Research on The Quantity of Taxicabs  
Yunfeng Ma  
School of Electric Power Engineering, North China Electric Power University, Baoding 071000, China.  
15233765487@163.com  

Keywords: Taxi arrival rate, Space driving rate, Taxi charge mechanism, Market competition.  

Abstract. We built the model of simplified model, which contains a university, a college, a city commercial center, mall and airport surrounding, taking Ithaca as a prototype. And we select the question 1.2.3 to analyze. We establish the concept of the taxi arrival rate (the times a taxi could arrive its destination average per hour), and connect it with the average waiting time of passenger. In the situation that the taxi effective driving time is confirmed (in view of the road condition, we take it as 30km/h), we can obtain the relationship between the variable quantity such as $W_h$ (carrying time per hour), a(arrival rate of the taxi),and k(space driving rate).The reasonable k should be in the range of 20%-35% via our analyzing. When k=0.25, 259 taxis is needed in our city model.  

Introduction  
Taxicabs traffic is an important component of urban passenger transport, which subjects to government regulation as a result of the existence of asymmetric information, transaction costs, excessive competition and other issues. Taxi market regulation can be traced back to the 17th century, and the United States began taxi market regulation in the late 1920s during the depression. The regulation mainly include the number of taxicabs and its price controls two aspect, which cannot lead to one consistent conclusion because of the distinction of the local economic development level, urban road length, number of residents, the taxi service quality, etc. The regulation of taxi also has a long history.  

Terminology and Definitions  
P$_1$ : total population of local residents.  
P$_2$ : total floating population.  
A$_1$: the per capita number of trip of local residents.  
A$_2$: the per capita number of trip of floating population.  
$\sigma_1$: share ratio of taxies of local residents.  
$\sigma_2$: share ratio of taxies of floating population.  
$\xi$: number of passengers carried per taxi.  
T: work hours of taxi drivers during the daytime.  
S: total road mileage of traffic area.  
N: the total number of the taxi.  
a: arrival rate of the taxi.  
t: average waiting time for the taxi of the passengers.  
k: space driving rate.  

Assumptions  
- In the process of simplification of the map, we only take the place with a large population into account, such as airport, university, arts college, mall and residential area into account, without accounting shops and theatres.  
- We abstract the residential area of local residents as a point and put it in the town center.
We abstract the residential area of students in the university and arts college as two points, and respectively put them in the center of university and arts college.

Only when local residents go to the airport, they take a taxi.

Solutions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$</td>
<td>Average trip distance of local residents</td>
</tr>
<tr>
<td>$D_2$</td>
<td>Average trip distance of floating population</td>
</tr>
<tr>
<td>$D$</td>
<td>Average trip distance of all the people in the town</td>
</tr>
<tr>
<td>$W_0$</td>
<td>Empty driving time per hour</td>
</tr>
<tr>
<td>$W_h$</td>
<td>Carrying time per hour</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Empty driving speed</td>
</tr>
<tr>
<td>$V_h$</td>
<td>Carrying running speed</td>
</tr>
<tr>
<td>$L_h$</td>
<td>effective trip distance of all the taxies in the town per day</td>
</tr>
<tr>
<td>$l_h$</td>
<td>effective trip distance of a taxi per day</td>
</tr>
</tbody>
</table>

Step 1

We will solve this question by analyzing and simplifying the Tompkins County. The distances between different places will depend on the real distances in Tompkins County [1]. The following is the county simplified. According to the definition, $D_1$=6.73km. The population in the university is 21593 [2], the population in the arts college is 6448, so we assign them different weights: $D_2$=3.96km.

Step 2

According to the definition, we set the optional function [3] as following:

$$a = \frac{60S}{tW_0V_0}$$  

$$k = \frac{W_0V_0}{W_hV_h + W_0V_0}$$  

$$W_0 + W_h = 60\text{min}$$  

$$a = \frac{W_h/60}{D/V_h}$$  

$$D = \frac{P_1D_1\sigma_1 + P_2D_2\sigma_2}{P_1D_1 + P_2D_2}$$  

From the data from Ithaca official website, we know that $S=300\text{km}$, $P_1=29287$, $P_2=28041$, $\sigma_1=0.02$, $\sigma_2=0.5$, $T=13\text{h}$. And in consideration of the criterion of space driving rate, we shall take the $k$ as about 0.25. What’s more, with a view to the fact that inside these zones, traffic speeds are restricted to $40\text{km/h}$, and traffic problems such as traffic jam, we shall assign $V_h=30\text{km/h}$. 

916
Thus all the unknown variables are determined. Eq. (1 to 5) is solved for $a, V_0, W_0, W_h$ and substituted in Eq.(6 to 8).

\[
0.9L_h = l_h \cdot N \\
l_h = W_h \cdot V_h \cdot T/60 \\
L_h = \frac{P_1D_1A_1\sigma_1 + P_2D_2A_2\sigma_2}{N}
\]

When $t=15\text{min}$, we draw the conclusion that $a=3.9247$, $V_0=12.0264$, $W_0=32.76$, $W_h=27.24$. Thus $L_h=56100$, $l_h=213$, $N=259$.

When the longest waiting time of 10% of the passengers is 25min, average waiting time for the taxi of the 90% of passengers is $(15-0.1\times25)/0.9=13.888\text{min}$, so we change the data of $t$ as 13.888 [4] and the Eq(6) is changed as following:

\[
0.9 \times 0.9L_h = l_h \cdot N
\]

Thus we obtain the numerical values of $N$ as 224.

**Conclusion**

By changing the number of $k$, we can draw different kinked lines as following. Seen from the lines, the average empty-load rate always keep in a lower number, that’s because when searching passengers, not all taxi drivers use the cruise way, some of them sit around for their customers, which leads to a lower average speed.

What’s more, when the effective driving speed is confirmed, $N$ and $k$ have a proportional relation basically, proving that when the taxi number get beyond reasonable value, the bigger value is, the higher space driving rate becomes, and more resource will be wasted.

**References**

[1] Http://www.cornell.edu/about/ithaca.cfm