Sparse-Based Classification with Sub-dictionary Division and Weighted Combination

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Abstract. A novel classification approach for hyperspectral data based on sparse representation framework is proposed in this paper. This new method divides all spectrums of hyperspectral samples to sub-dictionaries based on spectral characteristics of the data themselves. Meanwhile, a weighting algorithm based on minimum residual is adopted to combine sub-dictionaries. Experimental results show that our proposal obtained higher accuracy comparing with that of a whole dictionary.

1. Introduction

Sparse representation is a new emerging theory after wavelets in signal processing community [1][2]. It was rapidly interested in 2000's and developed in image applications successfully [3] [4] [5] [6]. In the decade of 2010s, sparse representation has gained considerable attention by the scholars around the world and employed in many research fields recently [7] [8].

Pixel classification is one of the most important parts in the research of hyperspectral remote sensing. Sparse decomposition algorithm is an effective framework and can be implemented to classify high dimensional data. The framework has the advantages that it does not need dimension reduction and model selection, as well as dictionary training. Each test data can be sparsely represented with only a few non-zero decomposition coefficients by a few atoms of an over-complete dictionary which are constructed by training samples directly. According to the reconstruction error, test data can be classified to the corresponding class. Several researchers have improved the classification accuracy of remote sensing data based on sparse representation framework [9] [10].

SR (sparse representation) algorithm can use the training set directly to form the over-complete dictionary for speeding and simplifying classification algorithm. However, simply putting the training samples of the same category with all of their components together to form a dictionary for hyperspectral classification is not a good idea because of the much higher dimension of such data. It couldn't capture the intrinsic spectral characteristics of the test data more precisely especially in a noise environment. In order to improve the classification accuracy, we proposed a method of dividing spectrum to sub-dictionaries based on the spectral characteristics of the data themselves, and then use a weighting algorithm to combine the sub-dictionaries in which the weight of each sub-dictionary is based on their minimum residual.

This paper is organized as follows. In section 2, introductions on sparse representation and its algorithm are given briefly at 2.1 and 2.2. Then our method is presented in detail at 2.3. Experiments and results on Botswana data of NASA EO-1 satellite which is compared with SR algorithm are showed in section 3. Finally, conclusions of the paper are revealed in section 4.

2. Methodology

2.1 Sparse Representation

Sparse representation can be addressed as follows: Let, $y \in R^n$ is an input pattern which can be represented approximately as:

$$y \approx D\alpha \quad (1)$$

where, $D \in R^{n \times m}$ is an over-completed dictionary of atoms $\{d_i\} (i=1,2,...,m)$, $\alpha \in R^m$ is a sparse vector with a few non-zero coefficients. The formula can be characterized as:

$$\underset{\alpha}{\operatorname{argmin}} \|y - D\alpha\| \quad \text{subject to } \|\alpha\|_0 \leq \Gamma \quad (2)$$

in which $\|\cdot\|_0$ is the zero norm and Γ is a constraint which limits the number of non-zero coefficients of sparse vector α .

A set of dictionaries can be constructed according to different class $c (c=1, 2 \dots k)$. Each dictionary gets the labelled samples of the same class from hyperspectral data to form the over-complete dictionary. Let $D_c = [d_1, d_2, \dots, d_m]$ ($D \in R^{n \times m}$) denotes the dictionary of c -th class.

According to the theory of sparse representation, each test data can be sparsely represented by a few training samples of an over-complete dictionary with only a few non-zero decomposition coefficients. We can get sparse decomposition coefficient according to formula (3).

$$\alpha_c(y) = \arg \min_{\alpha_c} \|y - D_c \alpha_c\|_2^2 + \lambda \|\alpha_c\|_1 \quad (3)$$

The above formula is an ℓ_1 -regularized least squares problem. Where $\alpha_c = [\alpha_{c1}, \alpha_{c2}, \dots, \alpha_{cm}]^T$ is the coefficient of sparse representation of y corresponding to the c -th dictionary D_c .

Regularization parameter λ is used to balance the reconstruction error and sparsity. The threshold of parameter λ is selected according to empirical value. Then the test data y is categorized to the corresponding class according to formula (4).

$$\min_c R_c(y) = \min_c (\|y - D_c \alpha_c\|_2) \quad (4)$$

where $R_c(y)$ is the residual to reconstruct y by training samples in the category c .

2.2 Implementation of SR Algorithm

Algorithm I Implementation of SR Algorithm

Input:

X : Training set (labelled example set)

Y : Test set (Unlabelled test data set)

Output:

$c(y)$: The category of each test data y .

Step 1 Each training sample $x \in X$ or test data $y \in Y$ is normalized.

Step 2 Use the training set of different categories to form the over-complete dictionaries D_c respectively.

Step 3 Use lasso algorithm to decompose the test data sparsely to get the sparse representation coefficient α_c .

Step 4 According to formula (4), calculate the residual error $R_c(y)$ of test data which was sparse approximation by D_c . Then the test data is categorized to the corresponding class according to the minimum reconstruction error.

Step 5 Stop repeat step 3 and step 4 until all the test data are categorized.

Thanks for the good performance of sparse representation, we could use the training set directly to form the over-complete dictionary for classification applications. However, for hyperspectral

data with high dimension, if the data were separated according to their spectral characteristics to form sub-dictionaries, we could depict the intrinsic structures of such data precisely. That means classification will be more accurate while the computation becomes a bit complicated.

We propose a method by dividing spectrum to sub-dictionaries following the spectral characteristics of the data themselves, then, assign different weights to different sub-dictionaries according to their residuals. With this method, we can get a new dictionary in which each sub-dictionary has its own weight.

2.3 SR Algorithm with Sub-dictionary Division and Weighted Combination

The sparse representation classification method for hyperspectral images with sub-dictionary division is implemented in two steps: firstly, calculate the mean spectra curve of training samples of k categories on all bands. Secondly, all bands are divided based on band number of peaks and troughs of mean spectra curve of k categories. Here, band division is equal to sub-dictionary division.

In our improved SR algorithm based on sub-dictionary division and weighted combination, weight is used to describe the contribution of each sub-dictionary to the classification of test data. Average weight is gotten rid of because of its inability. So we need a new way to make more sense on calculating the weight of each sub-dictionary to the final sparse reconstruction. The new dictionary in matrix form of our improved SR algorithm is listed in formula (5).

$$D_c = [w_1 D_c^1, w_2 D_c^2, \dots, w_j D_c^j, \dots, w_L D_c^L] \quad (5)$$

where $j = 1, 2, \dots, L$ indicates the index of band division. D_c^j is the j -th sub-dictionary of c -th category.

In SR algorithm, the minimum reconstruction error is used to classify the test data. So in the sub-dictionary division model, minimum reconstruction error of each sub-dictionary reflects the contribution of the specific sub-dictionary to the final classification [11]. So calculation of w_j is based on $\min R_c^j(y^j)$.

The minimum reconstruction error of each sub-dictionary is calculated and normalized respectively, and then the weight of each band region is obtained as follows:

$$\min_c R_c^j(y^j) = \min_c \left(\|y^j - D_c^j \alpha_c^j\|_2 \right) \quad (6)$$

$$w_j = \frac{1}{\frac{\min R_c^j(y^j)}{\min R_c^1(y^1)} + \frac{\min R_c^j(y^j)}{\min R_c^2(y^2)} + \dots + \frac{\min R_c^j(y^j)}{\min R_c^L(y^L)}}, \quad j = 1, 2, 3, \dots, L. \quad (7)$$

$$w_j = \frac{1}{\sum_{i=1}^L \frac{\min R_c^j(y^j)}{\min R_c^i(y^i)}}, \quad i = 1, 2, 3, \dots, L. \quad j = 1, 2, 3, \dots, L. \quad \sum_{j=1}^L w_j = 1 \quad (8)$$

where, y^j is the given band of test data corresponding to sub-dictionary D_c^j .

Algorithm II Algorithm of Our Method

Input:

The matrix of training samples $X = (X_1, X_2, \dots, X_k) \in R^{n \times m}$, which contain k classes. Test data $y \in R^n$ contained a reconstruction error $\varepsilon > 0$.

Output:

$c(y)$: the category of test data.

Step 1 Normalize the training samples X to make it ℓ_2 -norm.

Step 2 Use the training set to form the over-complete dictionary D_c .

Step 3 For $c = 1, \dots, k$, calculate mean spectrum curve of c -th category and then divide all bands of D_c into L sub-dictionaries based on band number of peaks and troughs of mean spectra curve.

Step 4 For $j = 1, \dots, L$, use lasso algorithm to decompose the test data y^j sparsely to get the minimum reconstruction error $R_c^j(y^j)$.

Step 5 According to formula (7), calculate weight of each sub-dictionary. Then the test data is classified to the corresponding class according to minimum accumulative residual error.

Step 6 Stop repeat step 4 and step 5 until all the test data are categorized.

Figure 1 presents the block diagram of the proposed method.

Fig. 1 The block diagram of the proposed method, y^j is the given band of test data corresponding to sub-dictionary D_c^j .

3 Experiment

3.1 Sub-dictionary Division

The NASA EO-1 satellite acquired a sequence of data over the Okavango Delta, Botswana in 2001-2004. The Hyperion sensor on EO-1 acquires data at 30 m pixel resolution over a 7.7 km strip in 242 bands covering the 400-2500 nm portion of the spectrum in 10 nm windows. Uncalibrated and noisy bands that cover water absorption features were removed, and the remaining 145 bands were included as candidate features. The data contains 14 typical ground objects and 145 bands.

Calculating mean spectrum curve of 14 categories on 145 bands, we can show the results in figure 2.

(a) Mean spectrum curve of 1st to 3th category.

(b) Mean spectrum curve of 4th to 6th category.

(c) Mean spectrum curve of 7th to 10th category.

(d) Mean spectrum curve of 11th to 14th category.

Fig.2. Mean spectrum curve of 1st to 14th category

In figure 2, the spectral bands of 14 different kinds of materials have the similar structures. Thus, band division and sub-dictionary division based on band number of peaks and troughs of 14 categories are listed in table 1:

Table 1 Sub-dictionary Division of Botswana Hyperspectral Data

Sub-dictionary	D1	D2	D3	D4	D5	D6
Band Region	1-24	25-47	48-62	63-81	82-112	113-145

3.2 Experiment Result and Comparison

In the experiment, firstly, 25% data samples are selected randomly into the test data set, the rest of 75% data are used as the pool of training samples. Then four different training sets with different proportion of samples selected randomly from the training samples pool are investigated, the rates are 5%, 15%, 30% and 50%. Each training sets with corresponding proportion are executed 5 times. Pixel values with respect to each band are normalized. The classification accuracy is the average of five experimental results. The classification method used for comparison is SR algorithm.

Table 2 compares the classification performances of SR algorithm with our improved algorithm at different sampling rates.

TABLE 2. CLASSIFICATION ACCURACY OF SR ALGORITHM AND OUR METHOD

Training rate	SR algorithm	Our method
5%	88.460	89.613
15%	92.323	93.256
30%	94.205	95.167
50%	95.208	96.256
75%	96.088	96.854

Figure 3 shows the experimental results.

Fig.3. The experimental results

The overall results in table 2 and figure 3 for the Botswana data clearly demonstrate that the proposed method outperforms a standard SR algorithm in terms of accuracy, irrespective of the number of training samples.

4. Conclusion

In this paper, in order to improve discrimination performance of single dictionary based on SR algorithm, an improved classification method of sparse representation of hyperspectral images based on sub-dictionary division and weighted combination is proposed. The classification effectiveness of the proposed algorithm is verified by experiments.

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