

Research on Internal Static Accessibility Evaluation of Urban Rail Transit Network

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Abstract. The topological structure of urban rail transit network directly determines the degree of difficulty of passenger transfer in the transit system. To evaluate the degree, we defined the internal static accessibility of urban rail transit network as the indicator. We then proposed specific methods to establish a dual-topology map for an urban rail transit network and to use the matrix for describing the connection relationships between transit lines within a network. Accordingly, we put forward our internal static accessibility assessment indicators and carried out case studies of rail networks in Shanghai. The proposed description method of the network topological structure and evaluating indicator of ISA of the URTN provided quantitative analysis tools for the evaluation, selection and optimization of network planning schemes.

Introduction

In urban transport planning, accessibility is an important concept that reflects the degree of difficulties of trips [1]. The overall public transport accessibility is used to evaluate the convenience of a traveler making a trip from the starting point to the destination by a public transit network. The public transport accessibility should include two sections:

1. The degree of convenience of accessing public transit network from the traveler's starting point and of reaching the traveler's destination once leaving public transit network. In our study this is defined as external accessibility.
2. The degree of convenience of transferring between transit lines, which this study defines as internal accessibility.

The external accessibility of a public transit network is an important indicator to reflect land utilization and the overall performance of a public transportation system during urban transport planning. In traditional studies of public transport accessibility, external accessibility is a key factor [2, 3]. Internal accessibility indicates the coordination between different lines within the transit network; this can be further divided into static accessibility and dynamic accessibility. The former is mainly dependent on the topological structure of the public transit network, while the latter is determined by the vehicle (train) running network.

Internal static accessibility (ISA) of an urban rail transit network (URTN) is a concept relating to whether passengers can successfully reach destinations using the URTN, and whether it is necessary to transfer within the network and the transfer times. Studies into ISA will show useful applicable value in terms of the design, evaluation and reliability analysis of URTN. However, there has been little research in the area. In this study, we have proposed the evaluation indicators of ISA, which can be used as the quantitative analysis tool of assessing, selection and optimization of a transit network. Internal static accessibility (ISA) of an urban rail transit network (URTN) is a concept relating to whether passengers can successfully reach destinations using the URTN, and whether it is necessary to transfer within the network and the transfer times. Studies into ISA will show useful applicable value in terms of the design, evaluation and reliability analysis of URTN. However, there has been little research in the area. In this study, we have proposed the evaluation indicators of ISA, which can be used as the quantitative analysis tool of assessing, selection and optimization of a transit network.

Topology Map and Matrix Description of URTN

Different topological structures found in URTNs determine various ISA, resulting in different convenience levels for passengers [4, 5]. Therefore, when studying ISA, it is first necessary to extract topological structures and construct topology maps. And based on the dual topology map of the URTN, we applied the concepts such as an adjacency matrix to describe the connection relationships between lines in the URTN and established a direct transfer matrix, accessibility matrix and a minimum transfer time matrix.

Topology Map of URTN

Differences between connection relationships in various public transport networks are the causes of different ISAs. To pave the way for the subsequent studies of ISA, we adopted the dual approach to generate a topology map of the URTN.

A dual-topology map of the URTN is denoted as an undirected graph $G(V, E)$, where: $V(G) = \{v_1, v_2, \mathbf{L}, v_n\}$ is a non-empty finite vertex set, v_i is vertex and V is vertex set. Also, $E(G) = \{e_1, e_2, \mathbf{L}, e_m\}$ is a finite edge set, e_i is edge and E is edge set. We abstracted rail lines as vertices regardless of directions. $V(G) = \{v_1, v_2, \mathbf{L}, v_n\}$ represented the lines set. n was the number of lines. Direct connection relationships between lines were represented by edges. If line i and line j intersect, there is an edge between vertex v_i and v_j ; namely they are adjacent. On the contrary, when v_i and v_j are not adjacent, there is no edge in between them.

Matrix Description of URTN

(1) Direct Transfer Matrix

For the undirected graph $G(V, E)$, $V(G) = \{v_1, v_2, \mathbf{L}, v_n\}$, $A(a_{ij})_{n \times n}$ is denoted as the adjacency matrix of $G(V, E)$, where

$$a_{ij} = \begin{cases} 1, & v_i v_j \in E(G) \\ 0, & v_i v_j \notin E(G) \end{cases} \quad (1)$$

The adjacent matrix of the undirected graph is a symmetric matrix with elements of either 0 or 1, and the diagonal elements are 0. After abstracting the URTN into the dual topology map G , the corresponding adjacency matrix $A(a_{ij})_{n \times n}$ can be denoted as a direct transfer matrix of the URTN.

$a_{ij}=1$ ($a_{ji}=1$) represents that line i and line j intersect directly. Passengers interchanging between these 2 lines need only one transfer. Conversely, $a_{ij}=0$ ($a_{ji}=0$) represents that line i and line j do not intersect. For passengers these 2 lines are not accessible to each other through transfer once.

(2) Accessibility Matrix

For an undirected graph $G(V, E)$, $V(G) = \{v_1, v_2, \mathbf{L}, v_n\}$, $P(p_{ij})_{n \times n}$ is denoted as the accessibility matrix of $G(V, E)$, where:

$$p_{ij} = \begin{cases} 1, & \text{there are paths between } v_i \text{ and } v_j, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

The accessibility matrix shows whether a path exists between any 2 nodes. Matrix A, A^2, \mathbf{L}, A^n (A^n refers to n -th power of the adjacency matrix A) can be changed to Boolean matrixes, denoted as $A(1), A(2), \mathbf{L}, A(n)$, then: $P = A(1) \vee A(2) \vee \mathbf{L} \vee A(n)$, where \vee represents Boolean sum. $p_{ij}=1$ ($p_{ji}=1$) represents that passengers can interchange between line i and line j via transfer. Contrarily, $p_{ij}=0$ ($p_{ji}=0$) represents that passengers cannot interchange between line i and line j via transfer.

(3) The Minimum Transfer Time Matrix

We then established matrix $R(r_{ij})_{n \times n}$ to record the minimum transfer times passengers can interchange between any 2 different lines. This is called the minimum transfer time matrix of the URTN, where

$$r_{ij} = \begin{cases} k, & i \neq j \text{ and } a_{ij}^{(k)} > 0 \text{ and } a_{ij}^{(m)} = 0; k < n; \\ & m = 1, 2, \mathbf{L}, k-1; \\ \infty, & i \neq j \text{ and } a_{ij}^{(n-1)} = 0; \\ 0, & i = j. \end{cases} \quad (3)$$

In the equation, $a_{ij}^{(k)}$ is the element on row i and column j of the A^k matrix. R is also a symmetric matrix. When $i \neq j$ and $a_{ij}^{(n-1)} = 0$, passengers cannot interchange between line i and line j via transfer, so we defined $r_{ij} = \infty$. When $i = j$, there is no need to transfer, so $r_{ij} = 0$.

Evaluation Indicators of ISA for URTN

When abstracting an actual transit network to a topology map, topological parameters are prevalently used to analyze the features of nodes and their connections. Different features stand for different network structures, and different structures cause different systematic functions [6, 7]. The most common topological parameters include degrees, clustering coefficients, average path lengths and betweenness. However, there is little significance to directly apply these parameters in an analysis of ISA. Depending on the topology map and the matrix description of the URTN, we have proposed 5 quantitative evaluation indicators to reflect ISA.

Network Connectivity

Network connectivity C indicates the connectivity between lines in the URTN.

$$C = \begin{cases} 1, & G \text{ is a connected graph,} \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

where G represents the corresponding topology map of the URTN. In the undirected graph G , if there is a connecting path from vertex v_i to v_j , then v_i and v_j are connected; if any 2 vertices in G are connected, then G is a connected graph.

$C = 1$ represents existing direct or indirect connectivity between any 2 lines in the URTN, and passengers can transfer between any 2 lines. $C = 0$ represents that there is at least one isolated line that is not connected to any other lines in the network, which means passengers cannot transfer from the isolated line to any others. Obviously, the ISA performance is better in a network of $C = 1$.

The Maximum Transfer Times

The maximum transfer times in a network (N_{\max}) refers to the maximum number of transfer times needed to complete a trip from one point on one line to another point on a different line in the URTN.

$$N_{\max} = \max_{i,j} \left\{ \min_l y_{ij}^l \right\}, \quad i = 1, 2, \mathbf{L}, n; \quad (5)$$

$$j = 1, 2, \mathbf{L}, n; l = 1, 2, \mathbf{L}, g.$$

where,

n --- the number of lines;

g --- the number of accessible paths from line i to line j ;

y_{ij}^l --- the number of transfer times along path l from line i to line j .

Set $A(a_{ij})_{n \times n}$ as the direct matrix of the URTN, element (i, j) in $A, A^2, \mathbf{L}, A^k, \mathbf{L}, A^{n-1}$ constitute vector $(a_{ij}, a_{ij}^{(2)}, \mathbf{L}, a_{ij}^{(k)}, \mathbf{L}, a_{ij}^{(n-1)})$. The superscript k of the first non-zero element $a_{ij}^{(k)}$ in the vector is the minimum number of transfer times from line i to line j ; namely, $\min_l y_{ij}^l = k$ in equation (5). Therefore, N_{\max} can be determined through the exponentiation of the direct transfer matrix; meaning N_{\max} is the largest element on minimum transfer time matrix R of the URTN. The smaller the N_{\max} , the lower the cost to achieve intercommunication between the rail lines, and the better the ISA of the URTN.

Direct Transfer Coefficient

The direct transfer coefficient (L) represents the occurrence probability that only a transfer is needed to complete a trip in the URTN.

$$L = \sum_{i=1}^n \sum_{j=1}^n a_{ij} / 2C_n^2. \quad (6)$$

where,

a_{ij} --- the element (i, j) in the direct transfer matrix $A(a_{ij})_{n \times n}$;

C_n^2 --- the number of all line-pair combinations in the URTN.

$L \in (0, 1]$. The larger the L , the better the ISA of the URTN.

Average Transfer Convenience Index

The average transfer convenience index (K) refers to the degree of convenience reflected by the transfer times in completing a trip in the URTN.

$$K = \sum_{t=1}^{N_{\max}} \left[\left(\frac{1}{2} \right)^{t-1} \times X_t \right] / C_n^2, \quad X_t = \frac{1}{2} (N_{r_{ij}} | r_{ij} = t) \quad (7)$$

where,

X_t --- the number of line-pair combinations that passengers can interchange between these pairs of lines through transfer t times; this equals half the number of elements $N_{r_{ij}} (r_{ij} = t)$ in the minimum transfer matrix R .

Set $d = \left(\frac{1}{2} \right)^{t-1} (t = 1, 2, \mathbf{L}, N_{\max})$ as the reduction coefficient, which reflects the reduction effects of increasing transfer times on transfer convenience. In particular, when $t = 1$, $d = 1$, when $t > 1$, $0 < d < 1$.

$K \in (0, 1]$. The greater the K , the better the ISA of the URTN.

Distribution of the Minimum Transfer Times

The distribution of the minimum transfer times ($P\{r_{ij} = t\}$) represents the probability that the minimum transfer times (r_{ij}) required for passengers interchange between two lines in the URTN equals t . This probability equals the ratio of the number of line-pair combinations with the minimum transfer times of r_{ij} to the number of all line-pair combinations in the URTN.

$$P\{r_{ij} = t\} = X_t / C_n^2, \quad X_t = \frac{1}{2} (N_{r_{ij}} | r_{ij} = t), \quad (8)$$

$t = 1, 2, \mathbf{L}, N_{\max}$

where the symbols have the same meaning as above.

Case Analysis of ISA of the Typical URTN in Shanghai

As shown in Figure 1, the URTN in Shanghai (2014) consists of 14 lines, which are Line 1, Line 2, Line 3, Line 4, Line 5, Line 6, Line 7, Line 8, Line 9, Line 10, Line 11, Line 12, Line 13, and Line 16.



Fig.1. A Sketch Map of Shanghai's URTN (2014)

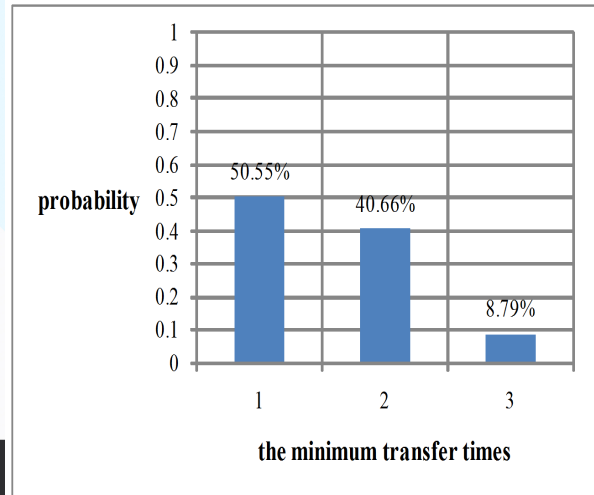


Fig. 2. The Distribution of the Minimum Transfer Times of Shanghai's URTN

Set the corresponding vertices in undirected graph as $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}$, and v_{14} . Thus, the direct transfer matrix, accessibility matrix and minimum transfer time matrix are respectively:

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

$$R = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 1 & 2 & 2 & 2 \\ 1 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 0 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 2 & 2 & 0 & 3 & 2 & 2 & 2 & 3 & 2 & 3 & 3 & 3 \\ 2 & 1 & 2 & 1 & 3 & 0 & 1 & 1 & 1 & 2 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 1 & 2 & 2 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 2 & 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 3 & 2 & 2 & 1 & 2 & 0 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 0 & 2 & 2 & 1 \\ 2 & 2 & 2 & 1 & 3 & 1 & 2 & 1 & 2 & 1 & 2 & 0 & 2 & 3 \\ 2 & 2 & 1 & 1 & 3 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 0 & 3 \\ 2 & 2 & 2 & 2 & 3 & 2 & 3 & 2 & 2 & 2 & 1 & 3 & 3 & 0 \end{bmatrix}.$$

Therefore, the ISA of the URTN in Shanghai (2014) can be calculated as shown in Table 1 and Figure 2.

Table 1. Calculation Result of ISA Evaluation Indicators of the URTN in Shanghai (2014)

Index	Value
Network Connectivity	$C^{\text{Shanghai}} = 1$
The Maximum Transfer Times	$N_{\max}^{\text{Shanghai}} = 3$
Direct Transfer Coefficient	$L_{\text{Shanghai}} = 92 / (2 \times C_{14}^2) \approx 0.505$
Average Transfer Convenience Index	$K_{\text{Shanghai}} = \left(46 + \frac{1}{2} \times 37 + \frac{1}{4} \times 8 \right) / C_{14}^2 \approx 0.731$

It can be seen that the ISA of the URTN in Shanghai demonstrated an overall good performance in the following ways:

(1) Regarding the number of maximum transfer times, in Shanghai's URTN passengers require to transfer at most 3 times to reach any destination point from one starting point..

(2) Regarding the direct transfer coefficient, the ratio of the number of direct intersecting line-pair combinations to the number of all line-pair combinations in Shanghai is high, indicating a high probability in Shanghai's URTN to transfer only once in order to reach the destination. According to the distribution map of minimum transfer times, this probability is 50.55%.

(3) Combining the average transfer convenience index and the distribution of the minimum transfer times, the ratio of the number of line-pair combinations that can be interactively reached through 2 or fewer transfers to the number of all line-pair combinations is 91.21% in Shanghai's URTN.

In summary, the travel convenience of Shanghai's URTN determined by the topological structure of URTN is rather good. In other words, because of the superior connection relationships between rail lines, the topological structure of Shanghai' URTN is more favorable for passengers to make trips.

Conclusions

The proposed description method of the network topological structure provided a distinctive tool to describe the connection relationships between lines in the URTN. And the proposed evaluating indicators of ISA of the URTN can be used in evaluation of network planning schemes and selection of the optimal scheme.

As mentioned previously, the overall transport accessibility of a URTN is composed of external accessibility, internal accessibility, static accessibility as well as dynamic accessibility. Each part interacts and jointly affects the travel convenience of a URTN. In the future study, we will continue research how to describe and evaluate the dynamic accessibility, and the interactions between the static accessibility and dynamic accessibility. We believe that these studies are significant for optimizing the urban rail transit network.

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