent.

Assumption 2: For the target group, the velocity of each target and its relative position attached to the target center are bounded. Namely, $\beta, \eta \in \mathbb{R}$ and $\beta, \eta > 0$ exist, such that $\|\dot{r}_l\| \leq \beta$, $\|r_l - \bar{r}\| \leq \eta$ for $l = 1, 2, \dots, m$.

IV. MAIN RESULTS

The current section is devoted to dealing with the problem, which consists of two aspects, namely, designing an estimator of the target center for each agent and developing a distributed control law for system (1) to achieve uniform circumnavigation.

A. Estimator of the Target Center

We first attempt to derive the estimator of the target center for each agent. Inspired by [7], we design the estimator as

$$\hat{x}_i = \Phi_i + \frac{n}{m} \sum_{R_l \in N_i^R} \frac{1}{|N_l^X|} r_l, \quad i = 1, 2, \dots, n, \quad (2)$$

where $\hat{x}_i \in \mathbb{R}^2$ is the estimator of the target center and

$$\Phi_i = \begin{cases} \alpha \sum_{j=1}^n a_{ij} \frac{\hat{x}_j - \hat{x}_i}{\|\hat{x}_j - \hat{x}_i\|}, & \hat{x}_j \neq \hat{x}_i, \\ 0, & \hat{x}_j = \hat{x}_i. \end{cases} \quad (3)$$

Here, $\alpha > 0$, $\Phi_i(0) = 0$ and a_{ij} is the (i, j) th entry of adjacency matrix A . The aim of (2) and (3) is to ensure the finite-time convergence of $\hat{x}_i$ to the target center.

Theorem 1: Suppose that Assumption 1 and 2 hold for system (1). If $\alpha > n(n-1)\beta$, then the estimator defined by (2) and (3) will converge to the target center $\bar{r}$ in finite time.

Proof. The proof is similar with [7] and hence is omitted here.

B. Distributed Control Law for Uniform Circumnavigation

In this subsection, we will develop a distributed control law to achieve uniform circumnavigation for the agents, with the aid of the aforementioned center-estimating algorithm. That is, we will find suitable control inputs u_i ($i = 1, 2, \dots, n$) to satisfy objectives (i)-(iii).

For each agent, which only has knowledge of its estimator of the target center, we must redefine its position relative to the estimated target center as $\hat{p}_i = x_i - \hat{x}_i$. Consider the polar coordinate centered at $\hat{x}_i$. The radius and angle can be redefined as $\hat{\rho}_i = \|\hat{p}_i\|$, $\hat{\theta}_i = \arctan2(\hat{p}_i^y, \hat{p}_i^x)$, in which $\hat{p}_i = [\hat{p}_i^x, \hat{p}_i^y]^T \in \mathbb{R}^2$. Then, we have $\dot{\hat{p}}_i = u_i - \dot{\hat{x}}_i$, that is, $u_i = \dot{\hat{x}}_i + \dot{\hat{p}}_i$. Here, $\dot{\hat{x}}_i$ can be easily derived because $\hat{x}_i$ has been given in (2). In order to obtain the control input u_i , we must derive $\dot{\hat{p}}_i$. Since $\hat{p}_i$ can be rewritten as $\hat{p}_i = [\hat{\rho}_i \cos \hat{\theta}_i, \hat{\rho}_i \sin \hat{\theta}_i]^T$, we derive $\dot{\hat{p}}_i = \dot{\hat{\rho}}_i \mathbf{r} + \hat{\rho}_i \dot{\hat{\theta}}_i \mathbf{s}$, where $\mathbf{r} = [\cos \hat{\theta}_i, \sin \hat{\theta}_i]^T$, $\mathbf{s} = [-\sin \hat{\theta}_i, \cos \hat{\theta}_i]^T$. By contrast, we observe that once $\dot{\hat{\rho}}_i$ and $\dot{\hat{\theta}}_i$ are designed, $\dot{\hat{p}}_i$ can be obtained and the control input u_i for system (1) can be specified as $u_i = \dot{\hat{x}}_i + \dot{\hat{\rho}}_i \mathbf{r} + \hat{\rho}_i \dot{\hat{\theta}}_i \mathbf{s}$. Therefore, the task of deriving the control input u_i becomes designing $\dot{\hat{\rho}}_i$ and $\dot{\hat{\theta}}_i$ to achieve objectives (i)-(iii).

Theorem 2: Suppose that Assumption 1 and 2 hold and that G is a tree. If $\alpha > n(n-1)\beta$, then the control input $u_i = \dot{\hat{x}}_i + \dot{\hat{\rho}}_i \mathbf{r} + \hat{\rho}_i \dot{\hat{\theta}}_i \mathbf{s}$ can achieve objectives (i)-(iii), where $\hat{x}_i$ is in the form of (2), and $\dot{\hat{\rho}}_i$, $\dot{\hat{\theta}}_i$ are designed as follows:

$$\dot{\hat{\rho}}_i = -\hat{\rho}_i(\hat{\rho}_i - d), \quad (4)$$

$$\dot{\hat{\theta}}_i = \frac{\gamma}{|N_i|} \sum_{j=1}^n a_{ij} \frac{\hat{\theta}_j - \hat{\theta}_i + f(\hat{\theta}_j - \hat{\theta}_i)}{j - i + g(j - i)}. \quad (5)$$

Here, for (4), $\hat{\rho}_i(0) > 0$, $d > \eta$ is the desired radius and η is a constant having been defined in Assumption 2. For (5),

$$f(\hat{\theta}_j - \hat{\theta}_i) = \begin{cases} 0, & \hat{\theta}_j = \hat{\theta}_i (j > i) \text{ or } \hat{\theta}_j > \hat{\theta}_i, \\ 2\pi, & \hat{\theta}_j = \hat{\theta}_i (j \leq i) \text{ or } \hat{\theta}_j < \hat{\theta}_i, \end{cases} \quad (6)$$

$$g(j - i) = \begin{cases} 0, & j > i, \\ n, & j \leq i, \end{cases} \quad (7)$$

$\gamma > 0$ and N_i denotes the set of neighbor agents of agent X_i .

Proof. The theorem is proved by the following three lemmas.

Lemma 1: If $\dot{\hat{\rho}}_i$ takes the form of (4) for any $\hat{\rho}_i(0) > 0$, then $\lim_{t \rightarrow +\infty} \|x_i - \bar{r}\| = d$ for $i = 1, 2, \dots, n$.

Proof. As (4) is a scalar differential equation, it is easy to verify $\lim_{t \rightarrow +\infty} \|x_i - \hat{x}_i\| = d$. Furthermore, by Theorem 1, $\hat{x}_i$ will converge to $\bar{r}$ in finite time. Thus, we can easily obtain $\lim_{t \rightarrow +\infty} \|x_i - \bar{r}\| = d$ and objective (i) is achieved.

Lemma 2: If $\dot{\hat{\theta}}_i$ is designed by (5), then it holds that $\lim_{t \rightarrow +\infty} (\hat{\theta}_i - \hat{\theta}_j) = 0$ for any $i, j = 1, 2, \dots, n$.

Proof. From (5), we have

$$\ddot{\hat{\theta}}_i = \frac{\gamma}{|N_i|} \sum_{j=1}^n a_{ij} \frac{\dot{\hat{\theta}}_j - \dot{\hat{\theta}}_i}{j - i + g(j - i)}. \quad (8)$$

Let $B = [b_{ij}] \in \mathbb{R}^{n \times n}$, $b_{ij} = \gamma a_{ij} / \{|N_i| \cdot [j - i + g(j - i)]\}$

when $i \neq j$, and $b_{ii} = -\sum_{j=1, j \neq i}^n b_{ij}$. Obviously, if $i \neq j$,

$b_{ij} \geq 0$. Then, (8) can be written as $\ddot{\hat{\theta}}_i = \sum_{j=1}^n b_{ij} \dot{\hat{\theta}}_j$. Based on

Lemma 1 [9], we can easily derive that $\lim_{t \rightarrow +\infty} (\dot{\hat{\theta}}_i - \dot{\hat{\theta}}_j) = 0$ for any $i, j = 1, 2, \dots, n$. Furthermore, relying on Theorem 1, we

also have $\lim_{t \rightarrow +\infty} (\hat{\theta}_i - \theta_i) = 0$. Therefore, it can be verified that $\lim_{t \rightarrow +\infty} (\dot{\hat{\theta}}_i - \dot{\theta}_j) = 0$, which accomplishes objective (ii).

Lemma 3: If G is a tree, then $\lim_{t \rightarrow +\infty} \psi_i = 2\pi/n$ for $i = 1, 2, \dots, n$, where ψ_i has been defined in objective (iii).

Proof. Depending on Lemma 2, we have $\lim_{t \rightarrow +\infty} (\dot{\hat{\theta}}_i - \dot{\hat{\theta}}_j) = 0$.

In general, assume $\lim_{t \rightarrow +\infty} (\dot{\hat{\theta}}_i - \gamma S) = 0$, where γ is a constant given in (5). Let $Z_{ij} = \hat{\theta}_j - \hat{\theta}_i + f(\hat{\theta}_j - \hat{\theta}_i)$. Here, $\hat{\theta}_i, \hat{\theta}_j \in [0, 2\pi)$. It follows that

$$\lim_{t \rightarrow +\infty} \left[\frac{1}{|N_i|} \sum_{X_j \in N_i} \frac{Z_{ij}}{j-i+g(j-i)} - S \right] = 0. \quad (9)$$

Besides, it is clear that $Z_{ij} + Z_{ji} = 2\pi$. In order to derive Z_{ij} and S in the stable state, consider the equations

$$\begin{cases} \frac{1}{|N_i|} \sum_{X_j \in N_i} \frac{z_{ij}}{j-i+g(j-i)} - s = 0, \\ z_{ij} + z_{ji} = 2\pi. \end{cases} \quad (10)$$

For analytical convenience, assume $z_{ij} = z_{ji}$ when $i < j$, and $z_{ij} = 2\pi - z_{ji}$ when $i \geq j$. Therefore, (10) can be written as the following liner system of equations:

$$\frac{1}{|N_i|} \sum_{\substack{X_j \in N_i \\ i < j}} \frac{z_{ij}}{j-i} + \frac{1}{|N_i|} \sum_{\substack{X_j \in N_i \\ i \geq j}} \frac{2\pi - z_{ji}}{j-i+n} - s = 0 \quad (11)$$

with unknown variables z_{ij} and s , where $i = 1, 2, \dots, n$ and $X_j \in N_i$. Especially, for (11), $z_{ij} = [j-i+g(j-i)] \cdot 2\pi/n$, $s = 2\pi/n$ is one of its solutions.

First, we prove that (11) has a unique solution if G is a tree. By the above observation, it is interesting to find that if G has μ edges $(X_{i_1}, X_{j_1}), (X_{i_2}, X_{j_2}), \dots, (X_{i_\mu}, X_{j_\mu})$, there are also μ variables $z_{i_1 j_1}, z_{i_2 j_2}, \dots, z_{i_\mu j_\mu}$ together with a variable s for (11). Without loss of generality, assume $i_w \leq j_w$ when $1 \leq w \leq \mu$. Then, (11) can be rewritten in a more concise form:

$$\sum_{w=1}^{\mu} c_{iw} z_{i_w j_w} - s = -\frac{1}{|N_i|} \sum_{\substack{X_j \in N_i \\ i \geq j}} \frac{2\pi}{j-i+n} \quad (12)$$

for $i = 1, 2, \dots, n$. Here,

$$c_{iw} = \begin{cases} [|N_i| \cdot (j_w - i_w)]^{-1}, & i = i_w, \\ \{-|N_i| \cdot (i_w - j_w + n)\}^{-1}, & i = j_w, \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

By the properties of the tree, G contains $n-1$ edges. Hence, (12) is a system of n equations with n unknowns $z_{i_1 j_1}, z_{i_2 j_2}, \dots, z_{i_{n-1} j_{n-1}}, s$. In order to prove that (12) has a unique solution, we only need to treat its coefficient matrix, denoted by

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n-1} & -1 \\ c_{21} & c_{22} & \cdots & c_{2n-1} & -1 \\ \vdots & \vdots & & \vdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn-1} & -1 \end{bmatrix}_{n \times n},$$

where c_{iw} ($i = 1, 2, \dots, n$ and $w = 1, 2, \dots, n-1$) have been given in (13). For matrix C , it is easy to discover that each column contains only two nonzero numbers with the exception of the last one. Besides, the signs of the two numbers are opposite. Let $C_1, C_2, \dots, C_n$ be the row vectors of matrix C . We can easily demonstrate that they are linearly independent. In other words, matrix C is invertible, which ensures the uniqueness of the solution of (11). From (9), it is obvious to see that the unique solution describes the final state of Z_{ij} and S , that is, $\lim_{t \rightarrow +\infty} Z_{ij} = z_{ij} = [j-i+g(j-i)] \cdot 2\pi/n$ for $i = 1, 2, \dots, n$, $X_j \in N_i$, and $\lim_{t \rightarrow +\infty} S = s = 2\pi/n$.

Next, we demonstrate that the final state of Z_{ij} guarantees $\lim_{t \rightarrow +\infty} \psi_i = 2\pi/n$. If $X_{i+1} \in N_i$, it is clear that $\lim_{t \rightarrow +\infty} \psi_i = 2\pi/n$. However, for the other cases, one path always exists that connects agent X_i with X_{i+1} . Then, it clearly holds that $\lim_{t \rightarrow +\infty} \hat{\psi}_i = 2\pi/n$, where $\hat{\psi}_i = \hat{\theta}_{i+1} - \hat{\theta}_i$ if $\hat{\theta}_{i+1} \geq \hat{\theta}_i$, and $\hat{\psi}_i = \hat{\theta}_{i+1} - \hat{\theta}_i + 2\pi$ otherwise. Also, by Theorem 1, we can derive that $\lim_{t \rightarrow +\infty} \psi_i = 2\pi/n$ for $i = 1, 2, \dots, n$. As a result, the agents can evenly distribute around the

circle as well as maintain a counterclockwise formation in the end. This makes objective (iii) satisfied.

Taken together, the control input $u_i = \dot{\hat{x}}_i + \hat{\rho}_i \mathbf{r} + \hat{\rho}_i \hat{\theta}_i \mathbf{s}$, subject to (2), (4) and (5), can achieve objectives (i)-(iii).

V. CONCLUSIONS

This study provided a cooperative control strategy to accomplish multi-agent uniform circumnavigation around moving targets. Under the proposed scheme, each agent individually determines its behavior based on local information only, namely, its estimator, and angles with respect to its neighbor agents. Therefore, a local sensor system can be adopted to generate the desired global behavior. The proposed scheme also reduces information requirements and becomes more flexible compared with the conventional ring topology frame. Lasalle's invariance principle and algebraic graph theory were applied to verify the convergence of the presented method. Simulations numerically illustrated the efficacy of the algorithm. Future research will extend the method to 3D space.

VI. SIMULATIONS

Several simulations are performed to illustrate the effectiveness of the proposed approach. We consider a network composed of six agents and four targets with the topology graph shown in Fig. 1. Take the system parameters $\alpha = 2.5$, $\gamma = 1$. Initially, the positions of the eight agents are selected

as $x_1(0) = [-15, 20]^T$, $x_2(0) = [19, -14]^T$, $x_3(0) = [-10, -8]^T$, $x_4(0) = [7, 13]^T$, $x_5(0) = [0, 0]^T$, $x_6(0) = [0, -22]^T$ and that of the four targets are selected as $r_1(0) = [12, 15]^T$, $r_2(0) = [7, -16]^T$, $r_3(0) = [-10, -15]^T$, $r_4(0) = [-8, 10]^T$.

We use $r_1(t) = [12 - 0.07t, 15 - 0.08t]^T$, $r_2(t) = [7 + e^{-0.01t}, -16 + e^{-0.07t}]^T$, $r_3(t) = [-10 + e^{-0.03t}, -15 - e^{-0.02t}]^T$, $r_4(t) = [-8 + 0.08t, 10]^T$ to portray the dynamic behavior of the four moving targets and select $d = 25$ as the ideal enclosing radius. The trajectories of the six agents are shown in Fig. 2. Eventually, the agents uniformly distribute along the circle centered at the target center and rotate around the targets in a counterclockwise formation. Fig. 3 further illustrates the finite-time convergence of the designed estimator. Fig. 4 reveals the evolution of the distances between each agent and its estimator, which indicates that the agents attain the desired distance $d = 25$. And changes of the angle $\hat{\psi}_i$ between agent X_i and X_{i+1} with respect to time are plotted in Fig. 5, which intuitively shows that $\hat{\psi}_i$ converges to a common value $2\pi/6$.

FIGURE I. TOPOLOGY GRAPH. THE SET OF BLUE NODES REPRESENTS THE MULTI-AGENT SYSTEM, WHEREAS THE SET OF RED NODES REPRESENTS THE TARGET GROUP. THE SOLID LINKS AMONG AGENTS INDICATE THE NEIGHBORING RELATIONSHIP OF THE AGENTS. THE DASHED LINKS BETWEEN THE AGENT AND TARGET INDICATE THAT THE TARGET CAN BE DETECTED BY THE AGENT.

FIGURE II. SIMULATED TRAJECTORIES OF THE SIX AGENTS CIRCUMNAVIGATING FOUR MOVING TARGETS. + DENOTES THE TARGET, · DENOTES THE AGENT AND ☆ DENOTES THE TARGET CENTER GIVEN BY THE ESTIMATOR.

FIGURE III. EVOLUTION OF $\hat{x}_i(t)$

FIGURE IV. EVOLUTION OF $\hat{\rho}_i(t)$

FIGURE V. EVOLUTION OF $\hat{\psi}_i(t)$

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