A Model of Sustainability Evaluation Based on PAC and AHP

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Abstract. We set a model of sustainability evaluation based on Principal Component Analysis (PCA) and Analytic Hierarchy Process (AHP). We defined sustainability index, which reflects the sustainable development level of a country. Through our model, as long as we know the relevant data of a country, it is capable for us to calculate the sustainability index exactly. And then we can judge if the country is sustainable development or not. Moreover, we also set up environment index, resource index, economy index and society index to find out what has blocked the sustainable development of a country, which is useful for us to determine policies as well as provide assistance.

Introduction
In order to understand and analyze the problem about sustainable development better, the following background is worth mentioning.

Over the past few years, "Sustainable Development" (SD) has emerged as the latest development catchphrase. A wide range of nongovernmental as well as governmental organizations have embraced it as the new paradigm of development [1].

Now, the International Conglomerate of Money (ICM) wants to use their extensive financial resources and influence to create a more sustainable world. They are particularly interested in developing countries, where they believe they can see the greatest results of their investments.

Modeling and Index Calculation

Determine evaluation factors
There are many factors related with the sustainability of a country or area. We divided all the factors into four categories. They are environment, resource, economy and society. All the factors we selected are presented in Table 1.

<table>
<thead>
<tr>
<th>Environment</th>
<th>GEF benefits index for biodiversity</th>
<th>y1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Carbon dioxide emissions</td>
<td>y2</td>
</tr>
<tr>
<td></td>
<td>Nitric oxide emissions</td>
<td>y3</td>
</tr>
<tr>
<td></td>
<td>Forest area (% of land area)</td>
<td>y4</td>
</tr>
<tr>
<td>Resource</td>
<td>Renewable internal freshwater resources per capita</td>
<td>y5</td>
</tr>
<tr>
<td></td>
<td>Energy production</td>
<td>y6</td>
</tr>
<tr>
<td></td>
<td>Alternative and nuclear energy</td>
<td>y7</td>
</tr>
<tr>
<td></td>
<td>Fossil fuel energy consumption</td>
<td>y8</td>
</tr>
<tr>
<td></td>
<td>GDP per unit of energy use</td>
<td>y9</td>
</tr>
</tbody>
</table>
After determining the factors, we need to select some typical countries to ensure the criterion of sustainable development. We choose 17 countries from different continents, including developed countries and developing ones, as the samples.

Table 2 The typical countries

<table>
<thead>
<tr>
<th>Developed countries</th>
<th>America</th>
<th>Japan</th>
<th>Germany</th>
<th>Norway</th>
<th>Canada</th>
<th>Spain</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developing countries (not include LDC)</td>
<td>Brazil</td>
<td>Columbia</td>
<td>Argentina</td>
<td>China</td>
<td>Kazakhstan</td>
<td>South Africa</td>
<td>Egypt</td>
</tr>
<tr>
<td>Least developed countries (LDC)</td>
<td>Sudan</td>
<td>Congo</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can see that the 17 countries come from different continents and their development level are different, so we can think the results is appropriate for all countries.

Ensure the sustainability indexes $I$ by PCA

There are several indicators in every categories. To use less variables represent the most change in the data, we use the method of principal component analysis[2].

First, calculate the data after pre-processing and obtain the coefficient of correlation matrix $R$. The formula is:

$$r_{ij} = \frac{\sum_{k=1}^{n} b_{ik} \cdot b_{kj}}{n - 1}$$

(1)

Where $r_{ij}$ is the coefficient of correlation of indicator $i$ and indicator $j$. $b_{ik}$ and $b_{kj}$ is the data after pre-processing.

And the accumulated rate of contribution.

$$\alpha_p = \frac{\sum_{k=1}^{p} \lambda_k}{\sum_{k=1}^{n} \lambda_k}$$

(2)

Where $\alpha_p$ is the accumulated rate of contribution. $n$ is the number of indicators.

Calculate the comprehensive evaluation value of each categories.

$$X_k = \sum_{k=1}^{p} w_i y_i$$

(3)
Where $X_k$ is environment index, resource index, economy index or society index. $w_i$ is the rate of contribution of each principal components, $y_i$ is the principal component.

After getting the four indexes of the four categories, we can obtain the sustainability indexes of the 17 countries.

$$Z_i = \sum_{j=1}^{4} e_i \cdot X_j$$  \hspace{1cm} (4)

Where $Z_i$ is the sustainability index I.

Ensure the sustainability indexes II by GRAP

Step 1. Ensure the reference sequence of each categories

We select the optimal data of different factors as the reference sequence.

Step 3 Calculate the grey correlation degree of each categories

$$r_i = \sum_{k=1}^{n} w_i \zeta_i(k)$$  \hspace{1cm} (5)

Where $w_i$ is the weight of every coefficient, $r_i$ is the grey correlation degree.

We finally get the correlation degree of each categories $r_i$.

Step 4 Calculate the gray correlation degree

$$R = r \cdot E^T$$  \hspace{1cm} (6)

Where $R$ is the gray correlation degree, $E$ is the weight of every category[3].

We regard the grey correlation degree as the sustainability indexes II($Z_2$).

Calculate sustainability index

We combine the sustainability index I and the sustainability index II. We set weight for the two factors. We assume that the weight of sustainability index I is $W$, the weight of sustainability index is $1-W$. The final result is:

$$Z = W \times Z_1 + (1-W) \times Z_2$$  \hspace{1cm} (7)

The Results

The result of sustainability index is as follow.

<table>
<thead>
<tr>
<th>Table 3 Some conclusions of sustainability index</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Average of 17 countries</td>
</tr>
<tr>
<td>The Highest (Norway)</td>
</tr>
<tr>
<td>The lowest (Saudi Arabia)</td>
</tr>
<tr>
<td>America</td>
</tr>
<tr>
<td>China</td>
</tr>
</tbody>
</table>

For environment index, resource index, economy index and society index, we can also get the highest value, the lowest value and the average value. We can calculate the Environment index, Resource index, Economy index and Society index:

$$z_i = W \times X_i + (1-W) \times r_i$$  \hspace{1cm} (8)

Where $z_i$ is the value of environment index, resource index, economy index and society index.

All results are shown in Table 4.
### Table 4 The index results

<table>
<thead>
<tr>
<th>Index</th>
<th>The highest</th>
<th>The lowest</th>
<th>The average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sustainability</td>
<td>Norway</td>
<td>Saudi Arabia</td>
<td>0.81</td>
</tr>
<tr>
<td>Environment</td>
<td>Brazil</td>
<td>Saudi Arabia</td>
<td>0.71</td>
</tr>
<tr>
<td>Resource</td>
<td>Norway</td>
<td>Kazakhstan</td>
<td>0.93</td>
</tr>
<tr>
<td>Economy</td>
<td>Norway</td>
<td>Congo</td>
<td>0.72</td>
</tr>
<tr>
<td>Society</td>
<td>Germany</td>
<td>Japan</td>
<td>1.60</td>
</tr>
</tbody>
</table>

#### Define When and How A Country is Sustainable or Unsustainable

To define how a country is sustainable or unsustainable, we only need to calculate the sustainability index of the country. The larger the index is, the more sustainable the country is.

The clear boundary between sustainable and unsustainable is not exist. Even if you can define a boundary, the result will be different by different person. So the sustainability is a concept which is relative. The magnitude of sustainability index can represent the level of the sustainable development of a country. You can also compare the indexes among different countries, or compare it with the countries which you know about well. Then, you can ensure the level of the country in sustainable development.

Meanwhile, if you find the sustainability index of a country is low, which suggests it is unsustainable. We can observe the environment index, the resource index, the economy index and the society index. We can compare these indexes with other countries or the average levels, and determine which development type is the country in. The unsustainable development countries can be divided into 3 mainly types.

- **Type 1 : Totally unsustainable.** All of the four indexes are low.
- **Type 2 : Naturally unsustainable.** The environment index and the resource index are low.
- **Type 3 : Artificially unsustainable.** The economy index and the society index are low.

Dividing the unsustainable countries into different types can help us make improving plan. It can make our work easier.

#### References

