# PID Control Parameters Design Analysis for First Order System

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**Abstract.** The PID controller design and parameter choosing problem are analyzed in detail. Especially, seven good conclusions are proposed for the choosing of coefficient of PID controller design for a first order system. Also single P control and PD control effect and the reason of steady state error are studied and calculation of steady state error is proposed.

## Introduction

PID control is the most widely used control law at present[1-4]. It is loved by the majority of engineers because it has a good universality, and also the demand for the precise knowledge of the model is not high. But it is worth mentioning the fact that the PID control is applied for a hundred years, but only few people tried to prove its stability[5-8]. Because of the difficulty of theoretical analysis, the research of the theory is less, so the parameters of the test is basically rely on the experience of the engineer, or rely on multiple simulation to select the optimal performance. Based on the above reasons, this paper is one of the few to try. In this paper, the stability analysis of P control, PD control and PID control, as well as the ability to resist the disturbance is analyzed. Finally, the appropriate interval of parameter selection is obtained.

## Model Description

To make the below research easy to read, a kind of first order system is chosen as a research object as following:

$$\dot{x} = f(x) + bu + F \tag{1}$$

Where the control object is to design a PID controller such that the control law can make the system state $x$ track the desired value $x^d$ under the existence of constant disturbance and analyze the system performance with PID control and disturbance.

## Analysis Of Steady State Error Of P Control Caused by Constant Disturbance

First, we discuss the linear system situation, then $f(x) = ax$. And define error variable as $e = x - x^d$ and design the PID control law as

$$u = k_p e + k_i \int edt + k_d \dot{e} \tag{2}$$

And substitute it the system model then it holds

$$\dot{e} = f(x) + bk_p e + bk_i \int edt + bk_d \dot{e} + F - \dot{x}^d \tag{3}$$

Without loss of generality, assume the desired value as a constant and $b = 1$, then it holds

$$\dot{e} = f(x) + k_p e + k_i \int edt + k_d \dot{e} + F \tag{4}$$

Then it can be written as

$$\dot{e} = ae + k_p e + k_i \int edt + k_d \dot{e} + F + ax^d \tag{5}$$

First , the integral item is neglected, then the integral coefficient is set as 0, then it holds:

$$\dot{e} = K_w e + F_w \tag{6}$$

 

where $K_w = (a + k_p)$ , $F_w = F + ax^d$ . Then the above system is stable if $K_w < 0$. But there is a steady state error caused which can be calculated as

$$e_{ss} = -F_w / K_w \tag{7}$$

Then the ratio between steady state error and input signal can be written as

$$\frac{e_{ss}}{x^d} = -\frac{F + ax^d}{x^d} = -\frac{F}{x^d} - a \tag{8}$$

Then below three conclusions can be made as following:

Conclusion 1: The constant disturbance will not affect the stability of the system but it will increase the steady state error.

Conclusion 2: if the input signal is increased, then the steady error will increase.

Conclusion 3: if the input signal is increased, then the ration between steady state error and input will decrease.

**Analysis Of Steady State Error of PD Control Caused By Constant Disturbance**

Second, we consider the control effect of the differential item, then $k_d \neq 0$ ,so it holds

$$(1 - k_d)\dot{e} = K_w e + F_w \tag{9}$$

Then the system can be written as

$$\dot{e} = \frac{K_w}{(1 - k_d)} e + \frac{K_w}{(1 - k_d)} F_w \tag{10}$$

So a conclusion can be made as following:

Conclusion 4: if the system is stable, then $\frac{K_w}{(1 - k_d)} < 0$ should be satisfied.

Consider the system should be stable under both PD control and P control, then the below condition should be satisfied $(1 - k_d) > 0$, it also can be written as $k_d < 1$.

So we can also have below conclusion 5 and conclusion 6 for PD controller design for first order linear system.

Conclusion 5: The steady state error of system will not change and it can be written as

$$e_{ss} = -F_w / K_w \tag{11}$$

Conclusion 6: if $0 < k_d < 1$, then the convergence speed of system will be increased, and if $k_d < 0$, then the convergence speed will be slower than P control, which is like the situation of systems with more damping ratio.

**Analysis Of Choosing Interval Of PID Control Parameters**

Third, we consider the situation that all coefficients are not zero then all items of PID control are effective. Then the system can be written as below second order system as follows:

Define a new variable as $o = \int edt$ , and the system can be written as

$$\dot{o} = e \tag{12}$$

$$\dot{e} = \frac{K_w}{1 - k_d} e + \frac{k_i}{1 - k_d} o + \frac{F_w}{1 - k_d} \tag{13}$$

Assume the system is stable by design proper control parameters as

$$\begin{bmatrix} 0 & 1 \\ \frac{k_i}{1 - k_d} & \frac{K_w}{1 - k_d} \end{bmatrix} \tag{14}$$

And also consider that the system should be stable with P control and PD control, then the parameter should be choose in the below interval

$$k_d < 1,\quad K_w < 0 \tag{15}$$

And the above system can be transformed as a differential equation as bellows:

$$1\ddot{o} - \frac{K_w}{1-k_d}\dot{o} - \frac{k_i}{1-k_d}o - \frac{F_w}{1-k_d} = 0 \tag{16}$$

Then the system is stable if the below conditions are satisfied:

$$\frac{K_w}{2(1-k_d)} < 0,\quad -\frac{k_i}{1-k_d} > 0 \tag{17}$$

Then the system is stable if $k_i < 0, k_w < 0$. So we can have conclusion 7 as follows:

Conclusion 7: Any first order system can always be stable if the PID control coefficients are set as a proper P coefficient and negative I coefficient and D coefficient smaller than 1, and the system will has no steady state error.

## Conclusion

In this paper, seven good conclusions are given for the PID parameters choosing problem and steady state error analysis which is very useful for engineers since PID parameters choosing intervals are given by using Lyapunov stability theory. Also the performance of anti-disturbance ability for system with PID controller and constant disturbance is analyzed according to three kinds of different situation. So this paper is very useful not only for beginners and also for experienced engineers since its novel theoretical analysis method.

## References


[1]Buschek H and Calise A J. Hypersonic flight control system design using fixed order robust controllers[A]. In:AIAA Guidance, Navigation and Control Conference[C]. Chattanooga, TN, AIAA 95-6062

[2]Buschek H, Calise A J. Uncertainty modeling and fixed-order controller design for a hypersonicvehicle model. Journal of Guidance, Control and Dynamics, 1997, 20(1):42-48.

[3]Heller M and Sachs G. Flight dynamics and robust control of a hypersonic test vehicle with ramjet propulsion[A]. In:AIAA International Space Planes and Hypersonic Systems and Technologies Conference[C]. Norfolk, VA, AIAA 98-1521

[4]M. Ohno, Y. Yamaguchi, T. Hata, M. Takahama, Robust flight control law design for an automatic landing flight experiment [J], Control Engineering Practice, 1999, 7(9):1143-1152

[5]Naidu S D, Banda S. S. and Buffington J L. Unified approach to $\mathrm{H}_2$ and $\mathrm{H}_\infty$ optimal control of a hypersonic vehicle[A]. In:Proceedings of the American control conference[C]. San Diego, California,1999,vol.4:2737-2741.

[6]Christopher I. Marrison. Synthesis of robust control systems for a hypersonic aircraft decision and contro1. IEEE, 1994.4:3324-3329

[7]E. Mooij. Linear quadratic regulator reentry control-performance assessment using a taguchi approach. AIAA, international space planes and hypersonic systems and technologies conference. 1998:665-677

[8]Kevin.P. Groves, Andrea. Serranti, Stephen. Yurkovich. Anti-Windup Control for an Air-breathing Hypersonic Vehicle Model. AIAA, 2006-6557:1-14.