A Novel Approach to Dynamic Output-Feedback Control for Interval Positive Systems

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Abstract. In this paper, a novel approach to dynamic output-feedback control is established for positive systems with interval uncertainties. Necessary and sufficient conditions for the existence of robust controllers are established.

Introduction

Positive systems exist in many branches such as industrial engineering, ecology [1-2]. Positive systems are special because they are defined on cones. Correspondingly, many methods for general systems cannot be used for positive systems. In recent years, many interesting results have appeared in the literature. For example, the problem of reachability and controllability have been studied for positive systems in [3-4]. Results on the stability for positive dynamic systems with delay can be found in [5–6]. The analysis and synthesis problems for 2-D positive systems have been investigated in [7]. A positive representation of a transfer function has been characterized in [8]. The problem of controller design has been dealt with for positive systems by the linear programming approach in [9]. Conditions for positive realizability have been proposed in terms of convex analysis in [10].

As is well known, many practical systems are usually influenced by variations or environmental changes and therefore there are uncertainties in system parameters. The parameter uncertainties lead to the complexity in the study of positive systems. Because of the complexity, the controller design problems for positive systems with interval uncertainties have not been fully investigated. Moreover, compared with state-feedback controllers, it is difficult to design output-feedback ones. The reason is that the problem becomes not convex. In all, for positive systems with interval uncertainties, the dynamic output-feedback controller design problem deserves to be tackled.

In this paper, the stabilizing dynamic output-feedback stabilization problem is investigated for interval positive systems. In detail, the aim of this paper is to design a dynamic output-feedback controller for a given positive system to ensure that the closed-loop system is robustly stable with the L_1-induced performance.

Preliminaries

First, some notations and results about positive systems are introduced.

$A \geq\geq 0$ means that for all i and j, $a_{ij} \geq 0$. The L_1-norm of x is defined as $\|x\|_{L_1} = \int \|x(t)\|_1 dt$. $\mathrm{R}^{n\times m}$ is the set of all real matrices of dimension $n \times m$. $A >> 0$ means that $a_{ij} > 0$. $\|\cdot\|$ denotes the Euclidean norm for vectors. For matrices $A, \underline{A}, A \in \mathrm{R}^{n\times m}$, $A \in [\underline{A}, A]$ equals to that $\underline{A} \leq\leq$

$A \leq\leq A$. The induced 1-norm of a matrix $Q = [qij\,] \in \mathrm{R}^{m\times n}$ is represented by $\|Q\|1 = \max(\sum |qij\,|)$.

In the section, we consider the positive interval system S_I :

$$S_I \quad \begin{aligned} \dot{x}(t) &= Ax(t) + B_u(t) + B_w w(t), \\ z(t) &= C_z x(t) + D_{zu}(t) + D_{zw} w(t) \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

Here, the system matrices A, B_w, C_z, D_{zw} and C belong to the domain:

$$A \in \left[\underline{A}, \overline{A}\right],\ B_w \in \left[\underline{B_w}, \overline{B_w}\right],\ Cz \in \left[\underline{C_z}, \overline{C_z}\right],\ Dzw \in \left[\underline{D_{zw}}, \overline{D_{zw}}\right],\ C \in \left[\underline{C}, \overline{C}\right]$$

Then, we introduce the following definition.

Definition 1 System (1) is positive if $x(t) \geq\geq 0$, $z(t) \geq\geq 0$ and $y(t) \geq\geq 0$ always hold for $x(0) \geq\geq 0$ and $w(t) \geq\geq 0$.

Next, we present some lemmas for positive systems and these results will be used later.

Lemma 1 The interval system (1) is positive if and only if A is Metzler, B_w , C_z, D_{zw}, *and C are all nonnegative.*

Lemma 2 Positive system (1) is stable if and only if there exists a nonnegative vector p satisfying:

$$p^T A \leq\leq 0.$$

Now, the definition of L_1-induced norm is proposed.
For a stable positive system S_I, it has L_1-induced performance at the level γ if we have

$$\|z\|_{L_1} < \gamma \|w\|_{L_1} \tag{2}$$

with zero initial conditions and here $\gamma > 0$ is given.

Main Results

In this section, the dynamic output-feedback stabilization controller is designed for interval positive systems. The general controller structure under consideration is of the form

$$\begin{cases} \dot{\hat{x}}(t) = F\hat{x}(t) + Gy(t) \\ u(t) = K\hat{x}(t) \end{cases} \tag{3}$$

Here, $F \in R^{n\times n}, G \in R^{n\times r}$, and $K \in R^{p\times n}$ are the controller matrices to be designed.

Define $e = x(t) - \hat{x}(t)$ *and* $\xi(t) = \left[x^T(t) e^T(t)\right]^T$, then the augmented system can be described by

$$\begin{cases} \dot{\xi}(t) = A_{\xi}\xi(t) + B_{\xi w}w(t), \\ z(t) = C_{\xi}\xi(t) + D_{\xi w}w(t), \end{cases} \tag{4}$$

where

$$A_{\xi} = \begin{bmatrix} A+BK & -BK \\ A-GC+BK-F & F-BK \end{bmatrix}, B_{\xi} = \begin{bmatrix} B_w \\ B_w \end{bmatrix},$$
$$C_{\xi} = \begin{bmatrix} C_z + D_z K & -D_z K \end{bmatrix},\ D\xi_w = D_{zw}.$$

Then, the dynamic output-feedback controller design (DOFCD) problem is formulated as follows.

Problem DOFCD: *Find a dynamic output-feedback controller (3) with Metzler* F , $G \geq\geq 0$ *and* $K \leq\leq 0$ *such that the system (4) is robustly stable, positive, and satisfies the performance in (2) under* zero initial conditions.

In this section, the specification $e(t) \geq 0$ facilitates the design of the dynamic output-feedback controller for interval positive systems, which may bring about some conservatism. However, the positive restriction on the error signals $e(t)$ will not affect that of the estimation $\hat{x}(t)$. Then, we establish the conditions to Problem DOFCD in the following.

Theorem 1

Suppose F is Metzler, $G \geq\geq 0$, $K \leq\leq 0$ and given S_I positive. The closed-loop system (4) is robustly stable, positive and satisfies $\|z\|_{L_1} < \gamma \|w\|_{L_1}$ for any $A \in [\underline{A}, \overline{A}]$, $B \in [\underline{B}, \overline{B}]$, $B_w \in \left[\underline{B}_w, \overline{B}_w\right], C_z \in \left[\underline{C}_z, \overline{C}_z\right], D_z \in \left[\underline{D}_z, \overline{D}_z\right], D_{zw} \in \left[\underline{D}_{zw}, \overline{D}_{zw}\right]$ *and* $C \in [\underline{C}, \overline{C}]$ if and only if there exist vectors $p_1 \geq\geq 0$, $p_2 \geq\geq 0$ satisfying

$$\underline{A} + \overline{B}K \text{ is Metzler}, \tag{5}$$

$$\underline{A} - G\overline{C} + \overline{B}K - F \geq\geq 0, \tag{6}$$

$$\underline{C}_z + \overline{D}_z K \geq\geq 0, \tag{7}$$

$$\mathbf{1}^T\left(\overline{C}_z + \underline{D}_z K\right) + p_1^T(\overline{A} + \underline{B}K)$$
$$+ p_2^T(\overline{A} - G\underline{C} + \underline{B}K - F) << 0, \tag{8}$$

$$-\mathbf{1}^T\ \overline{D}_z K - p_1^T\ \overline{B}K + p_2^T\left(F - \overline{B}K\right) << 0, \tag{9}$$

$$p_1^T\overline{B}_w + p_2^T\overline{B}_w + \mathbf{1}^T\overline{D}_{zw} - \gamma\mathbf{1}^T << 0. \tag{10}$$

Proof: Combining (5)–(7) with $G \geq\geq 0$, $K \leq\leq 0$ yields the following: for any $A \in [\underline{A}, \overline{A}]$, $B \in [\underline{B}, \overline{B}]$, $B_w \in \left[\underline{B}_w, \overline{B}_w\right], C_z \in \left[\underline{C}_z, \overline{C}_z\right], D_z \in \left[\underline{D}_z, \overline{D}_z\right], D_{zw} \in \left[\underline{D}_{zw}, \overline{D}_{zw}\right]$ *and* $C \in [\underline{C}, \overline{C}]$,

$$A + BK \geq\geq \underline{A} + \overline{B}K \text{ is Metzler},$$

$$A - GC + BK - F \geq\geq \underline{A} - G\overline{C} + \overline{B}K - F \geq\geq 0\ ,$$

$$C_z + DK \geq\geq \underline{C}_z + \overline{D}_z K \geq\geq 0\ , \tag{11}$$

which, together with $K \leq\leq 0$, imply

$$A_{\xi} = \begin{bmatrix} A+BK & -BK \\ A-GC+BK-F & F-BK \end{bmatrix} is\ Metzler,$$

$$B_{\xi} = \begin{bmatrix} B_w \\ B_w \end{bmatrix} \geq\geq 0,$$

$$C_{\xi} = \begin{bmatrix} C_z + D_z K & -D_z K \end{bmatrix} \geq\geq 0, D_{\xi w} = D_{zw} \geq\geq 0.$$

Consequently, the closed-loop system (4) is positive.

In addition, from (8)–(10), we have

$$\begin{aligned} & 1^T \left[\bar{C}_z + \underline{D}_z K - \bar{D}_z K \right] \\ & + \left[p_1^T \ p_2^T \right] \begin{bmatrix} \bar{A} + \underline{B}K - \bar{B}K & \\ \bar{A} - G\underline{C} + \underline{B}K - F & F - \bar{B}K \end{bmatrix} << 0, \\ & p_1^T \ p_2^T \begin{bmatrix} \bar{B}_w \\ \bar{B}_w \end{bmatrix} + 1^T \bar{D}_{\xi w} - \gamma 1^T << 0, \end{aligned}$$

which implies that the closed-loop system (4) is robustly stable and satisfies $\|z\|_{L_1} < \gamma \|w\|_{L_1}$. This proves sufficiency.

The necessity can be obtained by reversing the above procedure and the proof is omitted here.

Remark

It is noted that the inequalities in (11) do not necessarily hold if the assumption $K \leq\leq 0$ is not satisfied. It brings about difficulty in the controller design for interval positive system. The non-positiveness assumption on K makes the controller synthesis problems become easier. When B and D_z are exactly known, we can remove the constraint $K \leq\leq 0$ for this special case. Moreover, it needs further study to design a sign-indefinite K and it remains unsolved.

Summary

In this paper, the dynamic output-feedback controller synthesis problem has been studied for interval continuous positive systems under L_1-induced performance. Necessary and sufficient conditions for the existence of a desired robust controller have been established.

References

[1]. H. Caswell, *Matrix Population Models: Construction, Analysis and Interpretation*. (Sunderland, MA: Sinauer Assoc.,2001)

[2]. L. Caccetta, L.R. Foulds, V.G. Rumchev, A positive linear discrete-time model of capacity planning and its controllability properties. Math. Comput. Model. 40 (1–2), 217–226 (2004)

[3]. M.P. Fanti, B. Maione, B. Turchiano, Controllability of multi-input positive discrete-time systems. Int. J. Control 51 (6), 1295–1308 (1990)

[4]. M.E. Valcher, Controllability and reachability criteria for discrete time positive systems. Int. J. Control 65 (3), 511–536 (1996)

[5]. W. Haddad, V. Chellaboina, T. Rajpurohit, Dissipativity theory for nonnegative and compartmental dynamical systems with time delay. IEEE Trans. Automat. Control 49 (5), 747–751 (2004)

[6]. W. Haddad, V. Chellaboina, Stability theory for nonnegative and compartmental dynamical systems with time delay. Syst. & Control Lett. 51 (5), 355–361 (2004)

[7]. L. Benvenuti, L. Farina, A tutorial on the positive realization problem. IEEE Trans. Automat. Control 49 (5), 651–664 (2004)

[8]. L. Farina, On the existence of a positive realization. Syst. & Control Lett. 28 (4), 219–226 (1996)

[9]. H. Gao, J. Lam, C. Wang, S. Xu, Control for stability and positivity: equivalent conditions and computation. IEEE Trans. Circuits and Systems (II) 52 (9), 540–544 (2005)

[10]. M. Ait Rami, F. Tadeo, Controller synthesis for positive linear systems with bounded controls. IEEE Trans. Circuits and Systems (II) 54 (2), 151–155 (2007)