Mathematical Principle on Medical Apparatus of Imaging-diagnose

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Keywords: Medical aqpparatus, Imaging-diagnose, Mathematical porinciple, Chromatography

Abstract. Human thirst for exploring their body's internal structure for a long time, but it was the invention of CT and MRI technology which appeared until in 1970, that made the desire of people exploring the internal data of their bodies come true. And the basic method so-called chromatography imaging rely on that series of 2-dimension CT image or MRI image reorder three dimensional human structures, thus realized human visible on exploring the internal structure of it's body. Chromatography imaging technology, which is acquiring the plane images of objects by cutting the object with a group parallel planes. Firstly, getting each parallel's images of these objects, secondly, superimposing these graphic images, then 3-dimension stereo image of the object would be measured. This paper is intended to introduces mathematical principles on medical apparatus of imaging-diagnose, so as to help people understand "In essential, 'the high-technology' is a kind of mathematical technique" profoundly.

Introduction

Humans thirst for exploring their body's internal structure for a long time. While even though they want to know the image of object in their body in many situations, only method is measurement by non-directly, which is to detect objects by invisible ray, or explain by radiation of object. The information of non-directly measurement associated with image needed with some methods, its general purposes is processing these information, so as to form the intuitive image of measured objects, that said image reorder technology, thus people get on some understanding of the observe objects, the system using such technique is known as chromatography.

Modern medical science and clinical diagnosis need variety of medical images, such as pathology slice image, x-ray perspective image, X-CT scan image, g-ray of nuclear medical image, electromagnetic waves of magnetic resonance scan image(MRI), general B-type ultrasonic scan image, colored Dopple ultrasound wave scan image, infra-red hot imaging instrument image, and various electronic endoscope image. Using the light, sound, electric, and magnetic, etc. Various physical interactions mechanism, matches chemical staining agent or contrast agent, and by computer hardware and software technology, changes medical image diagnosis device, makes sure the medical diagnostic visibility [1].

In 1963, Cormack A. M. first set calculation method of multi-directional project which re-built slip images. In 1968, Hounsfield G. N. checked cranial CT device successfully [2]. Lucky, at the end of 1970's microcomputer image processing system got development, soon medical x-ray formed an industry, the progress medical imaging diagnostics from two-dimensional to three-dimensional imaging got the epoch-making significance.

We knows that slips can be used to understand bio organization and organ, for example, cutting staining sample into 1mm of slices, to observe the organization form structure of cross section under microscope. If cut sample parallel succession slice into dozens of, or hundreds of slices, we can observe each of them in proper sequence. According to the parallel slice digital image gotten by taking photos and sampling, then using computer to rebuild the three dimensional form of this organization or organ accurately. Here often call such plane figure as section figure.

Work Mode of Apparatus

Initially x-ray was used in medical imaging device. So the x-ray will be as example in the following discuss.

Basic problem of drawing computer section figure in medical is how to show human section figure of organization in computer terminal screen by many of rays located in same section but different direction with collection and recovery of information, the name of chromatography imaging system is Computer-Aided-Tomography-scanner, abbreviations for CAT-scanner, colloquially write as CT- scanning in 1971 the first commercial system for medical diagnose was born in the United Kingdom [3].

Usually the chest x-perspective of people, is parallel of x-ray beam vertical through human again projection to screen, due to each body part on ray absorption capacity is different, screen graphics displays out black and white brightness will be different. But this graphics only reflects infinite parallel slices of human organization of superimposed or average that perpendicular to the ray direction, it cannot show the space distribution of human organization.

X-ray section figure is different to perspective, it draws out this section figure of human organization structure according to tens of thousands and hundreds of thousands of very fine different ray located in the detection section, it utilize the change of ray strength through the measured objects. We knows the initial strength of all rays, measure each ray strength after through the measuring objects, and the results will transfer into computer, x-ray has two species to scan detection section, one is parallel way, the other is fan way, they respectively shows as Fig. 1 and Fig. 2. In parallel mode, as shown in Fig. 1, a x-ray source and a strength detector in depending on domain medium spacing synchronization translation, in different location on each other parallel of ray for repeatedly measurement and records, then source and detection device will common rotating a small of angle, again for repeatedly parallel measurement. The first system used this work way, source and detector each translation measurement 160 parallel rays, then every time rotating 1 degrees to new location, total rotation angle 180 degrees, it can be known the ray measured a total of $160 \times 180 = 2880$ in this way. In fan mode, as shown in Fig. 2, measured by its total number of 184320 rays.
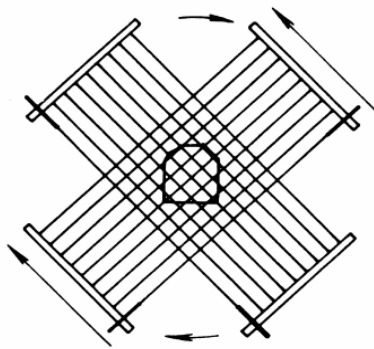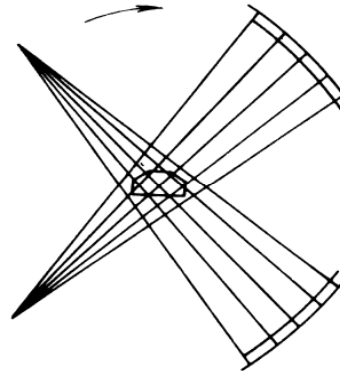

Fig. 1 Scan by parallel mode   Fig. 2 Scan by fan mod

Mathematical Principle

In order to illustrate the mathematical model and principle of drawing section figure by computer, first please reference schematic-Fig. 3. The sorption ability of x-ray for different organizations and organs of human body is different, which means that same strength of x-ray penetrate on same area or volume of different organization or organ, the strength produced is different. The different ratio of the original ray strength and the penetrating ray strength illustrate different properties of different organizations and organs, such a ratio can be called x-ray density of bio organizations or organs. In order to confirm scan section within all local of x-ray density, showed as Fig. 2, dividing whole visual domain into square networks, each grid is called an image point. The first device in the world has $80 \times 80 = 6400$ image points, and the later CT/T system of General Electrical Company

includes $320 \times 320 = 102400$ image points, the actual area of each image points is about 1mm × 1mm. Because the area is small, then we assumed that the x-ray density of organization or organ is in the same image points range, thus we can obtain x-ray density value of each image points from all measurement data, then displayed it proportional to the density value on screen corresponding location, the structure of bio organization and organ can be displayed clearly on the screen. This is the basic principle of CT-Imaging.

Here we use an extremely intuitive and simple way, according to the above principle, make mathematical model of this apparatus. According to a certain rule number all image points, for any image point *j*, define the x-ray density. Consider the situation of the single point as shown in Fig. 4.
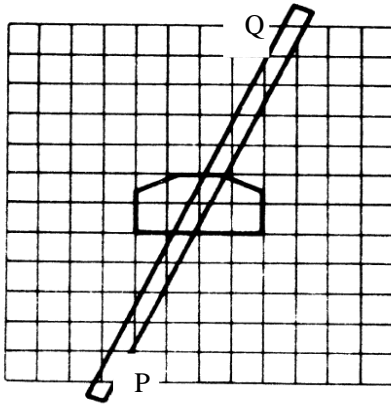

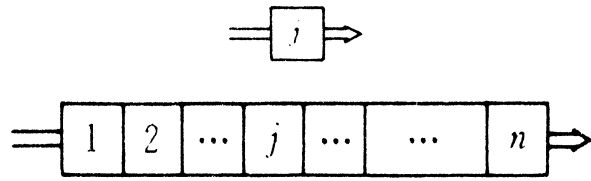Fig. 3 Illustrate schematic model.   Fig. 4 Ray transfer a single point and n points.

Let

$$y_j = \frac{the\ ray\ strength\ that\ in\ j\ image\ point}{the\ ray\ strength\ that\ out\ j\ image\ point} \tag{1}$$

Clearly, y_j depicts the features of image point *j*. Consider the n image points arranged as shown in Fig. 2, from the above definition, obtained easily

$$y_1 y_2 \cdots y_n = \frac{the\ ray\ strength\ that\ in\ 1\ image\ point}{the\ ray\ strength\ that\ out\ n\ image\ point} \tag{2}$$

Apparently right side quantity of formulation can be measured, its numerator can be measured in no objects by the same device, denominator can be measured in clinical manifestation. Assumed each ray give out an equation by above way, collection up will get a system of nonlinear equations for unknown as ray density of the image points, so solution is very difficult. In order to overcome this shortcoming, some mathematical skills are needed, so we define x-ray density of image point *j* as following:

$$x_j = \ln y_j \tag{3}$$

Due to ln*y* is monotone function, different *y* corresponds distinct *x*, thus image of the different tissues or organs can be clearly displayed on the screen can. Assume

$$b_i = \ln \frac{the\ i\ ray\ strength\ of\ no\ object}{the\ i\ ray\ strength\ after\ object} \tag{4}$$

If *i* ray rip into the *n* image points of serial number as i_1, i_2, …, i_n, to sum up, we get the following equation

$$x_{i_1} + x_{i_2} + \cdots + x_{i_n} = b_i \tag{5}$$

The quantity that appears at the left side of formulation is unknown number, we get the right side quantity by means of scanning. If we included n unknown numbers of n imagine points into third formulation in that field of vision, we can also get the following equation

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i \tag{6}$$

$$a_{ij} = \begin{cases} 1, if\ i\ ray\ through\ j\ point \\ 0, otherwise \end{cases} \tag{7}$$

The CT-Scanning operation way that mentioned above we can get to know that not every ray through a series of imagine points by normal incidence, the same ray in different image points has different passage length, therefore, the different absorption is distinguished by normal incidence. In order to make equation more accurate, equation should be amendment by coefficient a_{ij}, reflecting the effects of ray length on ray strength attenuation of each image point. The way to determine a_{ij} is followed:

(1) Centralization of image points, making assume:

$$a_{ij} = \begin{cases} 1, if\ i\ ray\ just\ though\ j\ point \\ 0, otherwise \end{cases} \tag{8}$$

(2) The length of central line, making assume:

$$a_{ij} = \frac{the\ length\ of\ i\ ray\ in\ j\ image\ point}{the\ side\ length\ of\ each\ image\ point} \tag{9}$$

(3) Area method, in this case, we should assume the ray has the width, making assume:

$$a_{ij} = \frac{the\ area\ of\ i\ ray\ in\ j\ image\ point}{the\ area\ of\ each\ image\ point} \tag{10}$$

Each kind of treatment, ultimately lead a large scale linear equations system, as follows:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \qquad\qquad \cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}a_n = b_m \end{cases} \tag{11}$$

where, m and n respectively represent the total number of rays and image points. The image reconstruction is the solution of the equation system. The equation number is far more over than unknown number, and the coefficient matrix is sparse, which contains a large number of zero elements.

Algebra Reconstruction Technique (Art)

The image reconstruction is of great significance to nondestructive testing (NDT). The reconstruction algorithms mainly include two methods: analytic reconstruction and iterative reconstruction. The basic algorithms of analytic reconstruction and iterative reconstruction are filtered back projection (FBP) and algebra reconstruction technique (ART) respectively. Once the projection data is incomplete, ART could reconstruct a better result than FBP. But in the actual ART is used less widely than FBP in industry and medicine, as it costs long time to calculate projection coefficients.

In recent years, many scholars researched on how to improve there construction speed of ART, and mainly focused on the calculation of projection coefficients, the selection of access mode and relaxation factor, and parallel computing.

The base of ART is to find the optimal solution by solving a set of equations, like the formula (11). Combined with Fig. 3, X-ray goes through the grid from point P to point Q, and the reconstructed image is divided into $n \times n$ pixels numbered from 0 to $n^2 - 1$. Where, in formula (11), b is the projection data read from the detector output under the angle of m. x_n is the corresponding gray value of each grid, and also be the solution of requested equations. a_{ij} is the corresponding projection coefficient representing the length of the X-ray through the grid. All the projection coefficients make a sparse matrix, which may be point model, line model or area model according to the selection of projective mode. In actual, the ordinary used model is line model line model considering the speed and accuracy as shown in Fig. 3. The iterative formula of ART is:

$$\overline{X}^{(k+1)} = \overline{X} + \lambda \frac{b_i - \overline{a}\overline{X}^k}{|\overline{a_i}|^2} \overline{a_i}^T$$

(12)

where, λ is the relaxation factor, and here $\lambda = 0.25$. $\overline{X}^0$ is the Assumed solution vector of an initial image with zero value. Through formula (12), $\overline{X}^1$, $\overline{X}^2$, …, can be calculated in turn until meet the given precision. a_i is the projection coefficient and the major factor to ART.

ART can reconstruct a good reconstruct result, but it cost a lot time, and the most spends on the computing of projection coefficients. The new imaging tools able to evaluate tissue perfusion (both by CT or MRI) and cellularity (Diffusion Weighted MRI) can be applied to evaluate vascular damage prior the ischemic changes and especially in differential diagnosis of fibrosis versus recurrent tumor [4].

As previously mentioned General Electric Company CT/T systems, $m = 184320$, and $n = 102400$. Owing to massive amounts of measured data, it requires a lot of computation, thus computer is an essential tool in image rebuilt technology. Such as the United States National Medical Library authorized Colorado University Medical School to establish the anatomy structure database of a male and a female body in 1991, they used the CT and MRI scanning the male body from scratch to feet, then cutting into 1878 slices, the thickness of each slice is 1mm, the amount of all data is 15 GB. The female body was cutting into more than 5,000 slices of 0.33mm each, the amount of all data is 59 GB [5].

Conclusion

Algebraic reconstruction is more suitable for image reconstruction than analytic reconstruction as its high contrast and precision for incomplete projecting data. The calculation of projection coefficients is the key step of algebraic reconstruction, which has crucial effect on the reconstructed quality and speed. The subsurface figure quality is the most important performance index to evaluate CT system. The advantages and disadvantages of subsurface figure is related to the quality of projection data in image reconstruction such as contrast ratio and edge features and noise of image, as well as depends on the image reconstruction algorithm. The numerical method to solve large sparse matrix contradiction algebraic equation set called algebraic reconstruction technique, this method is simple and operate easily, a new method is proposed to compute a quickly and accurately, it is still available in some areas. In addition, there are some better ways to handle it, such as Radon Transformation, this new theory was introduced by Austria mathematician Radon in 1917. Radon Transformation has a strong noise immunity and line detection, so it has been widely used in feature extraction of figure and pattern recognition.

This paper is intended to introduces mathematical principles on medical apparatus of imaging-diagnose, because there have been many advocates for increasing the use of computer algebra systems in industry. The primary idea for such an advocacy is that computer algebra systems represent real-world math more so than paper-and-pencil or hand calculator based

mathematics. This push for increasing computer usage in industry has been supported certain boards of education. It has even been mandated in the curriculum of certain regions. Computer algebra systems have been extensively used in higher education [6]. Many universities offer either specific courses on developing their use, or they implicitly expect students to use them for their course work. The companies that develop computer algebra systems have pushed to increase their prevalence among university and college programs. So this paper wants to help people understand "In essential, 'the high-technology' is a kind of mathematical technique".

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