

Research on grasping forces optimization of a soft-finger dexterous hand

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Abstract. To contact constraints model of soft finger, replacing paired friction cones by linear paired polyhedral, so as to realize the linearization of contact constraints model. The linear constraints inequalities of grasping forces is derived, and the objective function of the smallest sum of output torque squares is built, so the torque optimization is translated into quadratic programming. By the simplex method, quadratic programming is solved quickly, greatly improve the real-time performance. At last, the effectiveness of proposed algorithm was verified in numerical simulation with grasping a plane object when there is a soft finger contact.

Introduction

When dexterous hand is grasping objects, the grasping contact forces with grasping stability and satisfying the constraint condition aren't unique, joint torques also not only. To maximize the extent of each joint driving torque, it is need to distribute joint torques properly. The problem is reduced to the grasping forces optimization in joint space of dexterous hand.

Nakamura et.al [1, 3] scholars researched on the description of the nonlinear constraints, the calculation of initial contact forces, the real-time optimization of the algorithm and nonlinear intelligent optimization, etc. The disadvantage of nonlinear programming is that large amount of calculation will result in difficult in achieving real-time. Kumar et.al [4, 6] scholars put forward many optimization methods of contact force, researching on the simplification of point contact friction cone, the optimizing index of contact force, the linear programming and its real-time performance, etc. The disadvantage of linear programming is that contact forces of the simplified model are relatively conservative. LI Ji-ting took maximum relative carrying capacity of dexterous hand joint as objective function, and optimized the nonlinear joint torques by iterative algorithm. However, it brings difficulties to the optimization for converging and instantaneity, because of the nonlinear of constraints and complexity of objective function.

The paper replaces paired friction cones by linear paired polyhedral, with realizing the linearization of contact constraints model. Taking joint torque as variables, the linear constraints inequalities is derived, and the objective function of the smallest sum of output torque squares is built. Thus the joint torques optimization is translated into quadratic programming with completely linear constraints, and the effectiveness and real-time capacity of the optimization algorithm is illustrated by numerical simulation at last.

Linearizing the Constraints of a Soft Finger Contact Model

The contact force of constraints of a soft finger contact friction should satisfy the following conditions:

$$\sqrt{f_{ix}^2 + f_{iy}^2} / u_1 + |m_{iz}| / u_2 \leq f_{iz}, f_{iz} > 0, \quad (1)$$

where f_{iz} represents the contact normal force, f_{ix} and f_{iy} represents tangential force, m_{iz} represents

torsion moment, u_1 and u_2 represent coefficient of tangential friction and torsion moment respectively.

In order to linearizes the constraints of a soft finger contact model, Eq. (1) should be transformed equivalently as

$$\sqrt{f_{ix}^2 + f_{iy}^2} \leq u_1 f_{iz} - |m_{iz}| u_2 / u_2, f_{iz} > 0. \quad (2)$$

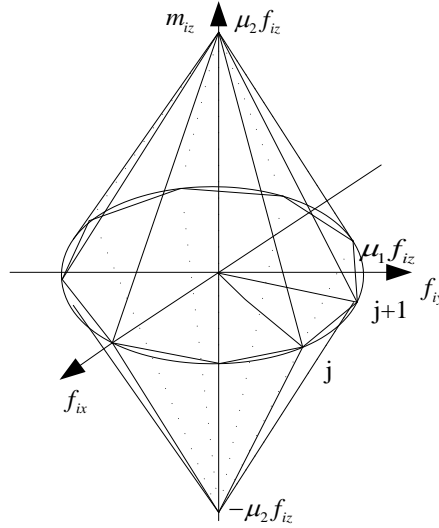


Figure 1. The linearized contact model of soft finger

In rectangular coordinate system $o_i f_{ix} f_{iy} m_{iz}$, as shown in Figure 1, f_{ix} , f_{iy} and m_{iz} with satisfying the constraints is included in bipyramidal. Linearize the round-bottomed to N-regular polygon so that it would be transformed into linear-polyhedral bipyramidal, where contact forces included in its 2N planes can be described as follows 2N linear inequalities,

$$\left. \begin{aligned} u_2(s_{j+1} - s_j)f_{ix} + u_2(c_j - c_{j+1})f_{iy} - u_1 u_2 s_N f_{iz} + u_1 s_N m_{iz} < 0 \\ u_2(s_{j+1} - s_j)f_{ix} + u_2(c_j - c_{j+1})f_{iy} - u_1 u_2 s_N f_{iz} - u_1 s_N m_{iz} < 0 \end{aligned} \right\}, \quad (3)$$

where $s_j = \sin(2\pi j / N)$, $c_j = \cos(2\pi j / N)$, $j = 1, 2, \dots, N (N \geq 3)$, $s_N = \sin(2\pi / N)$.

The contact normal force is always positive,

$$-f_{iz} < 0, \quad (4)$$

that is to say there is 2N+1 constraints inequalities in one contact point. When there are p contact points between soft fingers and objects, it can generate $4p$ contact forces \mathbf{f} , which can be described as

$$S_{(2Np+p) \times 4p} \mathbf{f}_{4p} < 0, \quad (5)$$

where $\mathbf{f} = [\mathbf{f}_1^T \quad \mathbf{f}_2^T \quad \dots \quad \mathbf{f}_n^T]^T$, $\mathbf{f}_i = [f_{ix} \quad f_{iy} \quad f_{iz} \quad m_{iz}]^T$, S represents the relevant matrix of $(2N+1)p$ inequalities' coefficient which works in concert with p contact points.

The more N gets, the more planes of polyhedral bipyramidal and constraint equations becomes, the more complex S will be, the calculation amount will increase, however, the more accurate linearization achieves. So, on the premise of assuring a certain calculation speed, it should choice a large N in calculations.

Building the Linear Constraints Inequalities of Joint Torque Section Headings

Besides the constraints of contact force within friction cone, joint torque must have been satisfying the Eq. (6) and Eq. (7) when a soft finger grasps objects stability,

$$\mathbf{G}\mathbf{f} = \mathbf{w}, \quad (6)$$

$$\mathbf{J}^T \mathbf{f} = \boldsymbol{\tau}, \quad (7)$$

where \mathbf{G} represents grasping matrix, \mathbf{J} represents Jacobian matrix, \mathbf{w} represents external wrench, $\boldsymbol{\tau}$ represents joint torque vector.

According to literature [7], contact forces, satisfying above equations, can be broken into an explicit function of \mathbf{w} and $\boldsymbol{\tau}$,

$$\mathbf{f} = \mathbf{M}_1 \mathbf{w} + \mathbf{M}_2 \boldsymbol{\tau}, \quad (8)$$

where \mathbf{M}_1 represents equivalent coefficient matrix of external force, \mathbf{M}_2 represents equivalent coefficient matrix of joint torque. Both of matrixes reflect the grasping configuration and geometric features of being grasped object, and can be obtained by \mathbf{G} and \mathbf{J} .

Take the \mathbf{f} that satisfies the Eq. (8) into the linear Eq. (5), it can derives Eq. (9) as

$$\mathbf{S}\mathbf{M}_2 \boldsymbol{\tau} < -\mathbf{S}\mathbf{M}_1 \mathbf{w}. \quad (9)$$

As can be obtained by the above equation, in the linear contact model of a soft finger, joint torque vector is not unique when the external wrench is kept constant. All vectors $\boldsymbol{\tau}$ with satisfying the above constraints constitute a feasible set in joint torque space, because of all constraints being linear, the feasible set is a convex set.

Building the Objective Function and Optimizing the Joint Torque

It should be rationally to build objective function of joint torque, for finding the optimal solution in convex set of joint torque vector.

Take the dimension limit of dexterous hand's structure and drive into consideration, the objective function, in which each joint torque being in its capacity, needs to choice driver and drive torque as small as possible for balancing the external wrench. Based on these considerations above, the objective function of the smallest sum of output torque squares is built as follows,

$$\delta = \sum_{i=1}^m \tau_i^2, \quad (10)$$

where τ_i represents the joint torque, m represents the number of joints.

The joint torque optimization can be boiled down to the following question,

$$\left. \begin{array}{l} \min \quad \delta = \sum_{i=1}^m \tau_i^2 \\ \text{s.t.} \quad \mathbf{S}\mathbf{M}_2 \boldsymbol{\tau} < -\mathbf{S}\mathbf{M}_1 \mathbf{w} \end{array} \right\}. \quad (11)$$

In grasping objects of two joints, the physical meaning of Eq. (11) can be visually described by simple graphic method: as shown in Figure 2, the shadow of polygon represents the feasible set of joint torque in which $\boldsymbol{\tau}$ satisfies the constraints of Eq. (11), because of all constraints being linear, it is a convex polygon.

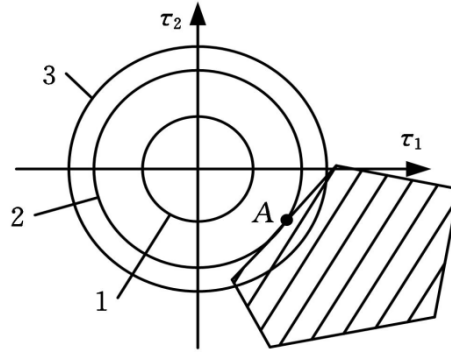


Figure 2. Optimal joint torque distribution in grasping with two joints

In Figure 2, several circles represent the distribution of joint torques in which τ meets the functional equation $\delta = \tau_1^2 + \tau_2^2$. There is no intersection between the circle 1 and feasible convex set, so the vector τ can't be obtained in circle 1. There are a plurality of intersection between the circle 3 and feasible convex set, the vector τ in intersections satisfy all the constraints but the smallest one δ , so it can't be the optimal solution. There is one tangent point A between circle 2 and feasible convex set, it is the optimum point with satisfying all constraints and making δ is smallest, the torque τ_1 and τ_2 corresponded to the point A are the optimal joint torque.

When Eq. (8) is popularized to more than one joint torque space, all of torques τ with satisfying the linear constraints constitutes a convex set in torque space, the objective function turns into a generalized sphere with changing diameter, the tangent point between convex set and generalized sphere is the optimal solution.

Analyzing the solving process of Eq. (8), all of constraints is linear, the objective function is a quadratic function, the optimal solution problem is translated into quadratic programming problem. The quadratic form of objective function is semi-positive definite and the feasible region is convex set, so it can also be translated into convex quadratic programming, and then, according to Kuhn-Tucker conditions, it can be directly translated into standard linear programming and solved with the simplex method quickly.

The Example

For the example that two-fingered of soft finger model for grasping plane object in literature[7], using optimization method in this paper to optimize the joint torque and compare the optimization result with example, verifying the correctness and real-time capacity of the proposed algorithm.

The example, as shown in Figure 3, shows the initial grasping configuration, and each finger has two joints. The tangential friction coefficients u_1 of two fingers are 0.5, the torsion moment coefficients u_2 are 0.2, the external wrench $w = [0 \quad -1N \quad 1N \quad 0.5N \cdot m \quad 0 \quad 0]$, the ultimate joint torque $\tau_{1max} = \tau_{3max} = 10(N \cdot m)$, $\tau_{2max} = \tau_{4max} = 5(N \cdot m)$.

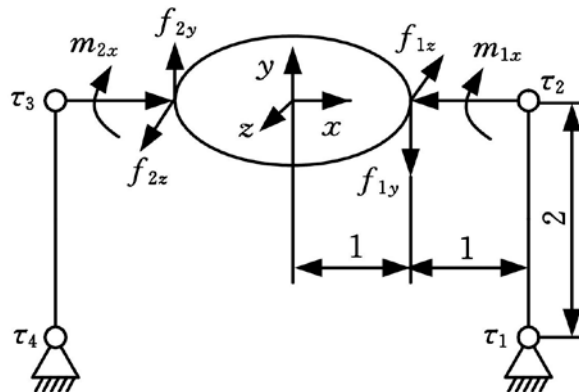


Figure 3 Grasping a plane object with two fingers in soft finger contact model

It should to calculate the grasping matrix \mathbf{G} first. Set that contact force $\mathbf{f} = [f_{1x} \ f_{1y} \ f_{1z} \ f_{2x} \ f_{2y} \ f_{2z} \ m_{1x} \ m_{2x}]^T$ and external force $\mathbf{w} = [F_x \ F_y \ F_z \ M_x \ M_y \ M_z]^T$. Based on the balanced relationship, list the components equations of contact force \mathbf{f} and external force \mathbf{w} respectively:

$$F_x = f_{1x} - f_{2x}, \quad (12)$$

$$F_y = -f_{1y} - f_{2y}, \quad (13)$$

$$F_z = f_{1z} - f_{2z}, \quad (14)$$

$$M_x = m_{1x} + m_{2x}, \quad (15)$$

$$M_y = -f_{1z} - f_{2z}, \quad (16)$$

$$M_z = -f_{1y} + f_{2y}. \quad (17)$$

According to the Eq. (6), organizing above equations to get grasping matrix and transposition of Jacobian matrix,

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{J}^T = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Linearizing the constraints of a soft finger friction and taking the $N=3$ to acquire the

$$\mathbf{S} = \begin{bmatrix} -4 & 0 & -1 & 0 & 0 & 0 & 5 & 0 \\ -4 & 0 & -1 & 0 & 0 & 0 & -5 & 0 \\ 2 & -3.5 & -1 & 0 & 0 & 0 & 5 & 0 \\ 2 & -3.5 & -1 & 0 & 0 & 0 & -5 & 0 \\ 2 & 3.5 & -1 & 0 & 0 & 0 & 5 & 0 \\ 2 & 3.5 & -1 & 0 & 0 & 0 & -5 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & -4 & 0 & -1 & 0 & -5 \\ 0 & 0 & 0 & 2 & -3.5 & -1 & 0 & 5 \\ 0 & 0 & 0 & 2 & -3.5 & -1 & 0 & -5 \\ 0 & 0 & 0 & 2 & 3.5 & -1 & 0 & 5 \\ 0 & 0 & 0 & 2 & 3.5 & -1 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}.$$

Based on the grasping matrix \mathbf{G} and transposition of Jacobian matrix \mathbf{J}^T , it can develop the equivalent coefficient matrix of external force and joint torque as follows,

$$\mathbf{M}_1 = \begin{bmatrix} 0.2 & -0.1 & 0 & 0 & 0 & -0.16 \\ 0.1 & -0.3 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 0.5 & 0 & -0.5 & 0 \\ -0.2 & 0.1 & 0 & 0 & 0 & -0.16 \\ 0.1 & -0.3 & 0 & 0 & 0 & 0.33 \\ 0 & 0 & -0.5 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}, \mathbf{M}_2 = \begin{bmatrix} 0.4 & -0.13 & -0.1 & -0.03 \\ -0.05 & -0.32 & -0.05 & -0.02 \\ 0 & 0 & 0 & 0 \\ 0.1 & -0.03 & 0.4 & -0.13 \\ -0.05 & 0.02 & -0.05 & 0.32 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Taking the above parameters into Eq. (10) and using the simplex method to achieve the final optimization outcome as follows,

$$\boldsymbol{\tau}_{\text{opt}} = [3.98 \quad 0.86 \quad -3.98 \quad -0.25]^T (\text{N}\cdot\text{m}). \quad (18)$$

The optimization is accomplished by a computer with Pentium-IV 2.8GHz, 1024 MB RAM, and the calculation time is less than 0.1s, so it possesses better real-time capacity.

If joint torque is optimized by the method in literature [7], joint forces will be

$$\boldsymbol{\tau}'_{\text{opt}} = [5.4 \quad 1.0 \quad -5.4 \quad -0.29]^T (\text{N}\cdot\text{m}). \quad (19)$$

Compare the data in Table 1, the four absolute values of joint torque obtained by this paper's method are less than the results of literature [7]. It illustrate that, under the same conditions, the joint torque obtained by this paper's optimization method is smaller, optimization results is more reasonable.

Table 1. Comparing the optimization results [N•m]

	τ_1	τ_2	τ_3	τ_4
Optimization results of literature[6]	5.4	1.0	-5.4	-0.29
Optimization results of this paper's method	3.98	0.86	-3.98	-0.25

Summary

For grasping objects of soft finger dexterous hand, linearizing the constraints of a soft finger contact model, the joint torque optimization method which is taking the smallest sum of output torque squares as objective function is proposed. The example illustrate that linearizing the soft finger contact constraints can greatly simplify the contact force constraints model, the optimization method can deduce the smallest joint torque quickly with satisfying the requests of joint structure and drive. So thus make each applied joint torque is smaller in grasping the same object and grasp heavier object by a dexterous hand with same configuration.

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