

The valuation of the loss caused by space debris

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Abstract. This paper main provides some mathematical methods to calculate the distribution of the space debris and the damage they brought each year. Through the statistical data recorded by NASA, we can get the flux and the amounts through Kessler model. When combined with NASA's Break up model, the debris' classification and the average loss of debris which can destroy the satellites in different orbits can be derived.

Introduction

The amount and the distribution of space debris has been a growing concern. The space debris, also called orbital debris, they have high velocity and are currently being tracked as potential hazards to space craft. This problem gradually triggered public attention since the Russian satellite Kosmos-2251 and the USA satellite Iridium-33 collided. Today, the number of tracked objects in orbit will grow exponentially over time due to satellite collisions producing large numbers of fragments. But, most people still think that the debris in the space are still less and only the debris whose diameters are bigger than 0.5 are dangerous. Therefore, to let the public know how much the debris in the orbits, how small a debris can destroy a satellite and how big the damage the debris caused a year is very important.

Temporal-Spatial Distribution of Space Debris

Prediction about the amount of space debris with different diameters

According to the large amounts of data, accumulated flux F of space debris can be fitted by Kessler model[1].

It is expressed as follows:

$$F(d, h, i, t, S) = H(d)\Phi(h, s)\Psi(i)[F_1(d)g_1(t) + F_2(d)g_2(t)]; \quad (5)$$

Where l is the diameter of space debris, t means time. h is height which ranges from 350 to 2000. S is the solar flux on 10.7cm wavelengths in the year of $t-1$, i is inclination.

$$H(d) = \left[10 \exp \left(- \frac{(\log_{10} d - 0.78)^2}{0.637^2} \right) \right]^{\frac{1}{2}}; \quad (1)$$

$$\Phi(hnS) = \frac{\Phi_1(h, S)}{\Phi_1(h, S) + 1}; \quad (2)$$

$$\Phi_1(hnS) = 10^{h/200 - S/140 - 1.5}; \quad (3)$$

$$F_2(d) = 8.1 \times 10^{10} (d + 700)^{-6}; \quad (4)$$

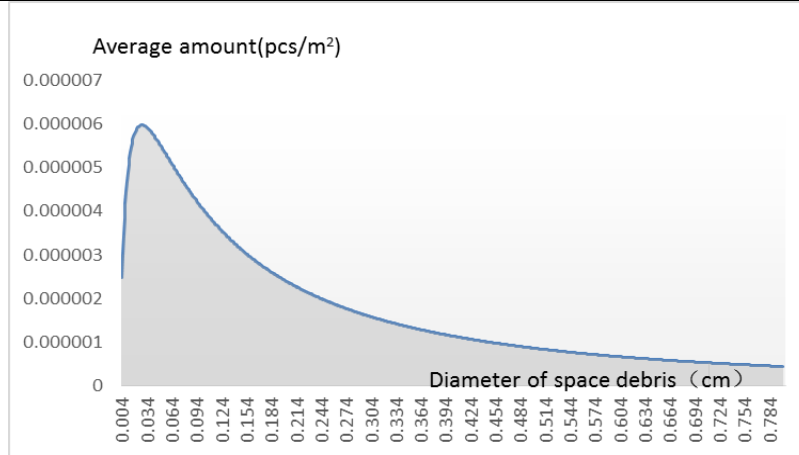
$$g_1(t) = (1 + q)^{t-1988}; \quad (5)$$

$$g_2(t) = 1 + p(t - 1988); \quad (6)$$

While other quantities are fixed, the diameter of space debris varies from 0.001 to 0.8. To realize the purpose, we use *MATLAB* to simulate the problem. The table 3 is shown as follows:

Table 3: The density of debris with different diameters

Diameter(cm)	0.001	0.002	0.003	0.004	...	0.799	0.8
Average amount(pcs/m ²)	4.57E-07	1.19E-06	1.89E-06	2.49E-06	...	4.46E-07	4.45E-07



Qualitative and Extended Analysis

Combined with the quantitative analysis above, we make a qualitative analysis. Evidence we had indicates mass of different sizes of space objects. We can observe it in following table:

Table 6: mass of different sizes of space objects

mass of different sizes of space objects				
size/cm	amount	amount rate	quality/t	quality rate/%
>10	8000	0.02	1998.6	99.93
1—10	110000	0.31	0.7	0.035
0.1—1	35000000	99.67	0.7	0.035
total	35118000	100	2000	100

According to the above data, we can see that most objects' size are less than 1cm. We reach a conclusion that debris are mainly concentrated in the range of 0 to 1. The prediction of NASA agrees well with our conclusion.

There are three orbits that we should consider. If we regard their heights are 5000km, 6000km and 36000km, then discuss the probability of collision happened in different orbits. From the conclusion above, diameters of most of debris are less than 10cm. In the paper we only consider most debris.

The cost of the space debris

According to the NASA's Breakup Model, area-mass ratio(A/M) is a normal distribution[2]:

$$D_{A/M} = \alpha(\lambda) \cdot N(\mu(\lambda), \sigma(\lambda)); \quad (7)$$

Where $\alpha(\lambda)$, $\mu(\lambda)$ and λ are:

$$\alpha(\lambda) = \begin{cases} 0 & \lambda \leq -1.95 \\ 0.3 + 0.4 \cdot (\lambda + 1.2) & -1.95 \leq \lambda \leq 0.55 ; \\ 1 & 0.55 \leq \lambda \end{cases} \quad (8)$$

$$\mu(\lambda) = \begin{cases} -0.3 & \lambda \leq -1.75 \\ -0.3 - 1.4 \cdot (\lambda + 1.75) & -1.75 \leq \lambda \leq 1.25 ; \\ -1 & 1.25 \leq \lambda \end{cases} \quad (9)$$

$$\lambda = \log_{10}(d) ; \quad (10)$$

From the law of gravitation and kinetic energy theorem:

$$V_{cir} = \sqrt{\frac{GM}{R}} ; \quad (11)$$

$$E_k = \frac{1}{2}mv^2 ; \quad (12)$$

We can calculate the speed of LEO, MEO and GEO orbits: 5.7138km/s(LEO), 5.4786km/s(MEO), 2.9682km/s(GEO). According to the consenses of NASA, the Unit destroy energy is 40.22kJ , it represents the energy that debris impact the satellite and destroy it. Using unit destroy energy combined with the Surround kinetic energy of each debris, the quality of the smallest debris in different orbits can be got: 0.6739g, 0.733g, 2.5063g. From this the shortest diameter which can destory the satellites in the obrits can be calculated: 0.35cm,0.425cm,1.15cm.

The flux in one year varies in different orbits: 0.0084 impact/yr, 0.0032 impact/yr, 0.0053 impact/yr. Plugging the shortest diameter into Kessler model, we obtain flux with different diameters: 0.0032 impact/yr, 0.0027 impact/yr, 0.0016 impact/yr.

On the basis of reference, the quality of satellite is 273kg. Combined with the empirical formulae of the satellite, we know the cross-sectional area of satellite:

$$X_{section} = 10^{(0.329 \log_{10}(M_{satellite}) - 0.86)} ; \quad (13)$$

Where $X_{section}$ is the cross-sectional area, $M_{satellite}$ represents the quality of satellite. The probability of a satellite being destroyed by debris:

To get $E(i)$, we scan the paper from others[5]:

$$E(i) = \frac{V_{cost}}{N_{total}} ; \quad (14)$$

$$N_{total} = N_{cata \log e} \cdot \frac{F}{F'} ; \quad (15)$$

Where $N_{total}=402000$, $F = 0.0084, 0.0032, 0.0053$ and $F' = 0.0032, 0.0027, 0.0016$. F and F' are obtained from model I. V_{cost} is the fixed cost of private firm. The satellites' numbers of LEO,MEO,GEO are 229,383,880 , each satellite costs 275 million dollors averagely. According to the formula 25, V_{cost} equals to \$164.349 , \$231.923 and \$315.779 in different orbits.

Conclusions

There are numerous dangerous debris in LEO, MEO, GEO. The debris whose diameters are bigger than 1cm with the mass less than 1g in LEO, MEO, GEO can cause big damage. The

mathematical method we use contained many experience formulate which makes our conclusion inexact, need to be improved.

References

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