Spatial Correlation Analytical Method of OD Flow in Freight Transport: a Case Study of China

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**Abstract:** Commodity and information interaction between regions are becoming more and more intense. The data of origin-destination flow (OD flow) is the observation of the interaction between paired regions. Each observed value of OD flow is related to an original point and a destination point in space, which brings enormous challenge in the establishment of mathematical model. In spatial interaction models, it is usually supposed that there is a certain relation (often the linear) between regional interaction and special distance, which may result in the inconsistence between data characteristics and model hypotheses. Based on the general form of traditional spatial autocorrelation indicators, a novel indicator system consisting of global and local indicators of spatial correlation for the origin-destination flow (OD flow) is proposed in this paper, and further the corresponding Z tests for the significance judgment are also derived under the null hypotheses of spatial uniformity and normal distribution. Finally, the feasibility and effectiveness of the proposed methodology are demonstrated by case studies of the railway freight exchange flows in Chinese mainland 2004. Through the proposed method, nodes in the freight transport network are classified to four types: global point, local point, generation point and attraction point.

**Introduction**

With the development of economy and society, commodity and information interaction between regions are becoming more and more intense. For the observation data of regional interactions, each observation value has a specific origin point and destination point, called origin-destination flow (OD flow). OD flow is widely used in the field of economy \cite{1-3}, transportation \cite{4-7} and international relationship \cite{8-11}. Generally, OD flows depend on not only the location of origin and destination but also the regional distances \cite{1, 12-14}, which brings additional challenges for mathematical modeling.

![Fig. 1 The relation of the average inter-city rail freight flow to the spatial distance](image)

Spatial interaction models assume that using distance as an explanatory variable in the linear regression function will eradicate the spatial dependence among the sample of OD flows between pairs of regions \cite{15, 16}, which has long been challenged \cite{14, 17}. For the relation between average inter-city rail freight flow and spatial distance in the mainland of China 2004 shown in Fig. 1, it can be
seen that the average inter-city rail freight decreases with the special distance and it shows a characteristic of non-linear attenuation.

In spatial autocorrelation theory, exploratory spatial autocorrelation indicators \cite{18-21} such as Moran’s I \cite{22-24} and Getis’ G statistics \cite{25-27} are widely used in the recognition of spatial correlation and spatial structure in the cross-section data \cite{28,29} and panel data \cite{30,31}. Many researchers \cite{32,33} use the exploratory spatial autocorrelation indicators to judge whether spatial correlation exists or not before the construction of spatial regressive models.

However, in the building process of space interaction models, there is a lack of indicators to make exploratory analysis of the correlation between regional interaction and spatial distance in OD flow data. Constructing spatial interaction models under linear relation hypotheses may encounter the problem that the data features do not match to the model hypotheses. Considering this situation, a spatial correlation indicator system consisting of global and local indicators is proposed to measure the correlation between spatial interaction and regional distance in this study.

The subsequent sections of this article are organized as follows. A novel spatial correlation indicator system is presented in Section 2 to measure the spatial correlation between regional interaction and spatial distance in OD flow data. In Section 3, the Z tests are derived for the significance judgment of spatial correlation indicators. The feasibility and effectiveness of the proposed methodology are demonstrated by the case studies of the railway freight exchange flows in Chinese mainland 2004 in Section 4. Finally, we conclude this study with Section 5.

**Spatial correlation indicators for OD flow**

In order to construct the indicator system for the spatial correlation of OD flows, it is necessary to establish the null hypotheses that the interactions between different spatial distances are equal and follow the same normal distribution, including the following two aspects.

1) Spatial uniformity hypothesis;

Spatial uniformity hypothesis means that the total OD flows generated (attracted) by a certain region are equally distributed to (from) all the other regions (including this region itself).

2) Normal distribution hypothesis;

Under spatial uniformity hypothesis, the OD flow between any pair of regions follows the same normal distribution.

The null hypothesis of spatial uniformity and normal distribution for OD flow are shown in Formula (1) and (2). Formula (1) (Formula (2)) indicates that the difference between the actual flow from region \(i\) to \(j\) (\(j\) to \(i\)) and the average generation (attraction) of region \(i\) follows normal distribution with expectation of zero and variance of \(\sigma^2\), called the left (right) null hypothesis. The constant \(n\) in the equations is the number of regions studied.

\[
\frac{\sum_{k=1}^{n} r_{ik}}{n} \sim N(0, \sigma^2) \quad (1)
\]

\[
\frac{\sum_{k=1}^{n} r_{ji}}{n} \sim N(0, \sigma^2) \quad (2)
\]

The general form of the traditional spatial autocorrelation indicators \cite{19,24} is described in Formula (3), where \(\Gamma\) is the global indicator of spatial autocorrelation and \(\Gamma_i\) is the local; \(w_{ij}\) is the spatial similarity between region \(i\) and \(j\); \(c_{ij}\) is the property similarity between region \(i\) and \(j\). There are different calculation ways of the property similarity \(c_{ij}\) for different spatial autocorrelation
indicators. For example, if set $x_i$ and $x_j$ are the attribute values of region $i$ and $j$, using $c_{ij} = x_i x_j$ yields a Moran-like measure, while taking $c_{ij} = (x_i - x_j)^2$ yields a Geary-like index [24].

$$\Gamma = \sum_{i,j} w_{ij} c_{ij}; \Gamma_i = \sum_j w_{ij} c_{ij}$$

(3)

The global and local spatial correlation indicators of OD flows under two null hypotheses (the left and right) are constructed respectively as Formula (4) and (6), where $w_{ij}$ is the spatial adjacency between region $i$ and $j$, and $r_{ij}$ is the flow from region $i$ to $j$. The statistic

$$r_{ij} = \frac{\sum_{k=1}^{n} r_{ik}}{n}$$

is used to replace $c_{ij}$ in the general form of spatial autocorrelation indicators.

For all the flow items generated (attracted) by region $i$, as the sum of the $n$ statistics

$$r_{ij} = \frac{\sum_{k=1}^{n} r_{ik}}{n}$$

is 0, so the degree of freedom is $n-1$ for the $n$ statistics. Therefore, the degree of freedom for $n^2$ statistics of flow items among all studied regions is $n(n-1)$ under the left or right null hypothesis. So, the estimated values of standard deviation under the two null hypotheses are derived with the freedom degree of $n(n-1)$ in Formula (5) and (7).

The global indicators $H$-left and $H$-right can describe the correlation between regional interactions and spatial adjacency of regions under the left and right hypotheses respectively, while the local indicators $h$-left, and $h$-right, describe the local correlation at the specific region $i$. Positive values of spatial correlation indicators mean that regional interactions between spatial neighbors tend to be greater than those between regions away from each other, while negative values of spatial correlation indicators illustrate that regional interactions between spatial neighbors tend to be weak.

As to the construction of spatial adjacency matrix on the basis of distance, there are the binary form, power function and exponential function generally as shown in Formula (8) to (10) respectively. Formula (8) is the commonly used binary spatial adjacency matrix; if the distance between region $i$ and $j$ is smaller than a certain threshold value $d$, the item $w_{ij}$ in the spatial adjacency matrix takes
value of 1; otherwise 0. While in Formula (9) and (10), \( w_{ij} \) is the power or exponential function of the distance between region \( i \) and \( j \), where different values of \( \alpha \) or \( \beta \) can be adapted to different decay rule of spatial correlation with the increase of spatial distance.

\[
\begin{align*}
\text{if } d_{ij} \leq d & : w_{ij} = 1 \\
\text{if } d_{ij} > d & : w_{ij} = d_{ij}^{-\alpha} \\
\text{if } d_{ij} \leq d & : w_{ij} = e^{-\beta d_{ij}}
\end{align*}
\]

(8)  
(9)  
(10)

**Significance test and verification**

In order to judge whether global and local spatial correlations are significant or not, \( Z \) statistics are constructed based on the statistical properties (expectations and variances) of spatial correlation indicators. Then, the Mont Carlo method is utilized in to validate the reasonability of \( Z \) test.

**Significance tests**

According to the left and right null hypotheses above, the difference between the OD flow from region \( i \) to \( j \) (or \( J \) to \( i \)) and the average generation (or attraction) of \( i \) follows normal distribution; with this difference divided by the estimated value of all OD flows’ standard deviation, the quotient follows t-distribution with the freedom degree of \( n(n-1) \). Two t-distributions under the left and right null hypotheses are shown in Formula (11) and (12) respectively.

**Formula (11)**

\[
\frac{(r_{ij} - \frac{1}{n} \sum_{k=1}^{n} r_{ik})}{\sqrt{\frac{\sum_{k=1}^{n} (r_{ij} - \frac{1}{n} \sum_{k=1}^{n} r_{ik})^2}{n^2 - n}}} = \frac{1}{\sigma} (r_{ij} - \frac{1}{n} \sum_{k=1}^{n} r_{ik}) - t(n^2 - n)
\]

**Formula (12)**

\[
\frac{(r_{ji} - \frac{1}{n} \sum_{k=1}^{n} r_{jk})}{\sqrt{\frac{\sum_{k=1}^{n} (r_{ji} - \frac{1}{n} \sum_{k=1}^{n} r_{jk})^2}{n^2 - n}}} = \frac{1}{\sigma} (r_{ji} - \frac{1}{n} \sum_{k=1}^{n} r_{jk}) - t(n^2 - n)
\]

As shown in Formula (11) and (12), the statistical properties (expectations and variances) of the global and local spatial correlation indicators under two null hypotheses (the left and right) can be achieved. The expectations and variances of the global and local spatial correlation indicators are shown in Formula (13) to (16). Because of the symmetry, the expectations or variances under the left and right null hypothesis are equal. Actually, it can be found that the statistical properties of spatial correlation indicators are mostly determined by spatial adjacency structure and the number of regions studied instead of the values of OD flows.

\[
E(H-\text{left}) = E(H-\text{right}) = 0
\]

**Formula (13)**

\[
D(H-\text{left}) = D(H-\text{right}) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij}^2}{(\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij})^2} \times D(t(n^2 - n)) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij}^2}{(\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij})^2} \times \frac{n^2 - n}{n^2 - n - 2}
\]

**Formula (14)**

\[
E(h-\text{left}_i) = E(h-\text{right}_i) = 0
\]

**Formula (15)**
\[ D(h - \text{left}_i) = D(h - \text{right}_i) = \frac{\sum_{j=1}^{n} W_{ij}^2}{(\sum_{j=1}^{n} W_{ij})^2} \times D(t(n^2 - n)) = \frac{\sum_{j=1}^{n} W_{ij}^2}{(\sum_{j=1}^{n} W_{ij})^2} \times n^2 - n - 2 \] (16)

For the significance judgment, Z statistics of the global and local spatial correlation indicators under two null hypotheses are shown in Formula (17) and (18) respectively. Generally, when the absolute value of Z statistic is more than 1.96, null hypothesis is rejected, which represents that the spatial correlation is significant; otherwise there is no significant interactive correlation between regions and their adjacency.

If the value of Z statistic is bigger than 1.96, the interactions between adjacent regions are significantly greater than those between regions away from each other in space. On the other hand, if the value of Z statistic is less than -1.96, the interactions between regions close to each other are less than those between regions with large distance significantly.

\[ Z(H\text{-left}) = \frac{H\text{-left} - E(H\text{-left})}{\sqrt{D(H\text{-left})}} \quad Z(H\text{-right}) = \frac{H\text{-right} - E(H\text{-right})}{\sqrt{D(H\text{-right})}} \] (17)

\[ Z(h\text{-left}_i) = \frac{h\text{-left}_i - E(h\text{-left}_i)}{\sqrt{D(h\text{-left}_i)}} \quad Z(h\text{-right}_i) = \frac{h\text{-right}_i - E(h\text{-right}_i)}{\sqrt{D(h\text{-right}_i)}} \] (18)

Monte Carlo simulation

In order to validate the correctness of theoretical statistical properties derived above, the Mont Carlo method is utilized to get the simulative values of expectations and variances of spatial correlation indicators. As shown in Fig. 2(a), a spatial configuration with 9×9 grid is selected as study object; and spatial adjacency rule is defined as Fig. 2(b) where spatial neighbourhood exists only between boxes with common edge.

![Spatial system of simulation](image1)

![Spatial adjacency rule](image2)

Fig. 2 Spatial configuration and adjacency rule of Mont Carlo simulation

According to the definition of spatial configuration and adjacency rule in the simulation case, the theoretical values of expectations and standard deviations are calculated for \( H\text{-left} \) and \( H\text{-right} \). Then normally distributed numbers are generated automatically to get the simulation values of expectations and standard deviations for \( H\text{-left} \) and \( H\text{-right} \) as well. The theoretical and simulative values are presented in Table 1.

For the three groups of simulative experiments with simulation times of 10000, 20000 and 50000 in Table 1, the differences between the theoretical and simulative expectation values are all less than \( 10^{-3} \), and the simulative values of standard deviation differ from the theoretical just 5% at most. Therefore, the theoretical estimations of expectations and standard deviations for spatial
correlation indicators based on normal distribution hypothesis in this study are effective and appropriate. So the significant tests for spatial correlation indicators under the normal distribution hypothesis are reasonable.

Table 1 Theoretical and simulative values of expectation and standard deviation

<table>
<thead>
<tr>
<th>Simulation times</th>
<th>Experimental index</th>
<th>Expectation</th>
<th>Standard deviation</th>
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</thead>
<tbody>
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<td></td>
<td></td>
<td>Theoretical value</td>
<td>Simulative value</td>
</tr>
<tr>
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<td>H-left</td>
<td>0.000000</td>
<td>-0.000765</td>
</tr>
<tr>
<td></td>
<td>H-right</td>
<td>0.000000</td>
<td>-0.000833</td>
</tr>
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<td>H-left</td>
<td>0.000000</td>
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<tr>
<td></td>
<td>H-right</td>
<td>0.000000</td>
<td>-0.000525</td>
</tr>
<tr>
<td>50000</td>
<td>H-left</td>
<td>0.000000</td>
<td>0.000119</td>
</tr>
<tr>
<td></td>
<td>H-right</td>
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<td>0.000121</td>
</tr>
</tbody>
</table>

Case Study

To illustrate the applicability and effectiveness of the proposed method, the rail freight flow data of 278 cities along the railway in mainland China 2004 are selected for case studies. The spatial distance between each pair of the cities is the shortest path length in railroad network calculated by Floyd algorithm. The global spatial correlation characteristic and local spatial correlation structure of the freight interactions between the studied 278 cities are analyzed with the proposed method in the previous sections.

Global spatial correlation

Take into account the binary form of spatial adjacency shown in Formula (9), the spatial adjacency matrix varies with the upper limit value of distance between spatial neighbors $d$. Furthermore, the spatial adjacency matrix in the form of power function and exponential function of distance in Formula (10) and (11) are adopted to study the decay characteristic of the global spatial correlation with the increase of regional distance.

![Fig. 3 The change of Z-statistics for global spatial correlation indicators with distance](image)

Fig. 3 shows the varying curve of the Z statistics for global spatial correlation indicators $H$-left and $H$-right with the upper limit value of spatial neighbors’ distance $d$. As shown in Fig. 3, with the increase of $d$, the value of Z-statistics for $H$-left and $H$-right increase sharply at first and gradually level out then. Besides, the Z-statistics are greater than 1.96 in the spatial range from 100km to 3300km, so there are significant freight interactions among the 278 cities in the distance range from 100 to 3300 km.

On the other hand, there are four obvious peak points of Z-statistics at 225 km, 525 km, 775 km and 1150 km for both $H$-left and $H$-right in Fig. 3. This illustrates that, 4 obvious modes of the freight interactions between the 278 cities exist, including: short distance interactions within 225 km,
medium-short distance interactions within 525 km, medium-long distance interactions within 775 km and long distance interactions within 1150 km. Among the 4 interaction modes, the short range interactions satisfying freight demands between cities within province are the most significant, while the long distance interactions are the second most significant and mainly serve the cross-province freight demands. The short-medium and long-medium range interactions are compromises between the short range and long range interactions, which can satisfy the freight exchange demands within and across provinces.

Fig. 4 shows the decay pattern of spatial correlation with spatial adjacency form of power function and exponential function of distance in Formula (10) and (11). In Fig. 4(a), Z-statistics vary along with the constant $\alpha$ in power function and reach the maximum values for both $H_{\text{left}}$ and $H_{\text{right}}$ when $\alpha$ equals 6.40, but there are no pronounced peaks at the maximum value. Fig. 4(b) shows the varying process of the Z statistics along with the constant $\beta$ in exponential function; and we can find that, the Z statistic reach the maximum values for both $H_{\text{left}}$ and $H_{\text{right}}$ when $\beta$ equals 0.025, which are obviously greater than the maximum values of the power function. So it can be concluded that, the freight interactions among the 278 cities decay exponentially, and the suitable value of $\beta$ in the exponential decay function is 0.025.

Local spatial correlation structure

Under different upper limit value of spatial neighbors’ distance $d$, the local spatial correlation indicators $h_{\text{left}}$ and $h_{\text{right}}$ for all 278 cities are calculated. If city $i$ has the maximal value of local spatial correlation indicator $h_{\text{left}}$ ($h_{\text{right}}$) at a certain value of $d$, this distance can be defined as generation (attraction) influence range of this city.

After calculation, the average generation and attraction influence range of the studied 278 cities are 768.57km and 848.50km respectively. Take the difference between the generation influence range and the average of all studied cites as abscissa, and the difference of the attraction influence range to
the average of all studied cites as ordinate, every city can be mapped to a point in this two-dimensional coordinate plane to get a scatter diagram as Fig. 5.

Fig. 5 Scatter diagram of attraction and generation influence range for studied 278 cities

As shown in Fig. 5, if Z-statistic of the maximal value of local spatial correlation indicator $h_{left}$ and $h_{right}$, for a city is greater than or equal 1.96, it can be called a significant point; otherwise, the city is a random point. The percentage of significant points is just 10%, which are mainly distributed around the bisector of the first and third quadrant; while most cities are random points, which indicates that there is no significant spatial dependence for freight interactions for most cities. In addition, the influence ranges of generation and attraction for the same city also tend to be close.

Furthermore, cites which have significant spatial correlation of freight interactions could be classified into 4 classes according to their positions in the scatter diagram of Fig. 5. City in the first quadrant is called global point, whose attraction and generation influence ranges are both greater than the average level. City in the second quadrant has attractive interactions with cities in a long distance range but generative interaction with neighboring cities, so it is called attraction point. The attraction and generation influence ranges of cities in the third quadrant are both less than the average level, so it exchanges freight with other cities in small distance and can be called local point. City in the fourth quadrant generates freight mostly to the cities within a large distance but attracts freight from neighboring cities, so it is called generation point. These four categories of cities are mapped in Fig. 6.

Fig. 6 Spatial position of the different categories of cities
In Fig. 6, cities like Guangzhou, Urumchi and Dalian are global points. These cities are important economic centers or port cities with large demand of freight transportation, so they play positive and dominant roles in the railway freight system. The local points in Fig. 6 can be divided into two groups; big cities like Hangzhou, Shijiazhuang and Guiyang are both important economic centers, but their freight interactions with other cities just within the local areas, which means that the economic influence effects of these cities are restricted within a small range of space; other local points like Tsitsihar, Mutankiang and Ankang are scattered in underdeveloped areas, which mainly serve the freight demands between cities in local areas.

Changchun, Harbin, Shenyang and Tianjin are generation points. The first 3 are located in northeast China which produce industrial materials and agricultural products but have few cities with strong demands, so these 3 cities generate freight mostly to the cities in long distance but attract freight only from neighboring cities. As an important harbor city, a large quantity of products and raw materials are transported to Tianjin from the ocean and then transferred to north and northwest China by rail, so it is also a generation point. Shanghai, Chongqing, Korla and Huaihua are attraction points. Among them, Shanghai is an important metropolis and port city, where not only great amount of supplies are demanded but also large quantities of commodities congregate here by rail to ocean transportation, so it plays an attractive role in the railway freight system. Chongqing, Huaihua and Korla Show features of attraction point because of the strong demands of commodities and insufficient supplies from their own and the surrounding areas.

Conclusions
Measurement of the spatial correlation for OD flows is necessary to recognize the relationship between regional interaction and spatial distance. A novel indicator system consisting of global and local indicators of spatial correlation for OD flows was presented in this article firstly; and then the relevant Z-statistics for significant test were also developed. As an application, the proposed methodology was applied to the spatial dependence analysis of railway freight flows in Chinese mainland 2004 to demonstrate the feasibility and effectiveness of the proposed methodology. Some conclusions are achieved as follows.

(1) The expectations and standard deviations of global spatial correlation indicators $H_{\text{left}}$ and $H_{\text{right}}$ are similar for Mont Carlo simulation and the theoretical value, which indicates that the significance tests for spatial correlation indicators under the normal distribution hypothesis are reasonable.

(2) For the studied 278 cities, their freight interactions appear decaying with the increasing of distance, and it follows exponential function of distance. There are four typical freight interaction modes: the short distance interactions within 225 km, medium-short distance interactions within 525 km, medium-long distance interactions within 775 km, and long distance interactions within 1150 km.

(3) For the local spatial correlation characteristics, cities with significant spatial correlation are separated into four types: global point, local point, generation point and attraction point, which can reveal the transportation and economy status of cities in freight interaction system and economy.

So the model proposed in this paper can effectively reveal the global and local spatial correlation of interactions among regions. The influence range determination and classification of regions are meaningful to recognize points with different characteristics in an interaction system. Further studies may be focused on the application of the proposed spatial correlation indicators to spatial interaction model improvement.
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