Research on the temperature of the bathtub water

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Abstract. When a person fills a bathtub with hot water and settles into the bathtub, the bath gets noticeably cooler. In order to study the temperature even throughout the bathtub and as close as possible to the initial temperature, we need develop a model of the temperature of the bathtub water in space and time. First, we figure out the heat loss, then replace three-dimensional partial differential equations with two two-dimensional equations. Based on the finite difference method, we make time domain and space domain discrete, list discrete node equations by using heat balance method and solve discrete node equations.

Introduction

We are discussing the problem basing on the problem A in MCM 2016. Firstly, both the person and the tub could effect the temperature, we simplify the motions made by the person with rolling motions. Secondly, counting various heat loss, by finite difference method, we build and solve two two-dimensional temperature models of different boundary conditions. Last, the results show the distributions of temperature are pretty proper, which proves the rationality of the two models.

Developing the mathematical model

On the basis of the fundamental of heat transfer, the three-dimensional partial differential equation of conduction heat transfer [1], which includes the third class boundary condition, is easily to be developed.

\[
\frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right)
\]

Due to the complexity of the solution of the three-dimensional partial differential equation, we replace it with two two-dimensional partial differential equations, which include the third class boundary conditions.

\[
equation I \quad \frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right)
\]
boundary conditions
\[
-\lambda_1 \left( \frac{\partial t}{\partial n} \right) = h_1 (t_1 - t_{w1}) + w_1 q_3
\]
\[
-\lambda_2 \left( \frac{\partial t}{\partial n} \right) = \frac{Q_2}{S} + w_1 q_3
\]

\[
equation II \quad \frac{\partial t}{\partial \tau} = a \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right)
\]
boundary conditions
\[
-\lambda_1 \left( \frac{\partial t}{\partial n} \right) = h_1 (t_1 - t_{w1}) + w_1 q_3
\]

By now, our work is to calculate the value of \( t_{w1}, h_1, Q_2, Q_3, S, a, \lambda_1, \lambda_2 \). During the research, we assume the movement of the person in the bathtub is the same as rolling motion. The intensity of motion made by the person can be expressed in \( \theta = \), which effects heat transfer coefficient of te
forced convection heat transfer by changing \( N_{u,w} \).

\[
N_{u,w} = 0.01526R_e^{0.082315}P_r^{0.4} \left( \frac{\nu}{uT_0} \right)^{-0.08614} \theta_m^{0.10369}
\]

Here we need make a hypothesis: Temperature of inside wall of tub \( t_{w1} = 39.78^\circ C \).

**Prove the following:**

There is a fundamental theorem\(^3\) saying that the heat leaking out into atmosphere through the bathtub is equal to the convective heat in the numerical value. We replace the leaking heat with \( q_1 \), and replace the convective heat with \( q_2 \).

\[
t_{w1} = \frac{t_w + t_i}{2}, \quad R_e = \frac{u \times r}{\nu}, \quad N_{u,w} = 0.01526R_e^{0.082315}P_r^{0.4} \left( \frac{\nu}{uT_0} \right)^{-0.08614} \theta_m^{0.10369} \quad h_1 = \frac{\lambda}{l} N_{u,w},
\]

\[
q_1 = h_1 \times (t_i - t_{w1}), \quad \lambda_1 = 0.6265 \quad \nu = 7.32 \times 10^{-7} \left( m^2 / s \right) \quad P_r = 4.865
\]

\[
\alpha_r = 3.46 \times 10^{-4} (K^{-1}) \quad \text{and} \quad R_e = 364188.16.
\]

Then, assuming \( u = 0.3 \text{ m/s} \), \( T_0 = 20 \text{s} \), \( \theta_m = 15 \)

\[
r = 0.4 \text{ m}, \quad \text{we work out} \quad N_{u,w} = 261.92, \quad h_1 = 207.90, \quad q_1 = 45.738.
\]

Similarly, the corresponding equations of calculating \( q_2 \) are

\[
t_{w2} = \frac{t_w + t_i}{2}, \quad t_{w2} = t_{w1} - \frac{q_2 \delta h}{\lambda}, \quad G_r = \frac{g \alpha_r \Delta t^3}{\nu^2}, \quad N_u = C(G_r, P_r)^n = CRa^n, \quad h_2 = \frac{\lambda}{l} N_u, \quad q_2 = \frac{t_i - t_{w1}}{1 + \frac{1}{h_1} + \frac{\delta}{h_2} + \frac{\lambda}{\lambda}}.
\]

In the same manner, the result is obtained

\[
\lambda_2 = 0.02588, \quad P_r = 0.7282, \quad \nu = 1.608 \times 10^{-5} \left( m^2 / s \right), \quad N_u = 108.51, \quad G_r = 1.96 \times 10^9, \quad \delta_0 = 0.3, \quad h_2 = 4.01, \quad q_2 = 44.47.
\]

Comparing \( q_1 \) with \( q_2 \), we found there is no significant difference between them. We can come to the conclusion that the hypothesis is correct \( t_{w1} = 39.78^\circ C \). Besides, \( h_1 = 207.90 \).

\[
Q_2 = \alpha \times y \times (0.0174\nu_f + 0.0229) \times (P_b - P_q) \times 760S \times \frac{B}{B}
\]

The formula is used to calculate the heat loss \( Q_2 \) of the water evaporation, which leaks out into atmosphere. the temperature of bathtub water is 40 degrees, \( \alpha = 4.1868/\text{kcal} \), \( y = 574.2/\text{kcal/kg} \), \( \nu_f = 0.3/\text{m/s} \), \( P_b = 40\text{mmHg} \), \( P_q = 15.6\text{mmHg} \), \( B = 761.84\text{mmHg} \), \( S = 1.36m^2 \), we can get \( Q_2 = 621.5919W \). The method of calculating \( Q_1 \) is the same as above. We can reckon \( q_3 = 1414.52 \) by the method of calculating \( q_1 \), we have calculated \( \lambda_1 = 0.6265 \), \( \lambda_2 = 0.02588 \), \( h_1 = 207.90 \), \( h_2 = 4.01 \). With the formula \( a = \frac{\lambda}{\rho e} \), the final mathematical model can be described as

**equation I**

\[
\frac{\partial t}{\partial \tau} = 0.000151276 \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right)
\]

**boundary conditions**

\[
\begin{cases}
-0.6265(\frac{\partial t}{\partial n}) = 207.9(40 - 39.78) + 0.9 \times 1414.52 \\
-0.02588(\frac{\partial t}{\partial n}) = \frac{621.5919}{1.36} + 0.1 \times 1414.52
\end{cases}
\]

**equation II**

\[
\frac{\partial t}{\partial \tau} = 0.000151276 \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right)
\]

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boundary conditions \(-0.6265\frac{\partial t}{\partial n} = 207.9(40 - 39.78) + 0.9 \times 1414.52\)

**Conclusion**

On the basis of finite difference method, we need to convert the derivative terms of the mathematical model to a differential expression. In accordance with the energy conservation equation, the discrete equations of internal nodes are listed by the method of heat balance.

\[
\frac{\lambda_{m+1,n} - t_{m,n}}{\Delta x} + \frac{\lambda_{m+1,n} - t_{m,n}}{\Delta x} + \frac{\lambda_{m,n+1} - t_{m,n}}{\Delta y} + \frac{\lambda_{m,n+1} - t_{m,n}}{\Delta y} = \rho \Delta x \Delta y \frac{t_{m+1,n} - t_{m,n}}{\Delta t}
\]

According to the Newton’s iteration method, with the value of node \(m\) and its neighboring nodes at some moment, we can reckon the value of node \(m\).

By using Matlab, in the case of selecting \(\Delta x = 0.0312, \Delta y = 0.0378, \Delta t = 0.1\), at the moment of \(t_1 = 11\text{min}\), the temperature distributions of the two-dimensional plane I and plane II can be simulated.

![Figure 1. The Temperature Distributions of the Two-dimensional Plane I](image1)

![Figure 2. The Temperature Distributions of the Two-dimensional Plane II](image2)

The two-dimensional temperature distribution can reflect the temperature changes in the three-dimensional space reasonably.

**References**

