

# Joint Optimal Transmitted Power and Transmitted Antenna Number for Energy-Efficient Massive MIMO Systems

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**Abstract.** In previous works, they mainly concern on optimizing transmitted antenna to maximize energy efficiency (EE). But the transmitted power optimization is also necessary for the improvement of EE. Because too large or too small transmitted power will lead to the decline of EE. This paper proposes a joint optimal transmitted power and transmitted antenna number iterative algorithm to maximize EE for massive MIMO system. Firstly a good approximation of multi-user interference (MUI) is derived. Then we demonstrate that EE is a concave function of the transmitted power and the transmitted antenna number. So for a given number of transmitted antenna, we can get the optimal transmitted power. An iterative algorithm that can maximize EE is proposed. The monte carlo simulations verify the effectiveness of the proposed algorithm.

## Introduction

With the development of mobile communication technology, the research of energy efficiency has attracted people's attention. Improving energy efficiency can reduce inter-user interference and environmental pollution [1]. The Massive MIMO system can significantly improve the energy efficiency and spectral efficiency by increasing the number of base station antenna [2]. However, the more base station antennas are activated, the higher complexity the system get and the more losses the radio frequency obtain [3]. Thus the transmitted antenna number is of great value to the energy efficiency optimization. On the other hand, when the transmitted power is too small, the data transmission time will be extended, which will lead to the increase of the circuit power consumption, corresponding to this, the energy efficiency will decline. While the transmission power is too large, the data transmission time will be shortened, the power consumption of circuit will decrease, but the power used for data transmission is very big, which will also lead to the decline of energy efficiency. So the transmitted power is also very important to optimization of energy efficiency.

In previous works, they usually assume that the total transmitted power is fixed, the problem is to find the optimal number of active antenna and the optimal antenna subset which can maximize energy efficiency [4]. The work in [5] shows how to select the transmit antenna number to maximize energy efficiency in multi-user MIMO system. The work in [6] demonstrates that the circuit power consumption should be taken into consider to ensure the energy efficiency of wireless communication in massive MIMO systems. The work in [7] explains that we can use a random antenna selection algorithm to verify the effect of transmit antenna number on energy efficiency. In [8], the author proposes a joint optimal number of RF chains and power allocation iterative algorithm to maximize the sum-rate.

Based on [8], this paper's aim is to maximize the energy efficiency. Through concave optimization theory analysis, we demonstrate the existence of the optimal value. So for a given transmitted antenna number, we derive the optimal transmitted power and then proposed a joint optimization iteration algorithm.

### System Model and Problem Formulation

Let us consider a massive MIMO system in the downlink of a single isolated cell as illustrated in Figure 1. The BS has  $N_{tx}$  transmit antennas serving  $K$  users and each user has only one receive antenna.

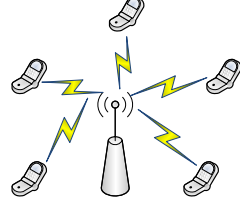


Fig.1. system model

The received signal vector at users is given by

$$\mathbf{y} = \sqrt{P_t} \mathbf{H} \cdot \mathbf{F} \cdot \mathbf{x} + \mathbf{n} \quad (1)$$

where  $\mathbf{y} \in \mathbb{C}^{K \times 1}$  and  $\mathbf{x} \in \mathbb{C}^{K \times 1}$  are the received and transmitted signal vector respectively.  $\mathbf{n} \in \mathbb{C}^{K \times 1}$  is the noise(AWGN) vector.  $P_t$  is the transmit power,  $\mathbf{H} \in \mathbb{C}^{K \times N_{tx}}$  is the slow fading channel matrix, the precoding matrix is  $\mathbf{F} \in \mathbb{C}^{N_{tx} \times K}$ , we use the conjugate precoding( $\mathbf{F} = \mathbf{H}^H / \|\mathbf{H}^H\|_F$ ). We assume the BS has perfect channel state information(CSI). The signal received by user  $k$  is given by

$$y_k = \sqrt{P_t} \mathbf{h}_{k,:} \mathbf{f}_{:,k} x_k + n_k + \sqrt{P_t} \sum_{i \neq k} \mathbf{h}_{k,:} \mathbf{f}_{:,i} x_i \quad (2)$$

where  $\mathbf{h}_{k,:} \in \mathbb{C}^{1 \times N_{tx}}$  and  $\mathbf{f}_{:,k} \in \mathbb{C}^{N_{tx} \times 1}$  are the  $k$ -th user's channel vector and precoding vector respectively.

The last term of (2) is the MUI. When  $K, N_{tx} \rightarrow \infty$ , MUI can be approximated by its expectation as[8]

$$\frac{1}{K} \sum_{i=1, i \neq k}^K |\mathbf{h}_k \mathbf{h}_i^H|^2 \approx E\{|\mathbf{h}_k \mathbf{h}_i^H|^2\} = N_{tx} \quad (3)$$

Also  $E\{|\mathbf{h}_k \mathbf{h}_k^H|^2\} = N_{tx}^2, \|\mathbf{H}^H\|_F^2 \approx K \cdot N_{tx}$ . So the average of received SINR for user  $k$  can be simplified as:

$$\overline{\text{SINR}}_k = E\left(\frac{\frac{P_t}{\|\mathbf{H}^H\|_F^2} |\mathbf{h}_k \mathbf{h}_k^H|}{\sum_{i \neq k} \frac{P_t}{\|\mathbf{H}^H\|_F^2} |\mathbf{h}_k \mathbf{h}_i^H|^2 + N_0 B}\right) = \frac{P_t \cdot N_{tx}}{(P_t + N_0 B) K} \quad (4)$$

So we obtain the sum-rate as a function of the transmitted power  $P_t$ , the transmitted antenna number  $N_{tx}$ , which can express as:

$$C = K \cdot B \log_2 \left( 1 + \frac{P_t \cdot N_{tx}}{(P_t + N_0 B) K} \right) \quad (5)$$

We define  $P_{sum}$  is the overall power consumption, which is written as

$$P_{sum} = P_t / \beta + P_b + P_c \quad (6)$$

where  $\beta$  is the power amplifier efficiency,  $P_c$  is the consumption of circuit power which was represent as[9]

$$P_c = K P_r + N_{tx} P_f \quad (7)$$

where  $P_f$  and  $P_r$  are the power consumed by each transmit and receive RF chain respectively.  $P_b$  is the baseband power consumption, which was presented in [10].

$$P_b = \chi (\text{Gflops}) / \rho (\text{Gflops/W}) \quad (8)$$

where  $\chi$  is the VLSI power-efficiency. Specific parameters and  $\chi$  are presented as[10]:

The energy efficiency is measured by throughput per Joule. Which can be expressed as:

$$U = C / P_{sum} = \frac{K \cdot B \log_2 \left( 1 + \frac{P_t \cdot N_{tx}}{(P_t + N_0 B) K} \right)}{P_t / \beta + P_{bb} + \alpha P_r + N_{tx} P_f} \quad (9)$$

Since the second order derivative of Eq.(9) with respect to  $P_t$  is negative, the denominator is a convex function with respect to  $P_t$ , so  $U$  is a pseudo-concave function of  $P_t$ . On the other hand, we can know that  $U$  is strictly concave with respect to  $N_{tx}$ , which has been demonstrated in [11]. So the problem of (9) is concave, therefore there exist the optimal  $P_t$  and  $N_{tx}$  to maximum EE.

### A Joint optimization iterative algorithm

To find the optimal  $P_t$  and the optimal  $N_{tx}$ , we propose a joint optimization iterative algorithm (JOIA). Firstly, we define  $\eta(P_t)$  by taking a derivative of  $U$  with respect to  $P_t$ . Then initialize three variables:  $low\_P_t$ ,  $mid\_P_t$ ,  $high\_P_t$ , which are the lower, the upper and the midpoint of the transmit power respectively. And  $mid\_P_t = (high\_P_t + low\_P_t) / 2$ . If  $\eta(P_t)|_{mid\_P_t} = 0$ ,  $P_t^*$  is found; If  $\eta(P_t)|_{mid\_P_t} < 0$ , replace  $high\_P_t$  with  $mid\_P_t$ ; otherwise, replace  $low\_P_t$  with  $mid\_P_t$ ; The iteration continues until  $high\_P_t - low\_P_t$  is sufficiently small to meet the convergence requirement. Thus we get the optimal transmitting power  $P_t^*$  for given  $N_{tx}$ . When firstly occur  $U(N_{tx}) < U(N_{tx}-1)$ ,  $N_{tx}$  is optimal value. The algorithm is as follows:

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**Algorithm 1: Joint Optimal Iterative Algorithm (JOIA)**

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for  $N_{tx} = 1:N$  do
  while( $high\_P_{tx} - low\_P_{tx} > 0.001$ )
    if  $\eta(mid\_P_{tx}) < 0$ 
       $high\_P_{tx} = mid\_P_{tx}$ 
    elseif
       $\eta(mid\_P_{tx}) > 0$ 
       $low\_P_{tx} = mid\_P_{tx}$ 
    else
       $P(N_{tx}) = mid\_P_{tx}$ 
    break;
  end if;
end while;
Break if  $U(N_{tx}) < U(N_{tx}-1)$ 
end
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**Simulation Results**

In this section, monte carlo simulations results are shown to prove the effectiveness of the proposed joint optimization iteration algorithm. For analysis convenience, we define BTPA ,OTPA and OTANA algorithm which respectively represent both transmit power and transmit number are not considered, only transmit power is considered and only transmit antenna number is considered.

Fig. 2 compares the energy efficiency of JOIA , BTPA , OTPA and OTANA when  $N_{tx}=600$ . We can see that more users always help improve the energy efficiency. And it is clear that the energy efficiency of JOIA is higher than the other three. Especially, when the number of users is 20, the energy efficiency of JOIA achieves nearly 12.8% gain compared with OTANA. Moreover, OTANA can get higher energy efficiency than OTPAE .

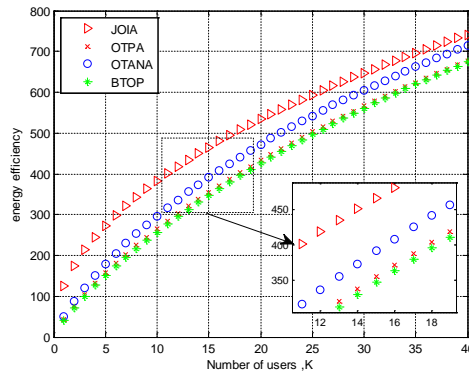


Fig . 2 Comparison between JOIA and the other three algorithm

Fig. 3 gives the relationship between the optimal transmitted power and transmitted antenna number. We can see that the optimal transmit power decreases firstly and then increases with the increase of transmit antenna number. And the optimal transmitted power increases with the increase of user number.

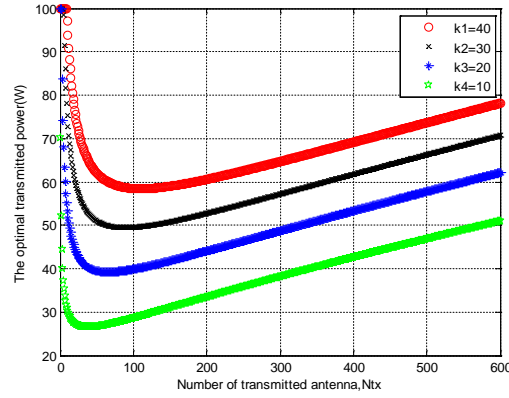


Fig.3 The optimal transmission power, as increasing number of transmitted antenna number

Fig . 4 is the relationship between the optimal transmitted power and the number of users. From Fig.4 we can see that when the number of users grows, the optimal transmitted power increases. That's because when  $k$  grows, the EE increases accordingly, so the optimal transmitted power increases to achieve EE. And the optimal transmitted power increases with the increase of transmitted antenna number.

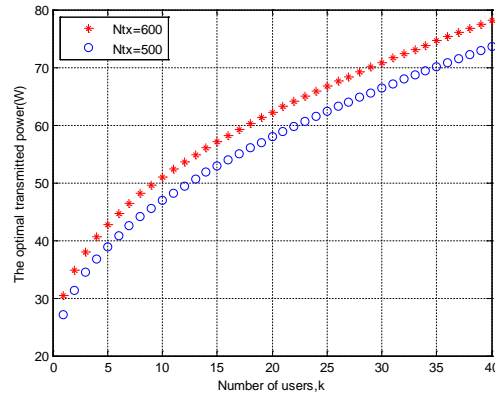


Fig . 4 The optimal transmission power, as increasing number of users

## Conclusion

This paper achieved the maximal EE by jointly optimizing the transmitted power and the transmitted antenna number. Through a good approximation of MUI, the expression of energy efficiency was derived. And we demonstrated that there existed the optimal transmitted power and the optimal transmitted antenna number because of the concave characteristic of EE. We obtained the optimal solution through iterative algorithm. The performance of proposed algorithm was demonstrated by comparing simulation results, and simulation results showed that the proposed algorithm obtain the significant EE gain.

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