Searching for the Accident Plane

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Abstract. It is difficult but important for the searchers to predict the accident plane. In order to imitate the crashed range of the aircraft, we devise the model of the process of the fragments’ falling. Because of the influence of the current, we are supposed to consider the fragments flow in the ocean current. After determining the scope of debris, we need to choose the best suitable searching plan from the several methods which adapt to different situations. We can estimate the searching time by using the data above.

1. Introduction

At 8th, March, 2014. From the China Civil Aviation Administration air traffic control board was informed that the Malaysia Airlines B777-200ER aircraft (fuselage number 9M-MRO), the implementation of MH370 (Kuala Lumpur to Beijing) flight mission, take-off time 00:42 (Beijing time). However, the aircraft lost contact and the radar signal with regulatory agencies in Ho Chi Minh control area at about 01:20. And after relevant regulatory authorities’ confirmation, the machine has not established contact with China’s Control Department or to enter China's ATC information region. The authorities were searching the fragments of MH370, so we establish models to offer a solution to the problems.

2. The analysis of the searching range

2.1 The crashed range of the plane

In order to estimate the falling range of the airplane, we need to imitate a plane to search the aircraft. The calculation needs many parameters. Firstly, we use figure (1) to calculate the maximum unpowered glide distance, \( d \) [²]. \( H \) is cruising altitude for missing aircraft.

\[
d = \frac{H}{\tan \theta}
\]  

Fig. 1 The practice definition of \( \theta \)

As the figure 2 shown, \( \theta \) expresses the angle between airplane and the horizon line. \( \theta \) depends on \( L/D \), which we called lift-drag ratio. We can calculate it by figure (2).

\[
\theta = \arctan \frac{1}{L/D}
\]  

We always choose Boeing 747 as long distance trans-ocean plane. They own more than 42 billion nautical miles in its lifetime [³]. Boeing 747 - 400 is one of the most common model in the Boeing 747 series. Its lift-drag ratio is 8. The cruise height is 35000 foots [⁴]. Due to its frequency of travel over
oceans, the 747-400 will be the first aircraft considered as the missing aircraft in this report. According to figure (2), $\theta$ is $7.125^\circ$.

According to papers we read, $H$ is about 10 km. We can get that $d$ is about 80 km according to (1). The speed of MH730 is about 850 km/h, so the horizontal velocity is 843.44 km/h. If we use a more simple model, we can define the search range as a circle, which the radius is $d$. Then they can make the search area smaller to improve their searching efficiency by using our method.

In order to calculate the search area $A$, we need the swerve radius $r$. At first, we assume that the plane starts flying in the $x$ positive direction. It swerves in the angular velocity of 1.5 degree per-second. It needs 60 seconds to swerve 90 degree. According to figure (3), we can get swerve radius $r$ by calculating the $x$ direction projection $V_x$ of the horizontal velocity.

\[ \begin{align*}
V_x &= v \cos \theta \\
r &= \int_0^{90} V_x \, dt 
\end{align*} \]

We can get that the swerve radius is 9.37 kilometers because the horizontal velocity is 843.44 km/h. If the airplane does not swerve, it can travel 80 kilometers from the last point of contact according to (1). But if it turned 90 degree, it can only travel another 65.29 kilometers in the straight direction because of the distance spent on turning. If it turned 180 degrees, it can only travel another 50.5 kilometers. Figure 2 demonstrates this fact visually.

**Fig.2** The loss in the swerving. The end of the line segment is Maximum flying distance of the plane

The destination of every line expresses the largest distance of the falling plane. We establish a plane right angle coordinate system, defining the swerving angle as $\alpha$. $hc(\alpha, r)$ is the arc length of the arc, which has a radius of $r$ and a central angle of $\alpha$. So we can get the coordinate of the destination point:

\[ \begin{align*}
x &= r \sin \alpha + \cos \alpha \cdot (d - hc(\alpha, r)) \\
y &= r \cdot (1 - \cos \alpha) + \sin \alpha \cdot (d - hc(\alpha, r))
\end{align*} \]

According to (4) (5), we can get a range depending on those destination points. The range includes all the places that the plane will possible appear. That is the search area $A$ as the figure 3 shown.

**Fig.3** Considering the swerving, the dark area is the region of plane crashed.
2.2 The ocean current’s affection on searching

The most important influencing factors of the fragments’ position is the ocean current. We divide the range in figure 4 as a searching net. Every small area in this net corresponds to a pair of longitude and latitude. We define the longitude of the set is $jd$ and the latitude of the set is $wd$. These coordinates change with the ocean current and make the searching area larger.

Due to the consistent change of the ocean current, we discretize the problem to simplify the calculation. We update the coordinate position in the interval of 12 hours. We use the Euler method to imitate the movement of the fragments. We can calculate the coordinates according to figure (6).

\[
(jd_n, wd_n) = (jd_0, wd_0) + \Delta t \left( \frac{\partial jd_1}{\partial time}, \frac{\partial wd_1}{\partial time} \right) + \cdots + \Delta t \left( \frac{\partial jd_{n-1}}{\partial time}, \frac{\partial wd_{n-1}}{\partial time} \right) \tag{6}
\]

3. Choose the suitable searching method

3.1 Searching method

We have to search in the visual way, because the accidental aircraft lost signal when it fall. Now we provide following methods:

(1) Sector search
(2) Extended square search
(3) Track line search
(4) Parallel line search
(5) Horizontal line search
(6) Moving rectangle search

The table 1 shows characters of those methods. We describe the practicability according to the covering range, the searching speed and accuracy and the target position it estimated.

<table>
<thead>
<tr>
<th>Searching method</th>
<th>Covering range</th>
<th>Speed</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector</td>
<td>Small</td>
<td>Slow</td>
<td>Certain</td>
</tr>
<tr>
<td>Extended square</td>
<td>Small</td>
<td>Fast</td>
<td>Right</td>
</tr>
<tr>
<td>Track line</td>
<td>Small</td>
<td>Normal</td>
<td>Accurate</td>
</tr>
<tr>
<td>Parallel line</td>
<td>Big</td>
<td>Fast</td>
<td>Accurate</td>
</tr>
<tr>
<td>Horizontal line</td>
<td>Long and narrow</td>
<td>Slow</td>
<td>Accurate</td>
</tr>
<tr>
<td>Moving rectangle</td>
<td>Small</td>
<td>Slow</td>
<td>Have errors</td>
</tr>
</tbody>
</table>

With table 1, we can use AHP (analytic hierarchy process) to analyze the advantages and disadvantages of those searching methods.

3.2 The summaries of searching methods

According to data we get, we can summarize that:

1. If the searchers need high speed, we should consider the track line search method, because the searching direction of this method is the accidental aircraft’s track line. This method can find target in the shortest time.
2. If the searchers need high accuracy, we should consider the sector searching method because the searching range of this method can cover most of the accidental range. So we can search the fragments more carefully by using this method.
3. If we can’t decide the target’s position, we should consider the parallel line search because this method can provide the biggest searching range. So these several searching members can search at the same time.

The standard of completing the search:

We can’t neglect any part of the searching range because there maybe someone needs us to rescue. We define that when we complete the whole searching area, we finish the search.

4. Result

In order to check whether the searching method and range we decide is right, we use our model to
imitate a real aircraft accident. We choose the Air France’s 447 scheduled flight in 2009 as our comparing model.

At first, we assume several searching requirements:
1. The flying height is 10 kilometers.
2. The flying speed is 450 kilometers per hour.
3. We have 10 searching planes (P-3C Orion)\(^5\)
4. The visibility in the process of searching is 10 kilometers
5. We use the method of parallel line searching
6. In order to guarantee the result, we keep the separate distance as 1 kilometer.

The searching area of AF447 is a circle with a radius of 73 kilometers. We get that the searching radius is about 80 kilometers when we use the AF447’s data. The result is acceptable because the control end received the signal from the aircraft. They can estimate the process of the plane accurately. So the error is acceptable \(^6\).

According to our model, the searching area is about 12077 square kilometers. We need about 12 days to complete the search.

5. Conclusion

The models in our model can help people find the lost plane in a comprehensive way. Our model can estimate the whole process of falling. We can also calculate the swerve radius of the accidental plane to get the maximum possible falling area. We forecast the probability. Considering the important influence of the ocean current, we can forecast the possible moving direction with the climate, weather and other geography conditions. Then we choose the best searching method according to different probability position, different climate condition and different requiring accuracy. We hope that our model can help those searchers.

Reference


