Price Decision in Asymmetrically Complementary Product Supply Chain

Lingzhi Shao*
Institute of System Engineering, Southeast University, Nanjing, China
shaolingzhi2005@126.com

Keywords: Asymmetrically complementary; Supply chain; Price decision; Discriminatory price

Abstract. This paper considers a decentralized asymmetrically Complementary Product Supply Chain consisting of two complementary suppliers and two competitive retailers. We will analyze the pricing game between the supply chain players and study the effects of pricing behavior on system’s performance. We also provide a number of numerical examples to demonstrate some important managerial insights.

Introduction
There are numerous examples about complementary products in our life. For example, computer hardwares and softwares, pencils and erasers, digital versatile disk (DVD) players and DVDs. There are many papers about the pricing decisions of centralized enterprises selling complementary products, such as [1]-[5]. A few researches consider the problem of two-stage supply chain. [5] Explores the pricing problems with regard to two complementary products in a supply chain including two manufacturers and one common retailer. [6] Considers the joint pricing and production decisions in a supply chain of complementary products supplied by n manufactures with uncertain demand. In another paper, [7] considered a model for partial complementarily products.

Different from the above researches, we consider an asymmetrically complementary product supply chain consisting of two oligopoly retailers selling substitute products (basic products and bundle product including basic and complementary product) and two monopoly asymmetrically complementary suppliers. Our main work is to investigate the pricing decisions of suppliers and retailers. Two scenarios will be considered in our paper including Retailers Alliance game (RA) and Retailers Decentralize game (RD). Moreover, in every scenario, we also consider the basic product supplier’s different pricing methods: common price or discriminatory price strategy.

Another related paper is [8]. They considered an assemble-to-order system including multi-suppliers supplying symmetrically complementary components and a common assembler producing substitute products. And they focused on the equilibrium solutions and coordination of the system. Differently, we try to study the players’ pricing strategy of players. [9] Also analyze the strategic interactions between two firms whose products are asymmetrically complementary, but they attempted to shed light on vertical competition among different layers of the computing industry by exploring the effects of changes in their product boundaries.

The rest of the paper is organized as follows: Section 2 will introduce the basic model and assumptions. Section 3 details the key analytical results. Some comparison and managerial implications will be presented in Section 4. The summary of results and some future research directions will be provided in Section 5.

Problem Description
In a supply chain, there are two independent suppliers providing complementary products 1 and 2. The product 1 provides some basic functions, and its quality is measured as \( s_1 \). The product 2 by itself does not provide any function, but enhances product 1’s performance. So the bundling product provides a higher performance level with higher quality \( s_b (s_b > s_1) \). In the market, one retailer sells
product 1 only to the customers with the retailing price \( p_1 \), while another retailer sells the bundle product consisting of the product 1 and complementary product 2 with the retailing price \( p_b \). The supplier 1 supplies the basic product 1(unit production cost \( c_{1i} \)) to retailer 1 and retailer 2 with the wholesale price \( w_1 \) and \( w_2 \). The wholesale price and unit production cost of the complementary product 2 supplied by supplier 2 to retailer 2 are \( w_2 \) and \( c_2 \). In reality, it is rational to assume that \( s_b - s_1 > c_2 \), \( \frac{s_b}{c_1 + c_2} < \frac{s_i}{c_1} \).

We assume that the primary demand during a single period is a fixed value (denoted 1). The customers are heterogeneous and the customer's type \( \theta \) is distributed uniformly on \([0,1] \). A customer of type \( \theta \) is willing to pay \( \theta s_i \) for a unit of product \( i (i = 1, b) \). There are three choices for an arriving customer, a basic product 1 from the retailer 1, a bundle product from the retailer 2 or nothing. Given the price \( p_1 \) and \( p_b \), a customer will decide her purchasing decision maximizing her surplus (\( \mu_i = \theta s_i - p_i, i = 1, b \)). When her surplus from the bundling product \( \mu_b = \theta s_b - p_b \) is larger than \( \mu_i = \theta s_i - p_i \), she will choose bundling product, so we get the threshold of the type \( \theta_b \) for bundle product and \( \theta_i \) for the basic product as \( \theta_b = \frac{p_b - p_i}{s_b - s_i}, \theta_i = \frac{p_1}{s_i} \). For \( \theta \in U [0,1] \), we can get the customer's market segmentation as follows: \( q_i = \frac{p_1 s_i - p_b s_b}{(s_b - s_i) s_i}, q_b = 1 - \frac{p_b - p_i}{s_b - s_i} \), when \( p_b - p_1 \leq s_b - s_i \) and \( \frac{p_1}{p_b} \leq \frac{s_i}{s_b} \).

**Analytical Results**

We assume that the suppliers and retailers have relatively equal market power, so they will make their price decision simultaneously. The game-theoretical approach is used to analyze the pricing decision models established in the following.

In RA- Common price scenario, the two retailers make their retail prices based on maximizing the total profit, and the basic supplier1 makes the same wholesale price for retailers. i.e. \( w_1 = w_2 \). We assume that the retailers can get unit profit \( m_1 \) and \( m_b \) from selling product 1 and bundling product. So the retailers' price decisions are equal to making decisions of \( \{m_1, m_b\} \). The retailers’ profit function is joint-concave and satisfy the second-order condition for a maximum. Thus solving the first-order conditions, we can get the retailers’ reaction functions: \( m_1^{al} = \frac{s_i - w_1^{al}}{2}, m_b^{al} = \frac{s_b - w_1^{al} - w_2^{al}}{2} \).

The profit function of suppliers is quasi-concave, and there is a unique maximizes: 
\[
\begin{align*}
    w_1^{al} &= \frac{s_i + c_1 - m_1^{al}}{2}, \\
    w_2^{al} &= \frac{s_b - s_i + c_2 - m_1^{al} + m_2^{al}}{2}
\end{align*}
\]

With the reaction functions, the equilibrium decisions can be obtained as the following proposition:

**Proposition 3.1.** In the RA-Common price game, the players' pricing game has a unique Nash equilibrium that is given as: \( w_1^{al} = \frac{2c_1 + s_i}{3}, \ w_2^{al} = \frac{2c_2 + s_b - s_i}{3}, \ p_1^{al} = \frac{2s_i + c_1}{3}, \ p_b^{al} = \frac{2s_b + c_1 + c_2}{3} \).

In RA- Discriminatory price setting, the supplier 1 uses discriminatory price strategy and the two retailers act as an alliance. He gives a different wholesale price \( w_1 \) for the retailer 2 who bundle the product 1 with the complementary product 2. We can get the following proposition 3.2.

**Proposition 3.2.** In the RA-Discriminatory price game, the players' pricing game has a unique Nash equilibrium: \( w_1^{al} = \frac{2c_1 + s_i}{3}, \ w_2^{al} = \frac{s_b - s_i + 3c_1}{4}, \ w_2^{al} = \frac{8c_1 - 3c_2 + 3s_b + s_i}{12}, \ p_1^{al} = \frac{2s_i + c_1}{3}, \ p_b^{al} = \frac{9s_b - s_i + 3c_1 + 4c_2}{12} \).
The RD game scenario arises in a market where the two retailers make decentralized decisions. When the supply 1 gives the common price for his two retailers, we have the following proposition.

**Proposition 3.3.** In the RD-Common pricing game, the players' pricing game has a unique Nash equilibrium that is given as: \( w_1^{p1} = \frac{4s_1s_b + (6s_b - 2s_1)c_1 - s_1c_2}{9s_b - s_1} \), \( w_2^{p1} = \frac{3s_1(s_b - s_1) + 6s_c c_2 - (s_b - s_1)c_1}{9s_b - s_1} \), \( p_1^{p1} = \frac{s_1(5s_b - s_1) + (3s_b + s_1)c_1 + s_1c_2}{9s_b - s_1} \), \( p_2^{p1} = \frac{2s_b(3s_b - s_1) + 4s_b c_1 + 3s_1c_2}{9s_b - s_1} \).

In RD- Discriminatory price setting, the retailers and suppliers make decisions independently, and the supplier 1 takes the discriminatory price strategy. We have the following proposition.

**Proposition 3.4.** The unique Nash equilibrium in DB-discriminatory pricing game are:
\( w_1^{p2} = \frac{5s_b + (8s_b - 3s_1)c_1 - s_1c_2}{2(6s_b - s_1)} \), \( w_2^{p2} = \frac{3s_1(s_b - s_1) + (9s_b - s_1)c_1 - (s_b - s_1)c_1}{2(6s_b - s_1)} \), \( w_3^{p2} = \frac{s_1(3s_b + 2s_1) + (7s_b - 2s_1)c_1 - 3s_1c_2}{2(6s_b - s_1)} \), \( p_1^{p2} = \frac{s_1(7s_b - 2s_1) + (4s_b + s_1)c_1 + s_1c_2}{2(6s_b - s_1)} \), \( p_2^{p2} = \frac{s_1(9s_b - 4s_1) + 5s_b c_1 + 3s_1c_2}{2(6s_b - s_1)} \).

Comparisons and Managerial Implications

In this section, we consider comparisons of four different scenarios and gain some insights into their managerial implications. The propositions indicate that the price decisions and profit of supply chain players will be affected by the different relationship between the two retailers and the price strategy used by the basic product supplier 1. The following propositions can be obtained.

**Proposition 4.1.** (1) In RA pricing game, when the supplier 1 gives discriminatory prices: the supplier 1 gives the same wholesale price for retailer 1, but take a higher wholesale price for the retailer 2, while the supplier 2 is forced to give a lower wholesale price; the retail price of the basic product will not be changed, while the retail price of the bundle product will become higher.

(2) In DA pricing game, when the supplier 1 gives discriminatory prices: the supplier 1 and supplier 2 will make a lower wholesale price, while the wholesale price of basic product for the retailer 2 is higher than the common price; the retail price of products will be higher.

(3) In asymmetry complementary product supply chain, when the two retailers make decisions indecently, the supplier 1 and supplier 2 will send higher wholesale prices for their products, while the retailer 1 and retailer 2 low their retail price.

**Proposition 4.2.** In retailer alliance setting, when the basic product supplier 1 uses discriminatory price strategy: (1) The supplier 2 and retailer 2 will get less profit, and the loss increases with the quality of bundling products; (2) The basic product retailer 1 will get more profit, and the additional profit increases with the quality of bundling products.

We also provide some numerical examples to compare the players' performances in different settings. Considering the case where the value of the model parameters as follows: \( c_1 = 2, c_2 = 1, a = 100, s_1 = 6, s_b \in \{7.1, 7.3, 7.5, 7.7, 7.9, 8.1, 8.3, 8.5, 8.7, 8.9\} \), we can get five figures. Fig 1- Fig 4 show that the profit of supplier 1, supplier 2 and retailer 2 will increase with the quality of bundling product in four different settings, but the retailer 1's profit will decrease with \( s_b \). Figs. 5 show that the profit of supply chain increases with \( s_b \). We also can get some insights.

**Insight 1.** Supplier 1 improves his profit using discriminatory price strategy, and in retailer decentralized game, the he gets a substantial increase of profit; the supplier 2's profit is reduced under supplier 1's discriminatory price strategy.

**Insight 2.** The retailer 1 will get increase of profit, when supplier 1 uses discriminatory price strategy. Different from retailer 1, the retailer 2 will get less profit, when supplier 1 uses discriminatory price strategy. The will also improve his profit in retailer alliance setting.
Insight 3. When the two retailers make decisions independently, the supply chain will get more profit because of the decrease of retail price. And the discriminatory price strategy of supplier 1 will damage the system profit of supply chain. The loss increases with the quality of bundling products.

Conclusion

This paper studies the pricing decisions of an asymmetry complementary products supply chain with two suppliers and two retailers. One retailer bundles the asymmetry complementary products supplied by two different companies, and another retailer sells the basic product only. We study four pricing games taking account of the relationship between the two retailers and the pricing strategy of the supplier 1, and get the unique Nash equilibrium in every situation. Then we attained some propositions and insights from the comparison of the equilibrium solutions and performance of supply chain players. We show that the basic product supplier 1 will make higher wholesale price for retailer 2 with discriminatory price strategy. The supplier 1 and retailer 1 will benefit from this price strategy, while the profit of the supplier 2, retailer 2 and supply chain will be damaged. Compared with retailer alliance, in retailer decentralized setting, the retail prices of two product will decrease, while the wholesale price increasing. When the two retailers make price decision independently, they will get less profit, while the other players and supply chain system will perform better. We also see
that the profit of supply chain players and system increase in the quality of bundling product, except for the retailer1.

References