Analysis of Dynamical Behavior in Hyper-chaotic Complex System

Liang Xue-feng

School of Mathematic and statistics, Tianshui Normal University, Tianshui, 741001,China

Liangxuefeng_123@126.com

Keywords: Hyper-chaotic complex Lü system; dynamical behaviour; Modified projective synchronization

Abstract: This paper investigates the complex dynamics of fractional-order hyper-chaotic complex Lü system via numerical simulations. It is found that the variation of both the system parameters and the fractional order can induce bifurcations, and the parameter ranges of chaos are different. Also, a controller of modified projective synchronization (MPS) is proposed, which can effectively achieve modified projective synchronization in a class of high dimensional fractional-order systems.

Introduction

In the past ten years, numerous studies and applications of fractional-order systems in many areas of science and engineering have been presented [1-2]. Moreover, rich dynamic behaviors, including different kinds of bifurcations and chaos, have been found in many fractional-order systems. In [3] the fractional-order Chua's circuit was investigated and a chaotic attractor exists in an order as low as 2.7. In [4] the chaotic behavior in a Duffing system with order less than 2 was found. In [5] the authors found that the lowest order in the Chen system is 2.1 and so on.

It is well known, synchronization of hyperchaotic systems is a very important nonlinear phenomenon, and has been studied to date mainly on real dynamical systems. Different regimes of chaos synchronization have been developed, including anti-synchronization (AS) [6], phase synchronization (PHS) [7] etc. In case of PS, the state variables of the drive and the response systems synchronize up to a constant scaling factor. The main aim of this paper is to study the fractional-order hyper-chaotic complex Lü system. Dynamic behaviors are numerically investigated with varying the system parameters and the fractional derivative orders.

Dynamical behaviors of hyper-chaotic fractional-order complex Lü system

The hyper-chaotic Lü system is described as following:

$$\begin{cases} \dot{y}_1 = a(y_1 - y_2) + y_4 \\ \dot{y}_2 = by_2 - y_1 y_3 + y_4 \\ \dot{y}_3 = y_1 y_2 - cy_3 \\ \dot{y}_4 = y_1 y_2 - dy_4 \end{cases} \tag{1}$$

We suppose the state variable vector of the system (9) are defined in the complex field, and the derivative orders are fractional, then we can describe the corresponding fractional-order hyperchaotic complex Lü system as

$$\begin{cases} D_*^{q_1} y_1 = a(y_1 - y_2) + y_4, \\ D_*^{q_2} y_2 = by_2 - y_1 y_3 + y_4, \\ D_*^{q_3} y_3 = \dfrac{1}{2}(\overline{y}_1 y_2 + y_1 \overline{y}_2) - cy_3, \\ D_*^{q_4} y_4 = \dfrac{1}{2}(\overline{y}_1 y_2 + y_1 \overline{y}_2) - dy_4, \end{cases} \tag{2}$$

where $y = (y_1, y_2, y_3, y_4)^{\mathrm{T}}$ is the state vector, $y_1 = x_1 + jx_2, y_2 = x_3 + jx_4$ are complex functions, and $y_3 = x_5, y_4 = x_6$ are real functions and $j = \sqrt{-1}$.

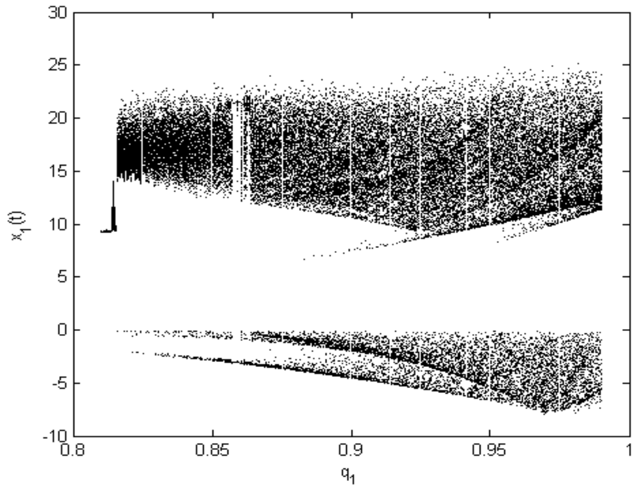

Fig.1 Bifurcation of the hyper-chaotic complex Lü system with $q_1 \in (0.81, 0.99)$

The bifurcation diagram with the variation of parameter a is depicted in Fig.2.

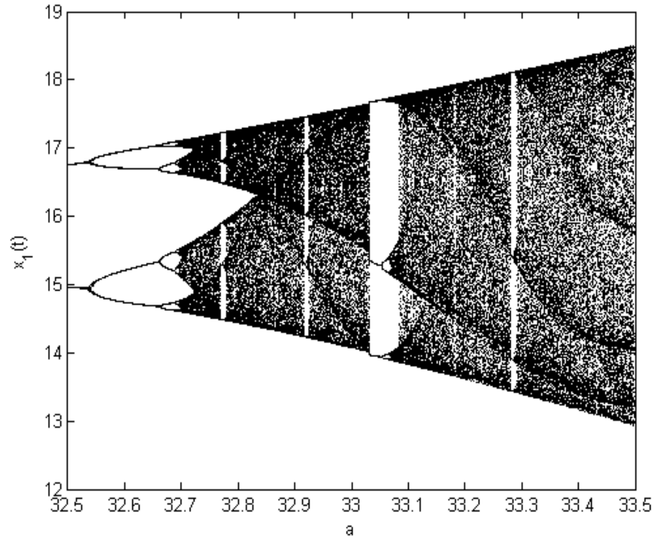

Fig.2 Dynamic behaviors of the system with $a \in (32.5, 35)$

We will investigate the dynamics of system (1) as the parameter d is varied when the derivative order $q = 0.95$. The phase diagram is depicted in Fig.3.

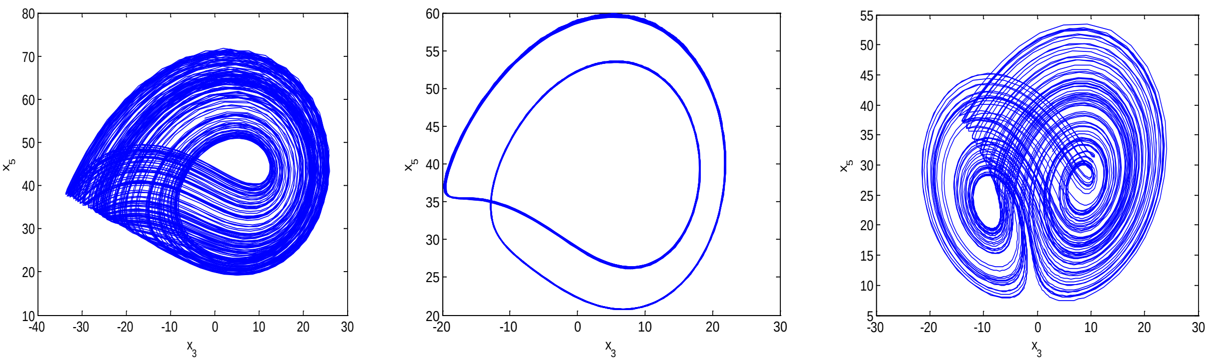

Fig.3. The phase portraits of the system for different values of d

Modified projective synchronization o hyper-chaotic complex system

The system (1) is taken as drive system, and the response system is

$$\begin{cases} D_*^q x_1' = a(x_3' - x_1') + x_6' \\ D_*^q x_2' = a(x_4' - x_2') + x_6' \\ D_*^q x_3' = bx_3' - x_1'x_5' + x_6' \\ D_*^q x_4' = bx_4' - x_2'x_5' + x_6' \\ D_*^q x_5' = x_1'x_3' + x_2'x_4' - cx_5' \\ D_*^q x_6' = x_1'x_3' + x_2'x_4' - dx_6' \end{cases}, \tag{3}$$

$z_1 = x_1' + ix_2', z_2 = x_3' + ix_4'$ are complex variables, $z_3 = x_5'$, $z_4 = x_6'$ is real variable, and $i = \sqrt{-1}$. Choose the arbitrary Hurwitz matrix $H = -I_6$ is given by:

$$\begin{cases} D_*^q x_1' = a(x_3' - x_1') + x_6' + m_1[a(x_3 - x_1) + x_6] - a(m_3x_3 - m_1x_1) - m_6x_6 + (a-1)e_1 + ae_3 - e_6 \\ D_*^q x_2' = a(x_4' - x_2') + x_6' + m_2[a(x_4 - x_2) + x_6] - a(m_4x_4 - m_2x_2) - m_6x_6 + (a-1)e_2 + ae_4 - e_6 \\ D_*^q x_3' = bx_3' - x_1'x_5' + x_6' + m_3[bx_3 - x_1x_5 + x_6] - bm_3x_3 - m_1m_5x_1x_5 - m_6x_6 + m_5x_5e_1 - (1+b)e_3 + m_1x_1e_5 - e_6 \\ D_*^q x_4' = bx_4' - x_2'x_5' + x_6' + m_4[bx_4 - x_2x_5 + x_6] - bm_4x_4 - m_2m_5x_2x_5 - m_6x_6 + m_5x_5e_2 - (1+b)e_4 + m_2x_2e_5 - e_6 \\ D_*^q x_5' = x_1'x_3' + x_2'x_4' - cx_5' + m_5[x_1x_3 + x_2x_4 - cx_5] - m_1x_1(m_3x_3 + e_3) - m_4x_4(m_2x_2 + e_2) - cm_5x_5 + (c-1)e_5 \\ D_*^q x_6' = x_1'x_3' + x_2'x_4' - dx_6' + m_6[x_1x_3 + x_2x_4 - dx_6] - m_1x_1(m_3x_3 + e_3) - m_4x_4(x_2m_2 + e_2) - dm_6x_6 + (d-1)e_6. \end{cases} \tag{4}$$

The simulation results are shown in the Fig.4

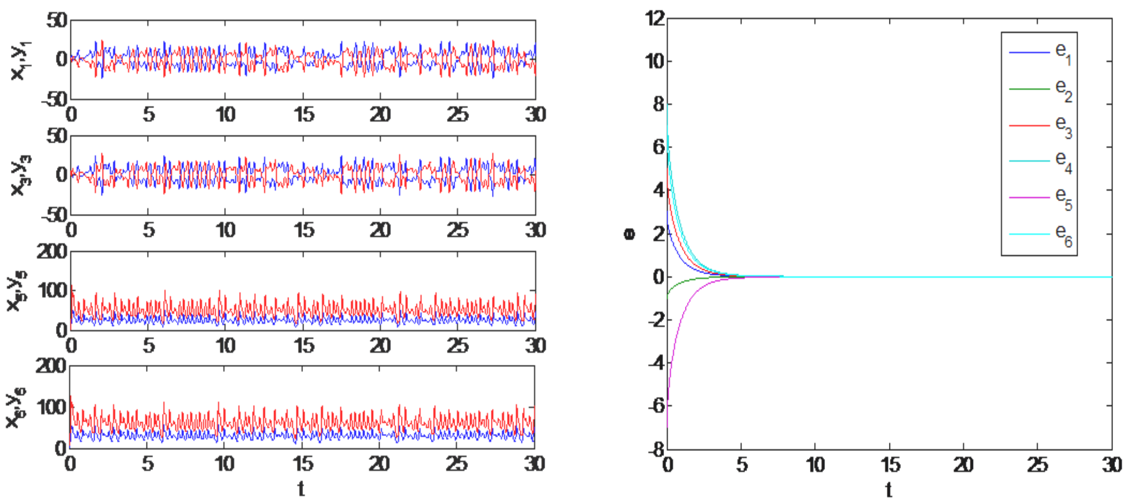

Fig.4. State variables and errors of drive system and response system

Conclusion

The dynamic behaviors of complex system Lü are studied through bifurcation diagram and the phase portrait, Various types of behaviors on the routes into and out of chaos in the system with the variation of both system parameters and derivative orders are observed. Moreover, the scheme of Modified projective synchronization (MPS) of the fractional-order hyper-chaotic complex Lü system is designed.

References

[1] B.M. Vinagre, V. Feliu: Modeling and control of dynamic systems using fractional calculus:application to electrochemical processes and flexible structures. In: Proceed-ings of 41st IEEE Conference on Decision and Control, Las Vegas (2002):214-239.

[2] Magin RL. Fractional Calculus in Bioengineering, Begell House, New York 32 (2004): 371-377.

[3] C.P. Li, W.H. Deng, D. Xu: Chaos synchronization of the Chua system with a fractional order. Physica A 360(2006) :171-185

[4] G. Mahmoud, S. Aly, M. AL-Kashif: Dynamical properties and chaos synchronization of a new chaotic complex system. Nonlinear Dynamics, 51(2008): 171-181.

[5]Lu JG, Chen, G.R. A note on the fractional-order Chen system, Chaos Solitons & Fractals 27(2007):13-17.

[6] P. Arena, R.Caponetto, L. Fortuna, and D. Porto: Chaos in a fractional order Duffing system. In: Proc ECCTD. Budapest, (1997) :1259-1262

[7] Y. Tang, J.A. Fang: General methods for modified projective synchronization of hyperchaotic systems with known or unknown parameters, Phys.Lett.A. 372 (2008):1816-1826

