Investigation of temperature field in the water

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Abstract. set up a dynamic model of adding hot water and divide it into two elementary models. Use the heat convection minimum entropy generation to prove that put the faucet at the bottom of the bathtub is good way for saving energy.

Introduction

The energy taken by hot water not only heats the water to maintain the initial temperature but heat the surface of human body. The dynamic model of heating the water divided into the lumped parameter model that human body absorb heat from the hot water and the varied heat quantity conductive process.

Assumptions

1. The shape and volume will not change when the temperature change.
2. The temperature of the surface of human body only determined by around temperature. Ignore the heat that created by the cell aerobic respiration.
3. Overflow when the height of the surface of the water overtops the height of the edge of the bathtub wall. Water is viscous fluid; the practical height of over flow is a little higher than the height of the bathtub wall.
4. Ignore the body's influence when people keep still in water. Convective heat transfer between water and air is very slow, the temperature difference between skin and surrounding water is small. But the velocity of the rising of the temperature is fast, so we cannot ignore body's effect.
5. Ignore evaporation.

Dynamic model of water heating process

Proof of the location of the hot water faucet

Entropy is the state parameter derived from the second law of thermodynamics, and the definition formula is as follows:

$$dS = \frac{\delta Q_{rev}}{T_r} = \frac{\delta Q_{rev}}{T}$$

In the formula, δQ_{rev} indicates the reversible process for energy transfer, and T_r is the heat source temperature.

Closed system within the irreversibility in the adiabatic processes entropy in the reason for the increase is due to the process of irreversible factors by the dissipation effect, the loss of mechanical power transformed into thermal energy (heat) consumption is refrigerant absorption. This part of the dissipation of entropy increment called entropy generation.

In the process of convective heat transfer of an incompressible fluid, the entropy of the fluid in the unit volume can be expressed as[1]:

$$S'''_{gen} = \frac{k}{T^2}(\frac{\partial T}{\partial x_i})^2 + \frac{\Phi}{T}$$

In the formula, k is thermal conductivity $(W/(m^2 \cdot k))$, T is temperature, x_i is Cartesian coordinate, Φ represents viscous dissipation in the process of fluid flow, the first item on the right of the formula (6) represents the entropy produced by the fluid heat transfer, and the second represents the entropy produced by the fluid flow.

The faucet's position is closer to the bottom surface, the higher the water temperature before heating. After the convective heat transfer, the higher the temperature, the smaller the entropy production, which means the less useful energy loss of the system, the higher the residual energy of the system.

Assume the time of the $i\ th$ time add water is t_{1i}, the variation of the height of the water surface is Δh_i, the temperature of the water surface after heating is T_{fi}.When adding hot water, consider the hot water flow is trickle and the temperature is T_{hw},water flow into the bathtub via the faucet, the sectional area of the faucet is very small compared with the area if the area of the surface of the water in the bathtub, so it has little influence to the flow state of water in the bathtub. Consider the intrant hot water as a thermal flow whose density of heat flow rate is $q\ J/(m^2 \cdot s)$, so the change of the water temperature field is caused by the heat flow. And add certain mass water (mass flow rate q_m, temperature T_{hw}) at the same time. The heat come with the water divided into two parts, act on water in the bathtub and human body respectively. Than establish two models to solve these two processes, hot water heat the water in the hub and hot water heat the body, respectively. Assume the height of the surface of the water is ascent momentary and solve the problem by the initial 3D thermal conduction model. So transfer the dynamic process of adding water into two dynamic processes and a static state.

According to the conservation of energy, the heat flow is absorbed by water and human body:

$$Q_{total} = Q_{hotwater} + Q_{body}$$

Q_{total} means the heat offered by the intrant water, $Q_{hotwater}$ means water absorb the heat to rise the temperature , Q_{body} means body absorb the heat for rising the temperature of skin surface.

Temperature rise on the surface of body

The process that surface of the human body heated by hot water is an unsteady heat conduction process. Skin of human exchange heat with water by convective heat transfer, so the temperature of the skin rise. Now we analyze the law how average temperature of skin changed with time by umped parameter model. The temperature of human body surface means the temperature of skin, subcutaneous tissue, muscle, etc. In fact, each of the parts of body has different temperature, so we use the temperature of skin to show the temperature of body surface only.

Here, we ignore the variation of the volume which changed with the ascent of the water surface, which means the volume human kept in water is a constant. The volume of human in water is V_{body}, the superficial area is S_{body}, the destiny of human body is ρ_{body}, specific heat capacity is c_{body}. The temperature of water before adding water is T_l (each time the temperature before add water is different). As for human body, the process can be seen as take the body from an initial environment temperature (T_l) into an environment temperature (T_{hw}).Obviously, $T_{hw} > T_l$, the surface heat transfer coefficient of the convective heat transfer between human body surface and water. We only consider convective heat transfer on the boundary. Heat flow expressed by Newton cooling formula:

$$l_{body} c_{body} V_{body} \frac{dT}{dt} = h_{watertobody} A_{body} (T_{hw} - T_l)$$

Simplify:

$$T_l = T_{hw} - c_0 \exp(-\frac{h_{watertobody} A_{body}}{l_{body} c_{body} v_{body}})$$

Boundary conditions: $t = 0, T = T_l$

Put the boundary conditions into (9), get the body temperature function of time t:

$$T_{body} = T_{hw} - (T_{hw} - T_l) \exp(-\frac{h_{watertobody} A_{body}}{l_{body} c_{body} v_{body}} t)$$

Heat be absorbed by human body:

$$Q_{body} = \int_{\Delta t} T_{body} dt$$

Water heating

The part of the heat flux q that is acting on the water is q_{water}, the heating process of water, in fact, is also a heat conduction process. The objective function and the initial three-dimensional heat conduction equation (1) are the same, just boundary condition is changed, namely heat conduction equation as follows:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

The boundary conditions are as follows:

$$\begin{cases} z = 0, -\lambda \dfrac{\partial T}{\partial z} = q_{water} \\ \qquad t_{1i} \end{cases}$$

Among them, t_{1i} is said to be heating during that time.

According to the calculation formula of heat, the following formula was established:

$$c q_m t (T_{hw} - \xi T_0) = q_{water} s t + Q_{body}$$

area of the

Among them, q_{water} indicates the effect of heat flux on water, s represents the bottom bathtub, T_{hw} express hot water temperature, ξ is a correction factor for the initial water temperature. ξ is to give the initial water temperature of a correction factor, the value of the closer to 1, the better(the correction coefficient is different each time adding water), which meet the subject requirements as far as possible close to the initial water temperature. According to the formula, it can be concluded:

$$q = \frac{c q_m (T_m - \xi T_0)}{s}$$

Summary

Complex heat and mass transfer process are simplified into several single processes, which is easily to be analyzed and understood as well as solved. We use rationable assumptionassumptions to make models simpler. In the essay, we set up several models to determine the best location of the faucet and analyse the rise of the temperature on human body. And finally we can get the conclusion about q: $q = \frac{cq_m(T_m - \xi T_0)}{s}$ and the heat absorbed by human body : $Q_{body} = \int_{\Delta t} T_{body} dt$

References

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