Estimation of Surface Fractal Dimension of Energetic Materials Based on MATLAB

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Abstract—The particle surface roughness which determined by the energetic materials surface morphology has greatly impact on thermal conductivity of multiple particles, which directly affect its sensitivity. But since the surface morphology is extremely irregular, it is difficult to describe the roughness accurately quantitatively using traditional methods. To solve this problem, the concept of surface fractal dimension based on fractal theory is given to describe the roughness in this paper. Choosing the particles of HMX and RDX as studying object, and the functions of MATLAB in image processing and numerical calculation were employed to get the surface fractal dimension of 2.794 and 2.772 respectively. A contrast experiment has been carried on between the calculation results of this method and the literature results. The results show that the calculation method is accurate and reliable, and the relative error is 0.106% and 1.042% respectively, the surface fractal dimension of HMX and RDX is 2.794 and 2.772 respectively.

Keywords—energetic materials; surface fractal dimension; MATLAB; images; friction sensitivity

1. INTRODUCTION

Fractal theory is a form of abstraction to describe the objective world which is not smooth, continuous or fragmented. In traditional European geometry, refer to the required independent or independent direction number to determine any point in the space as its space coordinates number, the point for zero dimension, a straight line for 1d, a plane for 2d and volume for 3 d. However, there are a lot of natural irregular and complex geometry which cannot be explained using conventional geometry. The emergence of the fractal theory has brought new changes for the traditional geometry. In fractal geometry, geometry dimension is continuous, it can also be an integer or a decimal fraction.

Fractal theory provides a scientific method to characterize the nature of irregularity and complexity. For a complex system with characteristic of fractal, the fractal dimension is a quantitative parameter to describe the degree of complexity of it, which measure the fractal space-filling capacity of the system and characterize the disorder of fractal systems based on measure theory and symmetry theory. It can be said, fractal theory is a new concept and methodology of science, which make us see the world from a different perspective.

Fractal theory has been applied in the field of energetic materials. Liu Limei, who use the fractal dimension to characterize the microstructure irregularities of the burning ore[1]; Lin Jinxing, who use fractal dimension to characterize the semi-solid morphology of A356 alloy[2]; Li Fengsheng, Song Xiaolan, who establish the heat conduction models of energetic particles with fractal characterization[3], including a simple model (containing only D) and mixed models (including D and Ds). In this model, using fractal dimension (D) to characterize the heterogeneous and complexity in the particle size; using a surface fractal dimension (Ds) to characterize the particle surface irregularities and complexity. Experimental results show that different D, Ds values have an impact on energetic materials thermal conductivity of the particles and its sensitivity. Energetic materials can be fired or exposed by accident when subjected to various external stimuli during the development, production, storage, transportation and using, causing major accidents and losses. Therefore, the sensitivity of energetic materials research has always been a subject which domestic and foreign counterparts focus on, this paper used a calculation method of image count box fractal dimension based on MATLAB, the energetic material surface fractal dimension (Ds) were calculated, you can then study its effect on the sensitivity.

2. THE PRINCIPLE OF BOX-COUNTING DIMENSION METHOD TO CALCULATE Ds

A. Binary Image

The digital image is composed of a series of pixels sequentially, stored in the form of a matrix. For a width of M pixels, high-N-pixel digital image, which can be regarded as a M×N matrix. Each element in the matrix corresponds to a pixel, the element value is the color or pixel color index. Pixel threshold is called gray level, usually expressed by 2n, the greater n is, brighter and
clearer the image is, the image looks soft and realistic. In
generally, the image gray level is 256 while n=8. To
extract those concerned and specific areas of physical
information in the image, the threshold segmentation
method usually be used to convert grayscale image into a
binary image, the binary image pixel is black or white,
which were characterized by 0 or 1 in the matrix. So we
get a numerical matrix containing only 0 and 1.

B. Meter Box Dimension

To give an accurate measurement to an object with
fractal characteristic, the calculation of fractal dimension is
particularly important. Fractal dimension has many
definitions according to the different measuring methods
[2], such as the Hausdorff dimension, similar dimension,
capacity dimension, information dimension and correlation
dimension. The Hausdorff dimension has won favor in
theory research for its universality of defining any fractal
characteristics, and the box dimension has been widely
used in engineering practice because it is easier to be
calculated and programmed implementation.

Fractal box dimension is also called the box-counting
dimension, which is one of the most widely used
dimension, it is generally used for its mathematics
calculation and experience estimation is relatively easier.

Set F as an arbitrary non-empty subset in \( \mathbb{R}^n \), \( N_k(F) \) as
the minimum number of subsets which cover F with the
largest diameter k, then the upper and lower box
dimension is respectively defined as

\[
Dim_F = \lim_{k \to \infty} \frac{\log N_k(F)}{-\log k}
\]

(1)

\[
Dim_F = \lim_{k \to 0} \frac{\log N_k(F)}{-\log k}
\]

(2)

If the two values are equal, we note the common value of
F the box dimension for

\[
Dim_F = \lim_{k \to \infty} \frac{\log N_k(F)}{-\log k}
\]

(3)

Box dimension has some equivalent definitions,
sometimes these definitions are more suitable for
application. In consideration of k-coordinate grid cube in
\( \mathbb{R}^n \), which of the following forms of cube:

\[
[m_1 k, (m_1 + 1)k] \times \cdots \times [m_n k, (m_n + 1)k]
\]

(4)

Of which \( m_1, \cdots, m_n \) is an integer, when calculating
the box dimension, \( N_k(F) \) is equivalent to the number of
net cubes which intersect with F with a side of k. Due to
the pixel is not infinitely small , as a result, when
calculating box dimension by the computer programming ,
the dimension can be estimated by the figure slope of
function \( N_k(F) \) relative to \( (-\log k) \).

Box-counting dimension is a commonly used method
of measuring fractal dimension of images with fractal
growth, especially two-dimensional random growth. Random
fractal is abundant in nature, so this kind of test
method has universality, but it is likely to underestimate
the fractal dimension of image when fractal dimension of
image is high. Specific computer algorithm is as follows[4]:

1) The image binarization.
The element value of matrix is either equal to 1 or 0.

2) Matrix partition.

Segment matrix, into several pieces with each
row=column=k (k=1,2,\cdots,4,2), and the matrix is divided
into \( 2^1 \times 2^1, 2^1 \times 2^2, \cdots, 2^1 \times 2^3, 2^2 \times 2^2 \) blocks. \( 2^1 \)
\( \leq \)image length.

3) Calculation of image cells.

Calculating the number of blocks contain 1 in image
matrix , and noting for \( N_1 \), to get a series of numerical values
\( N_1, N_2, \cdots, N_{(i+1)} \) and get the (i+1) couples of (k, \( N_k \)).

4) The curve fitting.

Using the least square to fit \((-\log k, \log N_k)\), a straight
line is obtained.

5) Calculation of image box dimension \( Db \).

Calculating the slope of of the fitting straight line,
image box dimension is equal to the slope of the straight
line.

It is important to note that the fractal dimension is a
collective term, the definition of fractal dimension with
different research objects are also different, that is to say,
for a certain definition of fractal dimension, is applicable
to some of the research objects, and may not apply for
other objects. Normally, adopting different definition and
algorithm of fractal dimension and the fractal dimension
value are also different. Therefore, in fractal dimension
calculation of the specific image, to choose suitable
algorithm according to the characteristics of fractal image,
determine the physical significance of definition of fractal
dimension you have used, and then examine the
relationships between the change of fractal dimension
values and the corresponding physical and chemical
phenomena. This paper firstly converts SEM images to
black and white binary image, and then the image fractal
dimension is calculated, as a result, the image fractal
dimension values is \( 1 < Db < 2 \). According to definition of
surface fractal dimension in fractal theory: \( 2 < Ds < 3 \) we
can approximately think that: \( Ds = Db + 1 \).

III. BOX DIMENSION CALCULATION BASED ON MATLAB

A. The Image Processing

Define abbreviations and acronyms the first time they
are used in the text, even after they have been defined in
the abstract. Abbreviations such as IEEE, SI, MKS, CGS,
sc, dc, and rms do not have to be defined. Do not use
abbreviations in the title or heads unless they are
unavoidable[5].

Original images of the Fractal images are generally
color images or gray images, color images can also be
processed into gray images. In calculating of fractal
dimension based on box dimension, we should firstly
extract the parts of the image we focus on through the
image segmentation threshold method, gray level
histogram plays a very important role in the image
segmentation. Gray scale histogram is a function about
gray scale distribution, is a statistics of the image grayscale
distribution. To get statistics of frequencies of all the pixels
in digital images according to the size of the gray value. For images with two-peaks gray level histogram. Generally, a threshold which has a guiding significance can be obtained according to the histogram. In a sense, it is most appropriate to take the gray value of low valley (point) between the two peaks as a threshold to determine the threshold segmentation, because the histogram is a derivative of an area function. Near the bottom, the histogram of the values are relatively smaller, that means the area of function changes slowly along with the grayscale threshold. To minimize the impact on the borders of an object we choose the bottom gray level as a threshold. “Fig .1” for the gray-level histogram of HMX and RDX[6].

![Gray histogram of HMX and RDX.](image1)

Then binarize the images to get the binary image and the corresponding binary matrix. These can be completed by using powerful image processing function of MATLAB. “Fig .2” are gray images and binary images obtained after the treatment of SEM images of HMX and RDX.

![Gray images and binary images of HMX and RDX.](image2)

**B. Implement of Box-counting Dimension on MATLAB**

With the binarized matrix, we can calculate fractal dimension Db through the MATLAB program, and then get the surface fractal dimension Ds, “Fig .3” shows the flow of the whole algorithm.

![Flow of the whole algorithm.](image3)
A function between \( N \) and \(-\log k\) was obtained according to MATLAB programming, image of the function shown in “Fig. 4”:

It can be seen that the surface fractal dimension of HMX and RDX is 2.794 and 2.772, respectively.

IV. VERIFY THE CALCULATION

In order to verify the above fractal box dimension calculation method is correct, for some known fractal dimension graphics, classic Sierpinski carpets and Koch snowflake curves were calculated results show Sierpinski carpets was 1.890, 0.106% errors with the theoretical value 1.892. Koch snowflake curve was 1.248, 1.042% errors with the theoretical value 1.261. In “Fig. 5”, the verified results described above based on the MATLAB show that the program's fractal box dimension calculation is stable and accurate.
V. CONCLUSIONS

This study showed that the use of image analysis methods can accurately research the fractal characteristic of the surface of energetic materials, and make it easier to operate and spread, having done preparatory work for researching the impact the fractal dimension have on the sensitivity by the image processing and MATLAB program linear regression calculated surface fractal dimension of HMX and RDX is 2.794 and 2.772, $2 < D_s < 3$, consistent with its definition[9].

Using Koch curve and Sierpinski carpet for verification shows that it is sufficiently accurate to calculate fractal box dimension by using MATLAB programs. In this paper the designed procedure is very effective not only for image processing, but for any image with fractal characteristic, and with this method the calculated fractal dimension is accurate[10].

REFERENCES