

Nonlinear Squeeze Film Behavior Between Hydromagnetic Parallel Circular Disks

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Abstract—A study of nonlinear transient behavior between hydromagnetic circular squeezing disks has been presented in this paper. Based on the hydromagnetic flow model together with momentum integral approach, a lubrication equation including the effects of local and convective inertia terms is obtained. The nonlinear motion equation of the upper disk is numerically evaluated by the fourth-order Runge-Kutta method. Comparing with the non-inertia case, the influences of total fluid inertia forces provide a longer squeeze film time for the hydromagnetic circular squeeze films.

Keywords—Nonlinear transient behavior; Hydromagnetic squeeze films; Inertia forces; Circular disks.

I. INTRODUCTION

Research of squeeze film behaviors plays an important role in many engineering applications, such as bio-lubrication, synovial joints, damping films, clutch plates and approaching surfaces. In general, the study of squeeze film performances was based on the use of a non-conducting lubricant, such as the parallel circular plates by Hamrock [1] and Lin [2].

Because of the requirements for machine systems operating under severe conditions, the use of electrically conducting fluids (such as liquid metals) as lubricants to avoid the change of viscosity with temperature has received great attention. Many investigators have applied the electrically conducting fluids with the presence of external magnetic fields across the lubricant film to study various kinds of hydromagnetic squeeze film mechanisms, for example, the circular plates by Shukla [3], the annular discs by Lin [4], the rectangular plates by Bujurke and Kudenatti [5], the sphere-plate mechanism by Chou et al. [6] and the conical plates by Vadher et al. [7].

In the analyses of circular plates [1-3], the effects of fluid inertia forces have been neglected as compared to the viscous forces. However, the influences of fluid inertia forces may become important when the motion speed of machine elements increases. Therefore, a further study is motivated.

In this paper, the effects of total fluid inertia forces on the nonlinear transient behavior between hydromagnetic parallel circular disks in the presence of external magnetic fields are mainly concerned. Based on the hydromagnetic flow model, a lubrication equation including the effects of local and convective inertia terms is obtained by using the momentum integral approach. The nonlinear motion

equation of the upper disk is numerically evaluated by the fourth-order Runge-Kutta method. Comparing with the non-inertia case, the height-time relationship of the hydromagnetic circular squeeze film is presented through the variation of the Reynolds number and the Hartmann number.

II. FORMULATION

Fig. 1. presents the squeeze film geometry of parallel circular disks lubricated with an incompressible electrically conducting fluid with the application of an external magnetic field B_0 in the z -direction. At the time instant t , the upper disk with radius r_c is moving toward the lower disk with a squeezing velocity $s = -dh/dt$, where h denotes the local film height.

Since the film height is thin, the thin film lubrication theory is applicable, $\partial p / \partial z = 0$. Based on the hydromagnetic flow model as Davidson [8], the hydromagnetic momentum equation and the continuity equation can be written in axially symmetric cylindrical coordinates as:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \eta \frac{\partial^2 u}{\partial y^2} - \sigma B_0 u \quad (1)$$

$$\frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

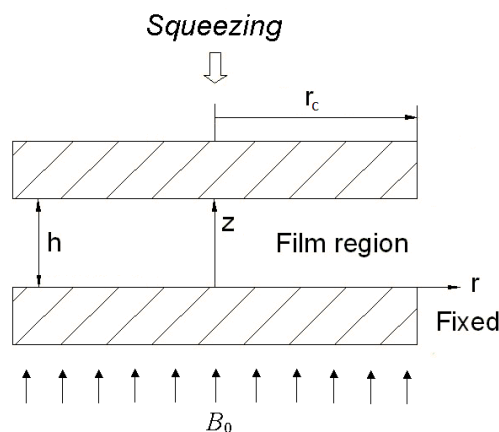


Figure 1.

$$2\pi \int_{z=0}^h r u dz = \pi r^2 s \quad (10)$$

Fig .1. Geometry of the parallel circular disks lubricated with an incompressible electrically conducting fluid in the presence of an external magnetic field.

In these equations, u and w denotes the fluid velocity in the r - and z - directions respectively, P is the pressure, ρ is the density, η is the viscosity and σ denotes the electrical conductivity of the fluid. The boundary conditions for the velocity components are

$$u|_{z=0} = 0, \quad u|_{z=h} = 0, \quad w|_{z=0} = 0, \quad w|_{z=h} = -s \quad (3)$$

Since the film height is thin, the inertia forces can be treated as constant across the film height, the inertia terms in the hydromagnetic momentum equation are then approximated by the momentum integral approach.

$$\frac{\rho}{h} \int_{z=0}^h \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) dz = -\frac{\partial p}{\partial r} + \eta \frac{\partial^2 u}{\partial z^2} - \sigma B_0^2 u \quad (4)$$

Applying the relationship of the continuity equation, the integrals in the LHS can be written as

$$\begin{aligned} \int_{z=0}^h \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) dz &= \frac{\partial}{\partial t} \int_{z=0}^h u dz + \\ &+ \frac{\partial}{\partial r} \int_{z=0}^h u^2 dz + \frac{1}{r} \int_{z=0}^h u^2 dz \end{aligned} \quad (5)$$

To obtain the radial velocity component, a modified pressure gradient function is introduced.

$$p_{mg} = \frac{\partial p}{\partial r} + \frac{\rho}{h} \left(\frac{\partial}{\partial t} \int_{z=0}^h u dz + \frac{\partial}{\partial r} \int_{z=0}^h u^2 dz + \frac{1}{r} \int_{z=0}^h u^2 dz \right) \quad (6)$$

Then the hydromagnetic momentum integral equation can be re-written as

$$\frac{\partial^2 u}{\partial z^2} - \frac{\sigma}{\eta} B_0^2 u = \frac{1}{\eta} p_{mg} \quad (7)$$

Integrating this ODE and applying the velocity boundary conditions, one can obtain the radial velocity component.

$$u = \frac{h_0^2}{\eta M^2} p_{mg} \cdot \left[\cosh\left(\frac{Mz}{h_0}\right) - 1 - \tanh\left(\frac{Mh}{2h_0}\right) \sinh\left(\frac{Mz}{h_0}\right) \right] \quad (8)$$

$$M = B_0 h_0 \sqrt{\frac{\sigma}{\eta}} \quad (9)$$

where h_0 denotes the initial film height and M is defined as the magnetic Hartmann number. The equation for squeezing motion is

Substituting the expression of u into this equation and performing the integration, one can derive

$$p_{mg} = -\frac{6\eta r s}{g_0} \quad (11)$$

$$g_0 = \frac{h_0^3}{M^3} \left[12M \frac{h}{h_0} - 24 \tanh\left(0.5M \frac{h}{h_0}\right) \right] \quad (12)$$

Combining equation (11) and equation (6), one can obtain the equation for the pressure gradient function.

$$\frac{\partial p}{\partial r} = -\frac{6\eta}{g_0} s \cdot r - \frac{\rho}{2h} \frac{\partial s}{\partial t} \cdot r - \frac{27\rho g_1}{hg_0^2} s^2 \cdot r \quad (13)$$

$$g_1 = \frac{h_0^5}{M^5} \left\{ \begin{aligned} &4M \frac{h}{h_0} + 4 \sinh\left(M \frac{h}{h_0}\right) \\ &+ \sec h^2\left(0.5M \frac{h}{h_0}\right) \left[2M \frac{h}{h_0} - 8 \sinh\left(M \frac{h}{h_0}\right) \right] \\ &- \sinh\left(2M \frac{h}{h_0}\right) \end{aligned} \right\} \quad (14)$$

III. NONLINEAR TRANSIENT ANALYSIS

The film pressure can be obtained by integrating the lubrication equation subject to the pressure boundary

condition: $p = 0$ at $r = r_c$.

$$p = \frac{1}{2} \left[\frac{6\mu}{g_0} \frac{dh}{dt} + \frac{\rho}{2h} \frac{d^2 h}{dt^2} - \frac{27\rho g_1}{hg_0^2} \left(\frac{dh}{dt} \right)^2 \right] \cdot [r^2 - r_c^2] \quad (15)$$

In these equations, the relationship of the squeezing velocity $s = -dh/dt$ has been applied. The hydromagnetic film force over the face of the disk can be obtained by integrating the film pressure.

$$F_h = -\frac{3\pi r_c^4}{2g_0} \frac{dh}{dt} - \frac{\pi \rho r_c^4}{8h} \frac{d^2 h}{dt^2} + \frac{27\pi \rho r_c^4 g_1}{4hg_0^2} \left(\frac{dh}{dt} \right)^2 \quad (16)$$

When the disk is subjected to an applied load, the equation of motion neglecting the buoyant force is:

$$F_a + W - F_h = m \cdot \left(-\frac{d^2 h}{dt^2} \right) \quad (17)$$

In these equations, F_a denotes the applied force, $W = mg$ is the weight and m is the mass of the disk. Introduce the nondimensional variables and parameters.

$$r^* = \frac{r}{r_c}, \quad h^* = \frac{h}{h_0}, \quad t^* = \frac{F_a h_0^2}{\eta r_c^4} t, \quad g_0^* = \frac{g_0}{h_0^3}, \quad g_1^* = \frac{g_1}{h_0^5} \quad (18\alpha)$$

$$W^* = \frac{W}{F_a}, \quad \text{Re} = \frac{\rho F_a h_0^4}{\eta^2 r_c^4}, \quad \text{Bi} = \frac{m h_0}{\rho r_c^4} \quad (18\beta)$$

where Re denotes the Reynolds number and Bi represents the body inertia parameter. As a result, the nondimensional motion equation of the upper disk can be written as:

$$\frac{d^2 h^*}{dt^{*2}} + \frac{12\pi h^*}{\text{Re}(\pi + 8\text{Bi}h^*)g_0^*} \cdot \frac{dh^*}{dt^*} - \frac{54\pi g_1^*}{(\pi + 8\text{Bi}h^*)g_0^{*2}} \cdot \left(\frac{dh^*}{dt^*}\right)^2 + \frac{8(1+W^*)h^*}{\text{Re}(\pi + 8\text{Bi}h^*)} = 0 \quad (19)$$

The above equation is a highly nonlinear ODE subject to the initial conditions: $h^*(t^* = 0) = 1$, $dh^*/dt^*(t^* = 0) = 0$. It can be numerically calculated by the fourth-order Runge-Kutta method using the software Matlab.

IV. RESULTS AND DISCUSSION

According to the nondimensional definitions, the Reynolds number Re depicts the inertia force effects of fluids and the magnetic Hartmann number M dominates the hydromagnetic effects of applied magnetic fields.

Fig. 2 illustrates the height versus time for different M with $\text{Re} = 0$ and $\text{Re} = 10$ under $W^* = 0.0015$ and $\text{Bi} = 0.005$. Comparing with the non-hydromagnetic non-inertia case ($M = 0, \text{Re} = 0$), the non-hydromagnetic inertia situation ($M = 0, \text{Re} = 10$) signify results in a larger film height. By the use of an electrically conducting fluid with an external magnetic field, the hydromagnetic inertia squeeze film ($M = 5, \text{Re} = 10$) yields a further increase in the film height. Increasing values of the magnetic Hartmann number ($M = 10, \text{Re} = 10$; $M = 15, \text{Re} = 10$; $M = 20, \text{Re} = 10$) increases the increments of the film height.

Figure 3 presents the height versus time for different Re with $M = 0$ and $M = 5$ under $W^* = 0.0015$ and $\text{Bi} = 0.005$. Comparing with the non-hydromagnetic non-inertia case ($M = 0, \text{Re} = 0$), the non-hydromagnetic inertia situation ($M = 0, \text{Re} = 5$) gives a larger film height. By the use of an electrically conducting fluid with an external magnetic field, the hydromagnetic inertia squeeze film ($M = 5, \text{Re} = 5$) yields a further increase in the film height. Increasing values of the Reynolds number

($M = 5, \text{Re} = 25$; $M = 5, \text{Re} = 125$; $M = 5, \text{Re} = 625$) increases the increments of the squeeze film height between the hydromagnetic parallel circular squeezing disks.

Comparing with the non-inertia situation, the effects of fluid inertia forces yield a larger film height for hydromagnetic circular squeeze films. In other words, the influences of total fluid inertia forces provide a longer squeeze film time for the hydromagnetic circular squeeze film as compared to the non-inertia case.

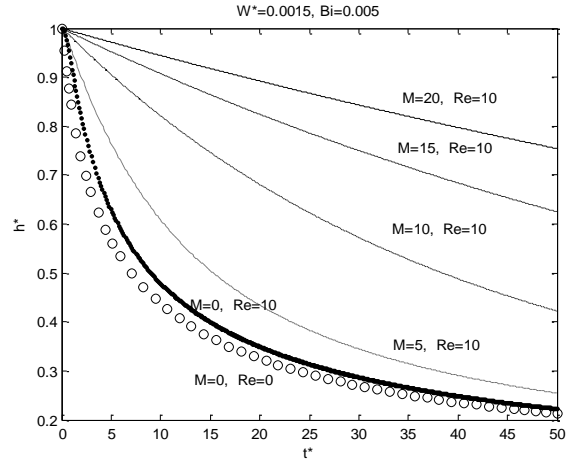


Figure 2. Height versus time for different M with $\text{Re} = 0$ and $\text{Re} = 10$.

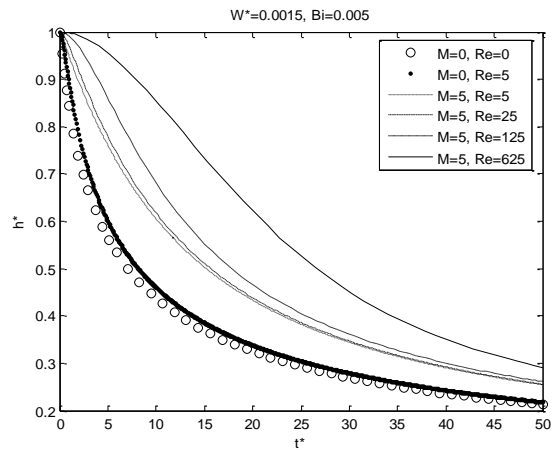


Figure 3. Height versus time for different Re with $M = 0$ and $M = 5$.

V. CONCLUSIONS

Based on the hydromagnetic flow model together with the momentum integral approach, a lubrication equation including fluid inertia terms is obtained to predict nonlinear transient circular squeeze film behavior.

The influences of total fluid inertia forces provide a longer squeeze film time for the hydromagnetic circular squeeze film as compared to the non-inertia case.

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