

Analysis of hybrid immunization strategy in complex networks

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Abstract. In this paper, based on the classic SIR epidemiological model, we consider introducing hybrid immunization strategy including targeted immunization and acquaintance immunization to research on the proportion of infected nodes when the spreading of disease ends and the time steps of disease spreading. We use Monte Carlo numerical simulation in this paper, and compare the result with that of single targeted immunization strategy and single acquaintance immunization. It can be concluded from the research that in hybrid immunization strategy, the influence of targeted immunization is greater than acquaintance immunization, and the image curve of simulation result is closer to that of single targeted immunization strategy.

Introduction

In the past several years, the networks information technology has developed rapidly. Safely and efficiently operation of complex networks is in an increase of need in our daily life and production [1-4].

One of the main reasons why the study of disease spreading in complex networks is put a lot emphasis on is that diseases spread in a rather high speed in some networks and thus cause numbers of social problems. To research on the topic of disease spreading in various kinds of complex networks, the spreading of diseases in given complex networks must be analogue simulated. Its aim is to stress on studying real dynamics of the disease and find out methods to control or even eradicate the spread of disease. Finding proper immunization strategy contributes to prevent and control infectious disease spreading. Targeted immunization and acquaintance immunization are two classic immunization strategies [5-7].

However, these two classic immunization strategies both have disadvantages. Targeted immunization strategy requires a precise understanding of networks' global information. Though acquaintance immunization strategy can avoid the problem of targeted immunization strategy, its immune effect differs pretty much in networks of different structures. So it occurs to us that there may have some new findings in analogue simulation if these two classic immunization strategies are mixed.

Therefore, according to SIR epidemiological model, we analogue simulate disease spreading in BA scale-free networks model and adopt mixed immunization strategy of targeted immunization and acquaintance immunization to immune different proportion of nodes in the networks. By changing the proportion of targeted immunization or acquaintance immunization in hybrid immunization strategy, it can be known that targeted immunization strategy takes the leading role in mixed immunization strategy.

The Simulation Algorithm

To find a proper immunization strategy is helpful to prevent and control the spread of infectious diseases. Targeted immunization and the acquaintance immunization strategy are both two classic immunization strategies. Could there be better effect or some characteristic of the hybrid immunization strategy if the two kinds of immunization are mixed? To find this we carried out a series of researches. The method of Monte Carlo numerical experiment is mainly used to analyze

the simulation experiments based on the existing networks structure.

Firstly, the BA networks model is set up as follows: the BA network is built from a network with 40 nodes and 20 random-wired links. Every time a new node is introduced into the network, it will attach to 20 existing nodes. The probability that an existing node is chosen to be connected by the new node is proportional to the degree of the existing node. This procedure is repeated for 4960 times. Thus, the BA network has 5000 nodes and the average degree is $\langle k \rangle = 39.76$.

Secondly, it comes to the SIR epidemiological model used in the simulation: In the SIR epidemiological model, each node of the networks is divided into the Susceptible node, Infected node and Removed node. The Infection node (disease seed node) is the source of disease transmission, which infects the Susceptible node with a certain probability of disease transmission. The Infected node is also removed with a certain probability as to obtain immunity or to die, which will not be infected with the disease again.

Infectious diseases transmit in the generated networks model in accordance with the SIR epidemiological model. Without loss of generality, we unify the spread of the disease as 0.1 and the proportion of the disease seed node as 0.05. At each time step, the Infected node will infect the Susceptible node randomly and then remove to be the Removed node.

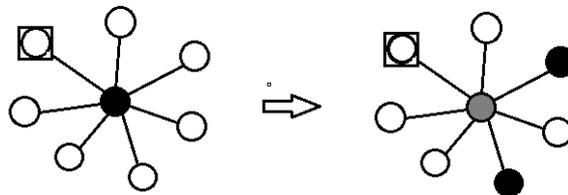


Fig.1. The illustration of SIR epidemiological model

In the Figure1, the black solid node represents the disease seed node or the Infected node, the white hollow node represents the Susceptible node, the gray solid node is represents the Removed node and the square frame of the white hollow node represents the immune node. When a time step begins, the disease seed node or the Infected node will infect all the Susceptible nodes which are connected with it into the Infected nodes randomly. But the immune node cannot be infected. When a time step ends, the original disease seed node or Infected node is removed into the Removed node and will be no longer infected in the next time step.

At the end of the simulation, all the time steps that were spent and the proportion of all the nodes that were infected by the disease are obtained.

The Analysis of Simulation Results

In each simulation, compare the results of a single immunization strategy with the hybrid immunization strategy including targeted immunization and acquaintance immunization.

In the simulation of single targeted immunization strategy, we choose the proportion of the nodes to be targeted immune according to the nodes' degree from large to small in the BA networks;

In the simulation of single acquaintance immunization strategy, we choose the proportion of the nodes in the BA networks to be acquaintance immune;

In the simulation of hybrid immunization strategy, firstly we choose the proportion of the nodes to be targeted immune according to the nodes' degree from large to small in the BA networks. Secondly we choose the proportion of the nodes in the BA networks to be acquaintance immune;

Then we choose the proportion of the nodes to be the disease seed nodes. Note at this point that the disease seed nodes are selected from the 5000 nodes randomly. So when the selected node has been immune, we need to choose the node randomly again until the nodes are not immune. After the selection of the disease seed nodes have been done, infectious diseases are spreading in the BA networks according to the SIR epidemiological model. At the end of the spreading, we obtain the time steps that were spent and the proportion of the nodes that were infected.

Every simulation is repeated 100 times to obtain the average the simulation results.

In the first kind simulation, the proportion of the acquaintance immune nodes is fixed to 0.1 in

the hybrid immunization strategy. Then we change the proportion of the targeted immune nodes.

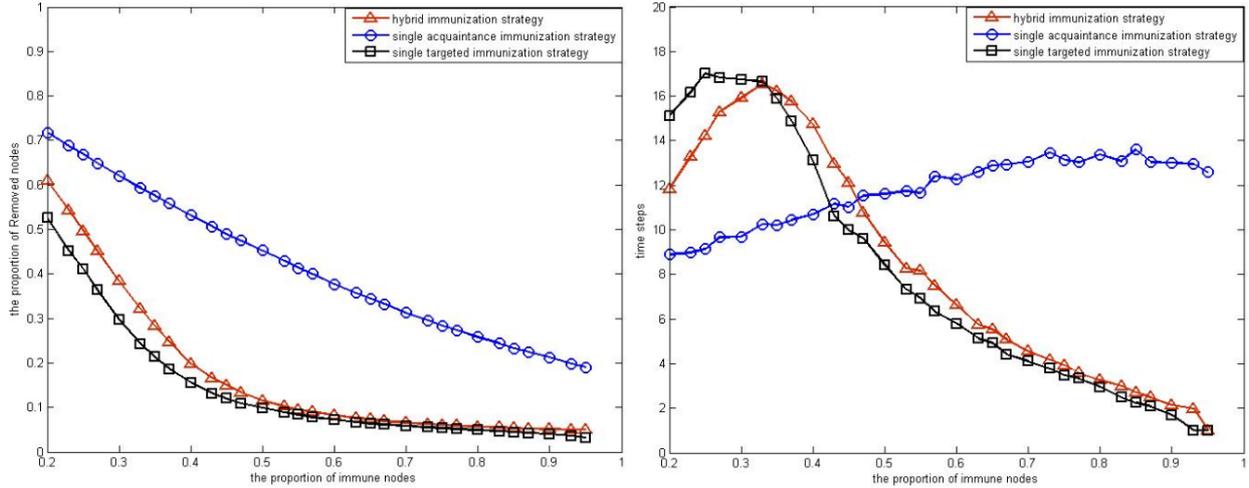


Fig.2. (Left) The proportion of Removed nodes curves (Right) The time steps curves

The results of the simulation of the SIR epidemiological model in the BA networks show in Figure2.

In Figure2 (Left), horizontal coordinate is the proportion of the immune nodes. In hybrid immunization strategy the horizontal coordinate means the sum of the proportion of the targeted immune nodes and the acquaintance immune nodes (all the below Figure are so). Longitudinal coordinate is the proportion of the Removed nodes at the end of the simulation. From Figure2 (Left), we can see that the curves of the proportion of the three kinds of immunization strategy descend with the increase of the proportion of the immune nodes. From top to bottom are the curves of acquaintance immunization, hybrid immunization and targeted immunization strategy. And the trend of the curve of hybrid immunization strategy is more close to the curve of targeted immunization strategy. The characteristic of the BA networks model is that the number of nodes with large degree (hub node) is much less than the nodes with small degree. In the targeted immunization strategy we choose the proportion of the nodes to be immune according to the nodes' degree from large to small and also nodes with large degree contributing to the disease spreading more than the small. So at the beginning the curve of the proportion of Removed nodes descends significantly. After most nodes with large degree have been immune, there is still a large part of nodes in the BA networks and most of them are the nodes with small degree which contribute to the disease spreading rather little. So when this part of nodes is chosen to be targeted immune, the proportion of Removed nodes descends insignificantly. For this reason, the curve of the proportion of Removed nodes appears to descend quickly at the beginning and then descend slowly. So we come to the conclusion that the targeted immunization plays the leading role in this simulation of hybrid immunization strategy from this simulation results. When the proportion of immune nodes in both hybrid immunization and single targeted immunization strategy are at the same, the number of nodes with large degree in hybrid immunization is a little less than single targeted immunization strategy. That is the reason why the curve of hybrid immunization is located at the top of the curve of single targeted immunization strategy.

In Figure2 (Right), horizontal coordinate is the proportion of the immune nodes. Longitudinal coordinate is time steps that have been spent at the end of the disease spreading. From Figure2 (Right) we can see the curve of time steps in the single acquaintance immunization strategy ascends with the increase of the proportion of the immune nodes and descends when the proportion is getting very large. The curve of single targeted immunization strategy is located at the top of the Figure2 (Right) at the beginning and descends after the fluctuation of proportion has been 0.25 to 0.35. Combining Figure2 (Left) and (Right) we find that in the targeted immunization part of nodes with large degree have been immune at the beginning, but there are still a few nodes with large degree so that the disease can infect some of them. However, the possibility of the disease transmission is greatly reduced and the transmission efficiency is reduced too, so the time steps are increasing. According to the characteristic of the BA networks model, when most nodes with large

degree have been immune at the first time step, some disease seed nodes that only connect with the nodes with large degree that have been immune cannot infect other nodes and will be removed into the Removed nodes at the end of the first time step. Some disease seed nodes that connect not only some nodes with large degree but also some nodes with small degree can infected these nodes randomly and then become the Removed nodes. Since the number of nodes that can contribute to disease spreading decreases with the increase of the proportion of immune nodes when most nodes with large degree have been immune, the disease spreading finishes quickly and time steps gets down. The trend of the curve of hybrid immunization strategy goes closely to the single targeted immunization and just to the right of some relative sliding. So we also get the conclusion that the targeted immunization plays the leading strategy. The analysis of the curve of hybrid immunization is almost the same with the analysis of the single targeted immunization. And the reason why the curve is just to the right of some relative sliding is also the same with the reason in Figure2 (Right).

To compare with previous simulation results, we conduct another disease spreading simulation where the proportion of targeted immune nodes is fixed and the proportion of acquaintance immune nodes is changed in the hybrid immunization strategy.

In the second kind simulation, the proportion of the targeted immune nodes is fixed to 0.1 in the hybrid immunization strategy. Then we change the proportion of the acquaintance immune nodes.

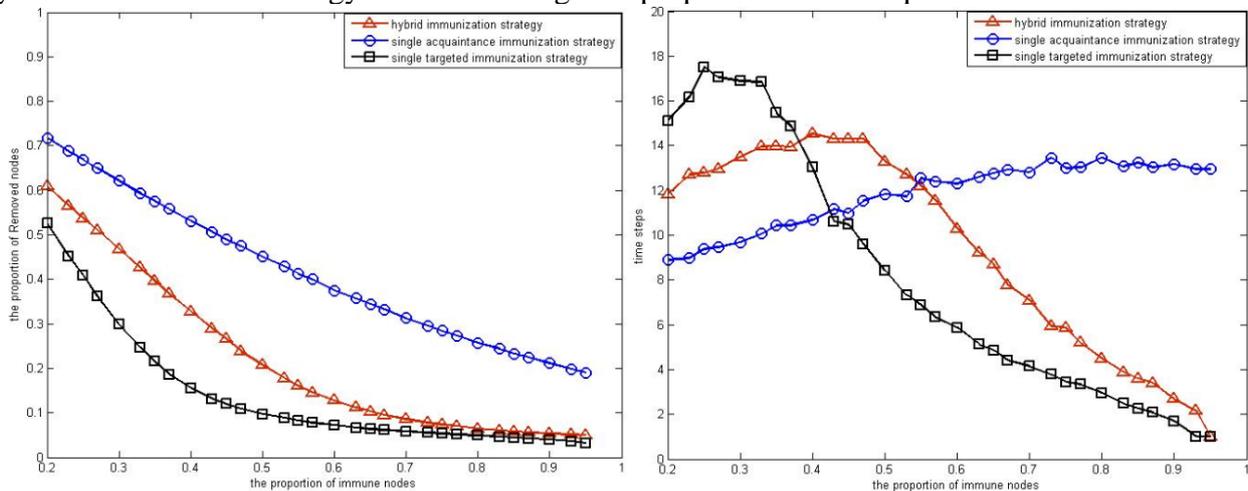


Fig.3. (Left) The proportion of Removed nodes curves (Right) The time steps curves

In Figure3 (Left) we can see that the trend of the curve of hybrid immunization strategy is similar to the curve in Figure2 (Left). The difference is that the curve in Figure2 (Left) is more of a deviation from the single targeted immunization strategy than the curve in Figure2 (Left). That means although in this simulation the proportion of the targeted immune nodes is fixed to 0.1, targeted immunization still plays the leading role. There are only few of nodes with large degree (hub nodes) in the BA networks so even if the proportion of the targeted immune nodes is small it still gets most hub nodes to be immune. At the beginning, with the increase of the proportion of immune nodes, though the curve of the proportion of the Removed nodes descends slower than the curve of single targeted immunization, it still descends faster than the curve of single acquaintance immunization. And it gets closer to the curve of single targeted immunization strategy as the proportion of immune nodes gets larger. So the targeted immunization in hybrid immunization strategy still plays the leading role than acquaintance immunization.

In Figure3 (Right) the trend of the curve of time steps in the hybrid immunization strategy is similar to the curve in Figure2 (Right). The difference is that the curve in Figure3 (Right) becomes smoother than the curve in Figure2 (Right) but it is still close to the curve in the single targeted immunization strategy. The analysis of the curve in Figure3 (Right) is almost the same with the analysis in Figure2 (Right). Because in Figure3 (Right) the proportion of targeted immune nodes is fixed to 0.1, if we want the same proportion of the nodes with large nodes to be immune we need more proportion of acquaintance immune nodes. So the curve of hybrid immunization in Figure3 (Right) descends later than the curve in Figure2 (Right). The reason why the curve is smoother is also that in Figure3 (Right) the increase of proportion is acquaintance immune nodes. So both the

disease transmission efficiency of lower level at the beginning and the difficulty increase of upper level in the networks are smaller than Figure2 (Right).

According to the previous simulations, we know that no matter the proportion of whether targeted immune nodes or the acquaintance immune nodes is fixed in the hybrid immunization strategy, targeted immunization plays the leading role than acquaintance immunization. So if we change the proportion of both immune nodes in the same way, what will be the simulation result?

In the third kind simulation, the proportion of the targeted immune nodes and acquaintance immune nodes are identical and are changed in the same way simultaneously in the hybrid immunization strategy.

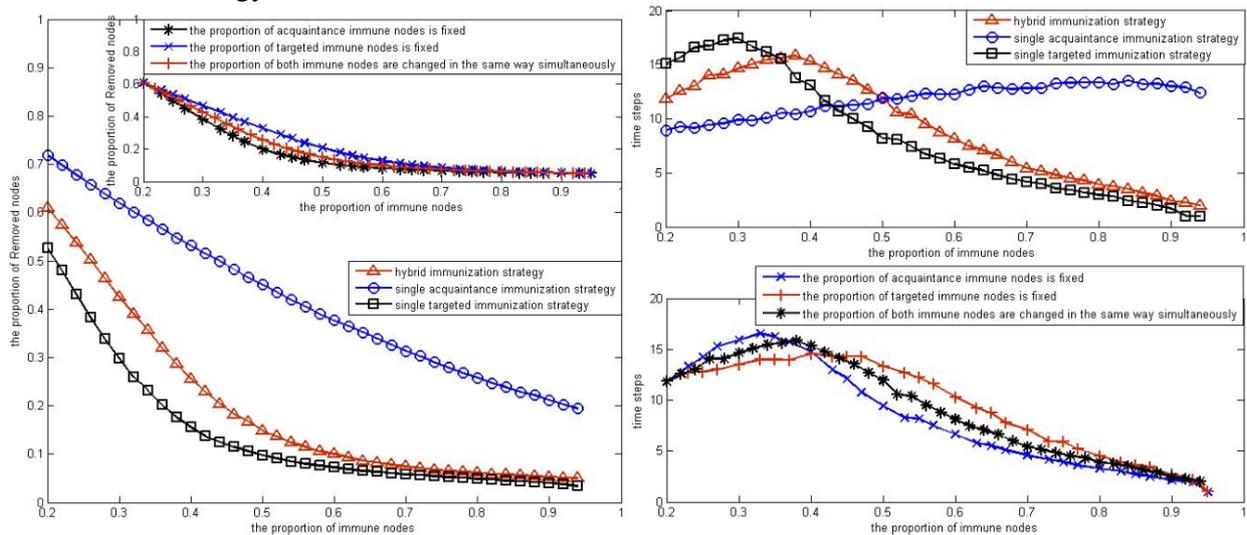


Fig.4. (Left) The proportion of Removed nodes curves (Right) The time steps curves

In Figure4, the proportion of the targeted immune nodes and acquaintance immune nodes are both initially 0.1 in hybrid immunization strategy simulation. And then both proportion increase 0.01 per time simultaneously.

In Figure4 (Left), the trend of curve of hybrid immunization strategy is similar to the curve in Figure2 (Left) too, which means that even when the proportion of the two immune nodes changes in the same way simultaneously, targeted immunization still plays the leading role in hybrid immunization strategy. The small picture in Figure4 (Left) shows the comparison of the curve of proportion of Removed nodes in the three kinds of simulations before. As can be seen, the larger the proportion of targeted immune nodes in hybrid immunization strategy is, the faster the curve descends.

In Figure4 (Right) above, the trend of the curve of time steps in the hybrid immunization strategy is similar to the curve in Figure2 (Right), which means that targeted immunization in hybrid immunization strategy determines the trend of the curve. In Figure4 (Right) below shows the comparison of the curve of time steps in the three kinds of simulations before. Among them the curve of which the proportion of acquaintance immune nodes is fixed is steepest and the curve of which the proportion of targeted immune nodes is fixed is smoothest. The curve of which the proportion of targeted immune nodes and acquaintance are changed in the same way simultaneously is between the two curves before. It means that the larger the proportion of targeted immune nodes in the hybrid immunization strategy is, the greater the change of the curve of time steps is.

The results of the three kinds of simulations are almost the same. In all these simulations, we change the proportion of the targeted immune nodes and acquaintance immune nodes to find the relation between the proportion of the Removed nodes or time steps with the proportion of immune nodes in hybrid immunization strategy. In the simulation below, the sum of the proportion of targeted immune nodes and acquaintance immune nodes is fixed. On this condition, we change the proportion of acquaintance immune nodes to observe the simulation results.

More in-depth theoretical research will be studied in the future such as the differential equations of disease transmission in the hybrid immune strategy.

Conclusion

From the 3 kinds of simulations we can get the conclusion that:

In the hybrid immunization strategy including targeted immunization and acquaintance immunization, the targeted immunization plays the leading role. In the 3 kinds of simulations, the trends of curves of the proportion of the Removed nodes or time steps are close to the curve of single targeted immunization. So the targeted immunization plays the leading role.

In the hybrid immunization strategy, the trend of the curve of time steps relates with the characteristic of the BA networks model. When the proportion of immune nodes is very small, there are still some nodes with large degree, disease can still infected parts of nodes but the efficiency of disease spreading decreases and the curve of time steps ascends. With the increase of the proportion of immune nodes, most of the nodes with large degree are immune. At this moment disease can only infect few of nodes before the disease spreading comes to the end. The curve of time steps descends.

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