Wave Equations for Prestack Reverse-time Migration

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Abstract. Numerical solutions of the wave equation have greatly aided geophysicists in both forward modeling and migration of seismic wave fields in complicated geologic media, and they promise to be invaluable in solving the full inverse problem. This paper compares time-stepping methods based on the solution of the acoustic wave equation with that based on the solution of the two way nonreflecting wave equation. In addition to the time-saving of implementation, it is imperative that one choose the most effective time-steeping method for a better imaging of the reflectors. With numerical tests on the Salt model, this paper shows that after a Laplace filtering the migrated results using the acoustic wave equation are superior to those using the two way nonreflecting wave equation.

Introduction

Reverse time migration, propagating the source wavefield forward and the recorded wavefield backwards in time [1] is quickly becoming the standard for high-end imaging. Recent interest in reverse-time migration has been focused on how to suppress or reduce the low frequency noises in the imaging [2,3]. The methods for suppressing low frequency noise in reverse time migration is mainly divided into three categories, suppressing in the wave extrapolation process, employing different imaging condions and performing flitering to the migrated results. For the wave extrapolation, two wave equations are frequently used, one is the acoustic wave equation [4], and the other is the two way nonreflecting wave equation [5].

With the acoustic wave equation, reverse time migration correctly handles both multi arrivals and the phase change due to caustics. Using the two way nonreflecting wave equation, we can avoid interlayer reverberations and thus reduce artifacts.

In this paper, we first give the acoustic wave equation and the two way nonreflecting wave equation and their solutions. Next, the imaging conditions and filtering methods are discussed. Finally, the results are shown by tests on the Slat model.

Wave equations

When density is constant, the acoustic wave equation reads

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 p}{\partial t^2}, \quad (1)$$

where $p = p(x, z, t)$ represents the pressure, $v(x, z)$ is the velocity. From the wave field given at t and $t - \Delta t$, we approximate its values at time $t + \Delta t$ by the expression

$$p(t + \Delta t) \approx 2p(t) - p(t - \Delta t) + v^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \right) \Delta t^2. \quad (2)$$

The space derivative terms in Eq.2 are computed by the Fourier method, the Hartley method, or the finite difference method. When propagating the recorded wavefield backwards,

$$p(t - \Delta t) \approx 2p(t) - p(t + \Delta t) + v^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \right) \Delta t^2 \quad (3)$$

is used. Four the Fourier method to keep the algorithm stable,

$$[(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2]^{1/2} v \Delta t \leq 2/\pi \quad (4)$$

is required.

The two way nonreflecting wave equation is

$$v \frac{\partial}{\partial x} \left(v \frac{\partial p}{\partial x} \right) + v \frac{\partial}{\partial z} \left(v \frac{\partial p}{\partial z} \right) = \frac{\partial^2 p}{\partial t^2}. \quad (5)$$

The forward and backward time-stepping scheme for this equation is

$$p(t + \Delta t) \approx 2p(t) - p(t - \Delta t) + \left(v \frac{\partial}{\partial x} \left(v \frac{\partial p}{\partial x} \right) + v \frac{\partial}{\partial y} \left(v \frac{\partial p}{\partial y} \right) + v \frac{\partial}{\partial z} \left(v \frac{\partial p}{\partial z} \right) \right) \Delta t^2 \quad (6)$$

and

$$p(t - \Delta t) \approx 2p(t) - p(t + \Delta t) + \left(v \frac{\partial}{\partial x} \left(v \frac{\partial p}{\partial x} \right) + v \frac{\partial}{\partial y} \left(v \frac{\partial p}{\partial y} \right) + v \frac{\partial}{\partial z} \left(v \frac{\partial p}{\partial z} \right) \right) \Delta t^2. \quad (7)$$

The stable condition is the same as the previous.

Imaging conditions and Laplace filtering

The conventional imaging condition for prestack reverse time migration is based on the cross correlation in time of the source wavefield with the receiver wavefield:

$$\text{Im } ag(x, z) = \sum_{i=1}^{nt} s(x, z, i\Delta t) p(x, z, i\Delta t) \quad (8)$$

The cross correlation image can be normalized by the square of the source illumination strength or the receiver illumination strength:

$$\text{Im } ag(x, z) = \frac{\sum_{i=1}^{nt} p(x, z, i\Delta t) s(x, z, i\Delta t)}{\sum_{i=1}^{nt} s^2(x, z, i\Delta t)}, \quad (9)$$

$$\text{Im } ag(x, z) = \frac{\sum_{i=1}^{nt} p(x, z, i\Delta t) s(x, z, i\Delta t)}{\sum_{i=1}^{nt} p^2(x, z, i\Delta t)}. \quad (10)$$

There are also some other imaging conditions such as excitation-time imaging conditions and receiver-/source-wavefield amplitude ratio. However, all conditions above can be seen as specific instances of cross correlation and unavoidably cause artifacts. The analysis of the production of the artifacts is given by Liu Faqi [6] who also presents an effective imaging condition for reverse-time migration using wavefield decomposition:

$$\text{Im } ag(x, z) = \sum_{i=1}^{nt} [s_d(x, z, i\Delta t) p_u(x, z, i\Delta t) + s_u(x, z, i\Delta t) p_d(x, z, i\Delta t)]. \quad (11)$$

To decompose the wavefield, first a two dimensional Fourier transform is performed. Then the decomposed wavefields are defined as

$$\begin{aligned} S_u(k_z, \omega) &= \begin{cases} S(k_z, \omega), & k_z \omega \geq 0 \\ 0, & k_z \omega < 0 \end{cases}, \quad S_d(k_z, \omega) = \begin{cases} S(k_z, \omega), & k_z \omega < 0 \\ 0, & k_z \omega \geq 0 \end{cases}, \\ P_u(k_z, \omega) &= \begin{cases} P(k_z, \omega), & k_z \omega \geq 0 \\ 0, & k_z \omega < 0 \end{cases}, \quad P_d(k_z, \omega) = \begin{cases} P(k_z, \omega), & k_z \omega < 0 \\ 0, & k_z \omega \geq 0 \end{cases}. \end{aligned} \quad (12)$$

where $S(k_z, \omega)$ and $P(k_z, \omega)$ are the 2D Fourier transform of the source wavefield and the record wavefield, respectively. After imaging, the Laplace filtering operator is performed to obtain the final migration images:

$$\text{Im}ag_L(x, z) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \text{Im}ag(x, z), \quad (13)$$

where $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} = \frac{4\omega^2 \cos^2 \theta}{v^2}$, θ is the incidence angle.

Numerical examples

To compare the migrated results of the two methods, we have tested the methods for the synthesized areal shot-record reverse time migration using the 2-D SEG/EAGE salt model. Depth and distance are measured in kilo feet. The plane waves are synthesized on the surface and the incidence angles of the waves are from -45° to $+45^\circ$ with interval 1° .

The migrated sections for the salt model using the acoustic wave equation are depicted in Fig.1a-b with crosscorrelation and the wavefield decomposition imaging condition, respectively. Fig.1c-d are the results after a performance of Laplace filtering on the migration results shown in Fig.1a-b.

Fig1.e-h provides corresponding migration images using the two way nonreflecting wave equation. From the figure, we see that reverse time migration using the two way nonreflecting can reduce artifacts before filtering and can not eliminate all artifacts. After filtering, the imaging of deeper reflector is a little better than that using the acoustic wave equation, however, the imaging beneath the salt is weaker than the previous since the method can not use multiarrivals to imaging.

Summary

Without the Laplace filtering, the method using the two way nonreflecting wave equation shows superior migrated images for the cross correlation imaging condition and the wavefield decomposition imaging condition. After the Laplace filtering, the method using the two way nonreflecting wave equation exhibits inferior migrated results compared to that using the acoustic wave equation. Since performing a Laplace filtering is a quick and time saving work, and the running time using the acoustic wave equation is similar to that using the two way nonreflecting wave equation, the acoustic wave equation is more adorable than the two way nonreflecting wave equation in prestack reverse time migration.

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Fig.1 Migration results.