RSS Based Location technique with the unknown transmit power and PLE using dimension reduction

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**Abstract:** Localization of mobile station (MS) has now gained considerable attention. Since the transmit power and path loss exponent (PLE) are two critical parameters in received signal strength (RSS) localization technique, many RSS based location methods considering the case that both of the transmit power and PLE are unknown have been proposed in the literature. However, one of major problems for these methods is computational complexity. Currently, MSs usually have insufficient power to afford the high computational burden. In order to meet the actual situation, the balance between positioning accuracy and computational complexity is required. This paper proposes a novel RSS based location method to reduce the computational complexity using dimension reduction technique. Simulations show that the proposed method can provide similar performance as the existing four-dimensional search methods whereas the computational burden of the proposed method is much less.

**I. Introduction**

Location estimation of a mobile station (MS) in wireless communication systems has gained considerable attention since the Federal Communication Commission passed a mandate requiring cellular providers to generate accurate location estimates for Enhanced-911 services [1]. Wireless location as an important public safety feature has created many potential applications to future cellular systems such as: location-sensitive billing, fraud protection, person/asset tracking, fleet management, mobile yellow pages, wireless network design, radio resource management, and intelligent transportation systems [2]. In a surveillance sensor network, it is crucial to know the location of an incident caught by a sensor, such as fire in a building or oil spill in coastal water. The demand is also high for mobile apps providing navigation and other location-based services (LBS) in hospitals, shopping malls, airport terminals, and campus buildings, to name a few. GPS is the most effective way to get location information but does not work indoors. Even for outdoor environments where this service is available, it is not energy-efficient to have to turn it on continuously all the time. Consequently, numerous efforts have been made towards GPS-free localization solutions.

Since the transmit power and path loss exponent (PLE) are two critical parameters in received signal strength (RSS) localization technique, many RSS based location methods considering the case that both of the transmit power and PLE are unknown have been proposed in the literature [3,4,5,6,7]. However, one of major problems for these methods is computational complexity. The basic idea of existing researches is to estimate the MS position by searching the location of MS in a four-dimensional space, which may cause the high computational complexity. Currently, MSs usually have insufficient power to afford the high computational burden. In order to ensure the mobile devices energy-efficient, the computational complexity of the existing location methods should be reduced. In order to meet the actual situation, the balance between positioning accuracy and computational complexity is required.

In this paper, an algorithm was derived to reduce the computational complexity. In the proposed algorithm, the location of MS is searched in a two-dimensional space rather than a four-dimensional space in the existing algorithm. The proposed algorithm reduces the computational complexity greatly. Simulations show that the proposed method can provide similar performance as the existing
four-dimensional search methods whereas the computational burden of the proposed method is much less.

II. System Model

The basic RSS model is briefly introduced in this section. Assuming that \((x, y)\) is the position of a MS to be estimated and the known coordinate of the \(i\)-th BS in a \(N - \text{BSs}\) system is \((x_i, y_i)\). Without loss of generality, the position of the first BS can be set to be \((0, 0)\). Denote the measurement with noise of \(*\) as \({k}\). The true distance between the \(i\)-th BS and MS can be modelled as:

\[
r_i^2 = (x_i - x)^2 + (y_i - y)^2 = k_i - 2x_i x - 2y_i y + k
\]

(1)

where \(k_i = x_i^2 + y_i^2\), and \(k = x^2 + y^2\).

Since the measured received power \(P_i\) at BS \(i\) (in decibel milliwatts) can be modeled as log-normal variable, the relation between \(P_i\) and \(r_i\) is:

\[
P_i = P_0 - 10\beta \log_{10} \left( \frac{r_i}{r_0} \right) + n_i
\]

(2)

where \(\beta\) is the path loss exponent (PLE), \(n_i\) is a zero-mean Gaussian random process with variance \(\sigma_i^2\) in decibel, \(P_0\) is the reference power at reference distance \(r_0\) and it depends on the transmit power. Typically, \(r_0 = 1m\).

The unknown vector is \(\theta = [x\ y\ P_0\ \beta]^T\). Currently, several search algorithms \([6,7]\) were proposed to solve (2) whose objective function is modelled as:

\[
\min_{(x, y, P_0, \beta)} \sum_{i=1}^{N} \left( P_i - P_0 + 10\beta \log_{10} \left( \frac{r_i}{r_0} \right) \right)^2
\]

(3)

It can be seen from the above equation that this method requires a search process in a four-dimensional space. The computational burden caused by the high-dimensional search may make these algorithms infeasible to be applied in a practical system. In this paper, a novel algorithm is proposed to reduce the computational complexity through dimension reduce technique.

III. The proposed method

This section proposes a novel RSS method for reducing the computational burden through dimension reduction technique.

For \(i = 1\), (2) becomes:

\[
P_1 = P_0 - 10\beta \log_{10} \left( \frac{r_1}{r_0} \right)
\]

(4)

To eliminate the unknown \(P_0\), (5) can be obtained from differencing (2) and (4):

\[
P_i - P_1 = 10\beta \log_{10} \left( \frac{r_i}{r_0} \right) - 10\beta \log_{10} \left( \frac{r_1}{r_1} \right) = 10\beta \log_{10} \left( \frac{r_i}{r_1} \right)
\]

(5)

It can be observed from (5) that the unknown parameters in (3) reduce from \(\theta = [x\ y\ P_0\ \beta]^T\) to \((x, y, \beta)\). Further simplification is necessary to reduce the computational burden. From (5), we have:
\[
\frac{P_i - P_j}{P_i - P_2} = \frac{\log_{10} \left( \frac{r_i}{r_j} \right)}{\log_{10} \left( \frac{r_2}{r_1} \right)}, \quad i = 3, \ldots, N
\]  

(6)

Let \( u_i = \frac{P_i - P_j}{P_i - P_2} \), (6) becomes:

\[
u_i \log_{10} \left( \frac{r_2}{r_1} \right) = \log_{10} \left( \frac{r_i}{r_j} \right)
\]

\[
u_i \left( \log_{10} \left( r_2 \right) - \log_{10} \left( r_1 \right) \right) = \left( \log_{10} \left( r_i \right) - \log_{10} \left( r_j \right) \right)
\]

\[
u_i \log_{10} \left( r_2 \right) - \left( u_i - 1 \right) \log_{10} \left( r_1 \right) = \log_{10} \left( r_i \right)
\]

(7)

With the measurement noise, a novel cost function can be modelled from (7):

\[
\min_{(x, y, z)} \sum_{i=3}^{N} \left[ \nu_i \log_{10} \left( r_2 \right) - \left( u_i - 1 \right) \log_{10} \left( r_1 \right) \right] ^2
\]

(8)

Subject to

\[
r_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}
\]

Compared the proposed objective function with that (3) of the existing algorithm, the computational complexity of the proposed algorithm is much less than that of (3) since the number of unknown parameters reduces from 4 to 2.

IV. Simulation Results

A square region of dimensions 40m \( \times \) 40m is considered for simulations, where the position of MS is uniformly distributed in the square space \( 0 \leq x, y \leq 40 \) m and the coordinates of BSs are BS1 \((0,0)\) m, BS2 \((40,40)\) m, BS3 \((40,0)\) m, BS4 \((0,40)\) m, BS5 \((40,20)\) m, BS6 \((20,40)\) m, BS7 \((0,20)\) m, BS8 \((20,0)\) m and BS9 \((20,20)\) m.

The RMSEs are defined as \( \sqrt{E \left[ (x - \bar{x})^2 + (y - \bar{y})^2 \right]} \) in the units of m, and are obtained from the average of 50 independent runs. To compare with the proposed method, LS method [8], high dimension search method (3), and CRLB are selected here.

Fig. 1 shows the RMSEs versus standard deviations (stds) of range measurements. Fig. 2 shows the computational complexity between proposed algorithm and high dimensional search algorithm when the step of the search is set to 0.2 m in the simulation. It can be seen from fig. 1 that the proposed method can provide similar performance as the four-dimensional search method. However, it can be seen from fig. 2 that computational complexity of the proposed algorithm is much less than the high dimensional search algorithm. The computational complexity of proposed algorithm is only 1.3 percent of original algorithm. Consequently, the proposed algorithm has the better holistic performance than the original algorithm.
V. Conclusions

RSS based location method is a very important geolocation technique due to its low complexity and cost of devices. The existing RSS location methods are based on high-dimension search. The inefficiency incurred by these algorithms may not be feasible to be applied in a practical system. In this paper, a novel RSS location algorithm is proposed to reduce computational complexity using dimension reduction technique. Simulations show that the proposed method can provide similar performance as the existing four-dimensional search methods whereas the computational burden of the proposed method is much less.

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References
