Mathematical Model on Reliability of Variation Process of Rolling Bearing Vibration Performance

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Abstract. A reliability evaluation model is created to analyze the variation process of time series based on the maximum entropy and grey bootstrap principles. The original sample data is collected by measuring the vibration acceleration of time series. Then the simulated vibration acceleration data is obtained using Monte Carlo method for different wear diameters. According to the maximum entropy principle, the estimated true value and confidence interval can be calculated for the reliability parameter of the intrinsic sequence. The grey bootstrap principle is applied to obtain a large number of sample data by resampling from the parametric sample data. The variation frequencies of time series can be achieved using Poisson counting principle. The estimated true value function and the upper and lower bound functions of reliability are achieved based on the variation frequencies. The variation probabilities can be calculated for different time series. The reliability evaluation model is proven to be able to be used to take intervention measures before the vibration performance fails, which is under the condition that the possibility distribution is unknown for sample data.

Introduction

Rolling bearing is one of the most commonly used mechanical equipment, which is an important supporting of shaft and other rotating components. Whether its performance is good or bad plays a very important role for the normal operation of the equipment. The prediction on performance reliability variation (degradation) process of rolling bearings is to predict the failure probability and performance reliability of rolling bearings according to the current performance data [1]. Then the variation process of bearing performance is studied to find the risks that the vibration performance will vary, so nasty accident will be avoided. Therefore, it is of great academic and application value to study the performance variation process of rolling bearings.

The available performance reliability prediction methods generally assume that the probability density functions and the performance failure thresholds of samples are known, so the performance reliability can be calculated and analyzed. For example, Ali proposed a method based on the data-driven prognostic approach and explored the combination of Simplified Fuzzy Adaptive Resonance Theory Map (SFAM) neural network under the condition of Weibull distribution [2]. Xia raised the bootstrap weighted-norm method and assumed that the life of rolling bearings is in accordance with three-parameter Weibull distribution to evaluate the optimum confidence interval of reliability [3]. Jiang put forward the modified MLE (MMLE) for estimating the parameters based on the zero-failure data and assumed that product lifetime followed a Weibull distribution [4]. Dan transformed the Weibull distribution into exponential distribution by using rolling bearing zero-failure data to calculate the characteristic life estimation values of Weibull distribution [5]. However, some probability distribution information on performance degradation is unknown. For example, the failure probability distributions of the vibration and noise, friction torque and fatigue life performances are still unknown or uncertain. Xia fused the grey bootstrap

principle into Poisson process and proposed the grey bootstrap Poisson method to forecast the vibration performance reliability variation process of rolling bearing with an unknown probability distribution [6].

For sample data with unknown probability distributions, it is difficult in using available methods to solve the reliability assessment problems. So far, it has been an important scientific and technical problem. In view of this, this paper fuses the maximum entropy principle [7,8,9] into the grey bootstrap principle [10,11] and the model for evaluating the variation process of vibration performance reliability of rolling bearings is proposed under the condition without any probability distribution or prior information on sample data. The evaluation model can be used to take intervention measures before vibration performance of bearings fails to avoid the occurrence of serious accidents.

Mathematical Model

The time sequences are obtained by periodic sampling of vibration acceleration of rolling bearings. The intrinsic sequence is marked as the first time series and expressed by the vector X_0 as follows:

$$X_0 = (x_1, x_2, ..., x_k, ..., x_N) \tag{1}$$

where x_k is kth performance data, $k = 1, 2, 3, ..., N(N \geq 1000)$; N is the number of performance data in the intrinsic sequence.

The nth time series is divided into D sequences, and the sample data of the dth sub sequence is expressed by vector X_{nd}.

$$X_{nd} = (x_{nd}(1), x_{nd}(2), ..., x_{nd}(l), ..., x_{nd}(L)) \tag{2}$$

where x_{nd} is the sample data of the dth sub sequence of the nth time series; d is the sequence number of sub series. $d = 1, 2, 3, ..., D$; $x_{nd}(l)$ is the lth data of the dth sub sequence of the nth time sequence; l is the sequence number of performance data in the sub sequence, $l = 1, 2, 3, ..., L$; L is the number of sample data in the sub sequence, $L \times D \geq 1000, D \leq 10$.

Maximum Entropy Principle. According to the maximum entropy principle, the probability density functions should meet the condition that the entropy value is the maximum.

$$H = -\int_\Omega f(x) \ln f(x) dx \rightarrow \max \tag{3}$$

where H is the information entropy; Ω is the feasible region for random variable x, $f(x)$ is the probability density function of continuous variable x.

Eq. (3) should meet the constraint conditions as follows:

$$\int_\Omega x^i f(x) dx = m_i; i = 0, 1, ..., m; m_0 = 1 \tag{4}$$

where i is the order of origin moment; m_i is the ith origin moment; m is the order of the highest origin moment.

All origin moments of the performance data in the intrinsic sequence are given by

$$m_i = \frac{1}{N} \sum_{k=1}^{N} (x_k)^i \tag{5}$$

where k is the sequence number of performance data, $k = 1, 2, 3, ..., N(N \geq 1000)$; N is the number of performance data of the intrinsic sequence; i is the order of origin moment.

The probability density function $f(x)$ can be expressed based on Lagrange multiplier method as follows:

$$f(x) = \exp\left(\sum_{i=0}^{m} c_i x^i\right) \tag{6}$$

where c_i is the ith Lagrange multiplier; i is the order of origin moment, $i = 0, 1, ..., m$; m is the order of the number of highest order of origin moment.

The first multiplier c_0 can be given by

$$c_0 = -\ln(\int_\Omega \exp(\sum_{i=1}^m c_i x^i) dx) \qquad (7)$$

Other m multipliers should meet the condition as follows:

$$m_i \int_\Omega \exp(c_i x^i) dx - \int_\Omega x^i \exp(\sum_{j=1}^m c_j x^j) dx = 0; i = 1, 2, ..., m \qquad (8)$$

Set a significant level, $\alpha \in [0,1]$, the maximum entropy estimated interval of continuous variable x can be calculated as follows:

$$[x_L, x_U] = [x_{\frac{\alpha}{2}}, x_{1-\frac{\alpha}{2}}] \qquad (9)$$

where x_L is the lower boundary value of estimated interval; x_U is the upper boundary value of estimated interval.

The estimated true value x_0 of performance parameter of time series is given by

$$x_0 = \int_\Omega x f(x) dx \qquad (10)$$

The maximum entropy estimation interval $[x_{L0}, x_{U0}]$ of the intrinsic sequence is calculated, where x_{L0} is the lower bound value of the estimated interval and x_{U0} is the upper bound value of estimated interval.

Prediction on the Variation Probability. Count the number N_{nd} that performance data of the dth sub sequence of the nth time series falls outside the estimated interval $[x_{L0}, x_{U0}]$. Based on the Poisson counting method, the frequencies that performance data falls outside the estimated interval can be given by

$$\lambda_{nd} = \frac{N_{nd}}{L} \qquad (11)$$

Grey Bootstrap Principle. The grey bootstrap principle fuses the bootstrap principle into grey prediction principle GM (1, 1). A data sequence vector λ can be given by

$$\lambda = \{\lambda_{n1}, \lambda_{n2}, ..., \lambda_{nD}; n = 2, 3, ..., r\}$$
(12)

B bootstrap resampling samples can be expressed by vector $Y_{Bootstrap}$ as follows:

$$Y_{Bootstrap} = (Y_1, Y_2, ..., Y_b, ..., Y_B) \qquad (13)$$

where Y_b is the bth sample; b is the order number of sample, $b = 1, 2, ..., B$.

$$Y_b = \{y_b(u)\}$$
(14)

where $y_b(u)$ is the uth re-sampling sample of Y_b; $u = 1, 2, ..., D$.

According to the grey prediction principle GM (1,1), the predicted value $\hat{y}_b(w)$ can be given by

$$\hat{y}_b(w) = \lambda_b(w) - \lambda_b(w-1); w = T + 1$$
(15)

where w and T are certain moments in the future.

Sequence vector $\hat{\lambda}_w$ can be obtained with B data as follows:

$$\hat{\lambda}_w = \{\hat{y}_b(w)\}; b = 1, 2, \cdots, B; w = T + 1$$
(16)

The Estimation of Reliability Functions. The estimated true value of reliability function can be expressed by $R_n(t)$ as follows:

$$R_{n0}(t) = \exp(-\lambda_{n0}t) \tag{17}$$

where λ_{n0} is the estimated true value of variation frequency of the nth sequence; $n = 1, 2, ..., r$.

The upper bound reliability function can be given by with $R_{n\mathrm{U}}(t)$:

$$R_{n\mathrm{U}}(t) = \exp(-\lambda_{n\mathrm{U}}t) \tag{18}$$

where $\lambda_{n\mathrm{U}}$ is the upper bound value of variation frequency of the nth sequence.

The lower bound reliability function can be expressed by $R_{n\mathrm{L}}(t)$ as follows:

$$R_{n\mathrm{L}}(t) = \exp(-\lambda_{n\mathrm{L}}t) \tag{19}$$

where $\lambda_{n\mathrm{L}}$ is the lower bound value of variation frequency of the nth sequence.

The Estimation of Variation Probabilities. The variation probabilities $P_n(t)$ of the time series are given by

$$P_n(t) = 1 - \left(\int_{t=0}^{t_n} f_1(t)\mathrm{d}t + \int_{t=t_n}^{+\infty} f_n(t)\mathrm{d}t \right) \tag{20}$$

where t is the time when bearings operate; t_n is the moment when the variant probability density functions of the nth sub sequence and the intrinsic sequence intersect, $f_1(t)$ and $f_n(t)$ are the variant probability density functions of the intrinsic sequence and the nth sub sequence, $n = 2, 3, ..., r$.

$$f_n(t) = \frac{\mathrm{d}(1 - R_n(t))}{\mathrm{d}t} = \lambda_n \exp(-\lambda_n t) \tag{21}$$

$$f_1(t) == \lambda_1 \exp(-\lambda_1 t) \tag{22}$$

Summary

According to the maximum entropy and grey bootstrap principles, a model is established to evaluate the variation process of vibration performance reliability of rolling bearings, which can be applied to calculate the estimated true value function and the reliability upper and lower bound functions. The reliability variation frequencies and variation probabilities can be obtained for different time series to study the variation process under different wear diameters.

For sample data with unknown probability distributions, the reliability evaluation model is proven to be scientific and reasonable in solving the reliability variation probability problems.

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