Compactly supported biorthogonal two-direction vector-valued wavelet Packet with arbitrary dilation

Lan Li^{1, a}

¹ School of Information Engineering, Xi'an University of Arts and Science, Xi'an 710065, P.R China

^alanli98@126.com

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Abstract. The notion of two-direction vector-valued wavelet packets which is the generalization of biorthogonal vector-valued wavelet packets is introduced. A method for constructing biorthogonal two-direction vector-valued wavelet packets is obtained and their properties are discussed by means of time-frequency analysis method. Biorthogonality formulas concerning these two-direction vector-valued wavelet packets are established.

Introduction

Wavelet packets, due to their good properties, have attracted considerable attention. They can be widely applied in science and engineering [1]. Z. Shen [2] generalized the concept of orthogonal wavelet packets to the case of non-orthogonal wavelet packets so that wavelet packets can be used in the case of the spline wavelets. Multivariate nonseparable wavelets have attracted the interest of many mathematicians [3-5]. Although separable wavelet bases have a lot of advantages, they have a number of drawbacks. One way to avoid this is through the construction of nonseparable wavelets. Li [6] generalized the two-scale refinable equation, established the orthogonality criteria for two-direction refinable function and two-direction wavelet packets. Vector-valued wavelets and multi-wavelets are different in the following sense. For example, prefiltering is usually required for discrete multiwavelet transforms but not necessary for discrete valued transforms, especially video images. Therefore, studying two-direction vector-valued wavelets is useful in multiwavelet theory and representations of signals. The objective of this paper is to generalize the concept of biorthogonal vector-valued wavelet packets to biorthogonal two-direction wavelet packets.

Preliminary

We begin with some notions and notations to be used in this paper. Denote by $\mathbb{R}$ and $\mathbb{C}$ the set of all real and complex numbers, respectively. Denote by $\mathbb{Z}$ and $\mathbb{Z}_+$ all integers and nonnegative integers, respectively. Let s be a constant, $s \geq 2, s \in \mathbb{Z}$. Denote I_s and O stand for the $s \times s$ identity matrix and the zero matrix, respectively.

The space $L^2(\mathbb{R}, \mathbb{C}^s)$ is defined to be the set of all vector-valued functions $P(t)$,

$$L^2(\mathbb{R}, \mathbb{C}^s) = P(\cdot) = (p_1(\cdot), p_2(\cdot), \dots, p_r(\cdot), \dots, p_s(\cdot))$$

where $p_r(\cdot)$ denotes the pixel on the r th row at the point t . $\|P\| := (\sum_{l=1}^s \int |p_l(t)|^2 dt)^{1/2}$,

$\int_{\mathbb{R}} P(t) dx := (\int_{\mathbb{R}} p_l(t) dt)_{l=1}^s$ and Fourier transform is $\hat{H}(w) := \int_{\mathbb{R}} P(\cdot) \cdot \exp\{-it \cdot w\} dt$. where $t \cdot w$ stands for the inner product vectors t and w . For two vector-valued functions $P, Q \in L^2(\mathbb{R}, \mathbb{C}^s)$, their symbol inner product is defined to be $\langle P, Q \rangle := \int_{\mathbb{R}} P(x) Q(x)^T dx$, where T means the transpose and the complex conjugate.

Two-direction vector-valued multiresolution analysis

The multiresolution analysis method is an important approach to obtain two-direction vector-valued wavelets. Let $\{V_j\}_{j \in \mathbb{Z}}$ be a sequence of the closed subspaces $L^2(R, C^s)$ satisfying

- (i) $\mathbf{L} \subset \mathbf{V}_{-1} \subset \mathbf{V}_0 \subset \mathbf{V}_1 \subset \mathbf{L}$;
- (ii) $\text{Clos}_{L^2(R, C^s)}(\bigcup_{j \in \mathbb{Z}} V_j) = L^2(R, C^s)$;
- (iii) $\bigcap_{j \in \mathbb{Z}} V_j = 0$;
- (iv) $f(\cdot) \in V_j \Leftrightarrow f(a \cdot) \in V_{j+1}, j \in \mathbb{Z}$;
- (v) There exists a sequence $\{\Phi(\cdot - n), \Phi(m \cdot) : n, m \in \mathbb{Z}\}$ is a Riesz bases of V_0 .

Then $\{V_j\}_{j \in \mathbb{Z}}$ is said to be a two-direction vector-valued multiresolution analysis (TVMRA) with arbitrary dilation. $\Phi(\cdot) \in L^2(R, C^r)$ is a corresponding two-direction vector-valued scaling function. Due to the facts that $V_0 \in V_1$, we know that $\Phi(\cdot)$ can be written as

$$\Phi(\cdot) = \sum_{n \in \mathbb{Z}} P_n^+ \Phi(a \cdot - n) + \sum_{n \in \mathbb{Z}} P_n^- \Phi(n - a \cdot) \quad (1)$$

Where sequence of constant $s \times s$ matrices $\{P_n^+\}_{n \in \mathbb{Z}}$ and $\{P_n^-\}_{n \in \mathbb{Z}}$ are called the positive-direction vector-valued mask and negative-direction vector-valued mask, respectively. Applying of the Fourier transform (1), we have

$$\Phi(\cdot) = P_0^+ (e^{\frac{-iw}{a}}) \hat{\Phi}(\frac{w}{a}) + P_0^- (e^{\frac{-iw}{a}}) \hat{\Phi}(\frac{w}{a})$$

where $P^+(w) = \frac{1}{a} P_n^+ \cdot \exp\{-it \cdot w\}$, $w \in R$ and $P^-(w) = \frac{1}{a} P_n^- \cdot \exp\{-it \cdot w\}$, $w \in R$ are called positive-direction vector-valued mask symbol and negative-direction vector-valued mask symbol, respectively.

We rewrite (1) as

$$\Phi(\cdot) = \sum_{n \in \mathbb{Z}} P_n^+ \Phi(-a \cdot - n) + \sum_{n \in \mathbb{Z}} P_n^- \Phi(n + a \cdot) \quad (2)$$

From (1) and (2), we have

$$X(\cdot) = \begin{cases} \Phi(\cdot) \\ \Phi(\cdot) \end{cases} = \sum_n \begin{bmatrix} P_n^+ & P_n^- \\ P_{-n}^- & P_{-n}^+ \end{bmatrix} \Phi(a \cdot - n) \quad (3)$$

Let W_n , $n \in \mathbb{Z}$ be the orthocomplementary subspace of V_n . Assume that there exist two two-direction vector-valued functions $\Psi_n(t) \in L^2(R, C^s)$, $v = 1, 2, \dots, a-1$, $\Psi_0 \in W_0 \subset V_1$, there exist two finite supported sequences of constant $s \times s$ matrices $\{Q_n^{+(n)}\}_{n \in \mathbb{Z}}$ and $\{Q_n^{-(n)}\}_{n \in \mathbb{Z}}$ such that

$$\Psi_n(t) = \sum_{n \in \mathbb{Z}} Q_n^{+(v)} \Phi(a \cdot - n) + \sum_{n \in \mathbb{Z}} Q_n^{-(v)} (n - a \cdot), \quad n = 1, 2, \dots, a-1 \quad (4)$$

Implementing the Fourier transform of (4) gives

$$\Psi_v(\cdot) = Q_0^+ (e^{\frac{-iw}{a}}) \hat{\Phi}(\frac{w}{a}) + Q_0^- (e^{\frac{-iw}{a}}) \hat{\Phi}(\frac{w}{a}), \quad n = 1, 2, \dots, a-1.$$

where

$$Q^{+(v)}(w) = \frac{1}{a} \sum_{n \in \mathbb{Z}} Q_n^+ \cdot \exp\{-it \cdot w\}, \quad Q^{-(v)}(w) = \frac{1}{a} \sum_{n \in \mathbb{Z}} Q_n^- \cdot \exp\{-it \cdot w\}, \quad w \in R, v = 1, 2, \dots, a-1.$$

we have

$$F_v(\cdot) = \begin{cases} \Psi_v(\cdot) \\ \Psi_v(\cdot) \end{cases} = \sum_n \begin{bmatrix} Q_n^{+(v)} & Q_n^{-(n)} \\ Q_{-n}^{-(v)} & Q_{-n}^{+(n)} \end{bmatrix} \Phi(a \cdot - n)$$

Similarly, for $\hat{\Phi}(\cdot)$ we have

$$\hat{\Phi}(\cdot) = \hat{P}_0^+ (e^{\frac{-i\omega}{a}}) \hat{\Phi}(\frac{W}{a}) + \hat{P}_0^- (e^{\frac{-i\omega}{a}}) \overline{\hat{\Phi}}(\frac{W}{a}) \quad (5)$$

We rewrite (5) as

$$\hat{\Phi}(\cdot) = \sum_{n \in \mathbb{Z}} P_n^+ \hat{\Phi}(-a \cdot -n) + \sum_{n \in \mathbb{Z}} P_n^- \hat{\Phi}(n+a \cdot) \quad (6)$$

From (5) and (6), we have

$$\hat{X}(\cdot) = \begin{cases} \hat{\Phi}(\cdot) \\ \hat{\Phi}(\cdot) \end{cases} = \sum_n \begin{vmatrix} \hat{P}_n^+ & \hat{P}_n^- \\ \hat{P}_{-n}^- & \hat{P}_{-n}^+ \end{vmatrix} \hat{\Phi}(a \cdot -n) \quad (7)$$

Assume that there exist two two-direction vector-valued functions $\Psi_n(x) \in L^2(R, C^s)$, $v=1, 2, \mathbf{L} a-1 \in W_0 \subset V_1$, there exist two finite supported sequences of constant $s \times s$ matrix $\{\mathcal{Q}_n^{+(n)}\}_{n \in \mathbb{Z}}$ and $\{\mathcal{Q}_n^{-(n)}\}_{n \in \mathbb{Z}}$ such that

$$\Psi_n(t) = \sum_{n \in \mathbb{Z}} \mathcal{Q}_n^{+(v)} \Phi(a \cdot -n) + \sum_{n \in \mathbb{Z}} \mathcal{Q}_n^{-(v)} \Phi(n-a \cdot), \quad n=1, 2, \mathbf{L} a-1 \quad (8)$$

Implementing the Fourier transform of (4) gives

$$\Psi_v(\cdot) = \hat{P}_0^+ (e^{\frac{-i\omega}{a}}) \hat{\Phi}(\frac{W}{a}) + \hat{P}_0^- (e^{\frac{-i\omega}{a}}) \overline{\hat{\Phi}}(\frac{W}{a}), \quad v=1, 2, \mathbf{L} a-1.$$

where

$$\mathcal{Q}^{+(v)}(w) = \frac{1}{a} \mathcal{Q}_n^+ \cdot \exp\{-it \cdot w\}, \quad \mathcal{Q}^{-(v)}(w) = \frac{1}{a} \sum_{n \in \mathbb{Z}} \mathcal{Q}_n^- \cdot \exp\{-it \cdot w\}, \quad w \in R, v=1, 2, \mathbf{L} a-1$$

we have

$$\hat{P}_v(\cdot) = \begin{cases} \Psi_v(\cdot) \\ \Psi_v(\cdot) \end{cases} = \sum_n \begin{vmatrix} \mathcal{Q}_n^{+(v)} & \mathcal{Q}_n^{-(n)} \\ \mathcal{Q}_{-n}^{-(v)} & \mathcal{Q}_{-n}^{+(n)} \end{vmatrix} \hat{\Phi}(a \cdot -n)$$

The properties of biorthogonal two-direction vector-valued wavelet packets

Set $\Psi_0(t) = X(t)$, $\Psi_m(t) = F_m(t)$, $\hat{\Psi}_0(t) = \hat{X}(t)$, $\hat{\Psi}_m(t) = F_m(t)$, $P_k^{+(0)} = P_k^+$, $P_k^{+(m)} = Q_k^{+(m)}$, $P_k^{-(0)} = P_k^-$, $P_k^{-(m)} = Q_k^{-(m)}$, $\hat{P}_k^{(0)} = \hat{P}_k^+$, $\hat{P}_k^{(m)} = \hat{Q}_k^{(m)}$, $\hat{P}_k^{-(0)} = \hat{P}_k^-$, $\hat{P}_k^{-(m)} = \hat{Q}_k^{-(m)}$, $m \in 1, 2, \mathbf{L} a-1$.

Then, equations (1) and (4) can be written as follows:

$$\Psi_m(t) = \sum_{n \in \mathbb{Z}} M_k^{+(m)} \Psi_0(a \cdot -k) + \sum_{n \in \mathbb{Z}} M_0^{-(m)} \Psi_0(k-a \cdot), \quad m \in 1, 2, \mathbf{L} a-1 \quad (9)$$

Similarly,

$$\hat{\Psi}_m(t) = \sum_{n \in \mathbb{Z}} \hat{M}_k^{+(m)} \hat{\Psi}_0(a \cdot -k) + \sum_{n \in \mathbb{Z}} \hat{M}_0^{-(m)} \hat{\Psi}_0(k-a \cdot), \quad m \in 1, 2, \mathbf{L} a-1 \quad (10)$$

For any $a \in \mathbb{Z}_+$ and the given biorthogonal scaling functions $\Psi_0(x)$, $\hat{\Psi}_0(x)$, we have

$$\Psi_v(t) = \Psi_{ab+m}(t) = \sum_{k \in \mathbb{Z}} P_k^{+(m)} \Psi_b(a \cdot -k) + \sum_{k \in \mathbb{Z}} P_k^{-(m)} \Psi_b(k-a \cdot), \quad (11)$$

$$\hat{\Psi}_v(t) = \hat{\Psi}_{ab+m}(t) = \sum_{k \in \mathbb{Z}} \hat{P}_k^{+(m)} \hat{\Psi}_b(a \cdot -k) + \sum_{k \in \mathbb{Z}} \hat{P}_k^{-(m)} \hat{\Psi}_b(k-a \cdot), \quad (12)$$

where $m \in 0, 1, 2, \mathbf{L} a-1, b \in \mathbb{Z}_+$ is the unique element such that $v = ab + m, m = 0, 1, 2, \mathbf{L} a-1$.

Definition 1 Two-direction vector-valued functions $\{\Psi_{ab+m}(t), b \in \mathbb{Z}_+, m = 0, 1, 2, \mathbf{L} a-1\}$ and $\{\hat{\Psi}_{ab+m}(t), b \in \mathbb{Z}_+, m = 0, 1, 2, \mathbf{L} a-1\}$ are called biorthogonal two-direction vector-valued wavelet packets with respect to biorthogonal matrix scaling functions $X(\cdot)$, $\hat{X}(\cdot)$, respectively, where $\Psi_{ab+m}(t)$ and $\hat{\Psi}_{ab+m}(t)$ are given by (9) and (10), respectively.

Fourier transform for the both sides of equation (9) and (10)

$$\Psi_{ab+m}(w) = P^{+(m)}\left(\frac{w}{a}\right)\Psi_b\left(\frac{w}{a}\right) + P^{-(m)}\left(\frac{w}{a}\right)\hat{\Psi}_b\left(\frac{w}{a}\right), \quad m \in 0, 1, 2, \mathbf{L}, a-1. \quad (13)$$

$$\hat{\Psi}_{ab+m}(w) = P^{+(m)}\left(\frac{w}{a}\right)\hat{\Psi}_b\left(\frac{w}{a}\right) + P^{-(m)}\left(\frac{w}{a}\right)\Psi_b\left(\frac{w}{a}\right), \quad m \in 0, 1, 2, \mathbf{L}, a-1. \quad (14)$$

Where

$$P^{+(m)}(w) = \frac{1}{a} \sum_{k \in Z} P_k^{+(m)} \cdot \exp\{-ik \cdot w\}, \quad P^{-(m)}(h) = \frac{1}{a} \sum_{k \in Z} P_k^{-(m)} \cdot \exp\{-ik \cdot h\}, \quad m \in 0, 1, 2, \mathbf{L}, a-1.$$

$$P^{+(m)}(w) = \frac{1}{a} \sum_{k \in Z} P_k^{+(m)} \cdot \exp\{-ik \cdot w\}, \quad P^{-(m)}(h) = \frac{1}{a} \sum_{k \in Z} P_k^{-(m)} \cdot \exp\{-ik \cdot h\}, \quad m \in 0, 1, 2, \mathbf{L}, a-1.$$

Lemma 1 Let $\Psi(x) \in L^2(R, C^{s \times s})$, Then it is an biorthogonal vector-valued function if and only if

$$\sum_{u \in Z} \hat{\Psi}(w+2up) \hat{\Psi}(w+2up)^* = I_s, \quad w \in R.$$

Lemma 2. Assume that $\Psi_v(x) \in L^2(R, C^s)$ and $\hat{\Psi}_v(x) \in L^2(R, C^s)$, $v \in 0, 1, 2, \mathbf{L}, a-1$ are vector-valued wavelets associated with biorthogonal scaling function $\Psi_0(x)$ and $\hat{\Psi}_0(x)$. Then for $m, n \in \Lambda_0$, we have

$$\sum_{s \in \Lambda_0} P^{(m)}(Q^{-T}(w+2sp)) P^{(n)}(Q^{-T}(w+2sp))^* = d_{m,n} I_s.$$

Theorem 1 Assume $\{\Psi_a(x), a \in Z_+^d\}$ and $\{\hat{\Psi}_a(x), a \in Z_+^d\}$ are two-direction wavelet packets with respect to biorthogonal two-direction vector-valued function $\Psi_0(x)$ and $\hat{\Psi}_0(x)$, respectively. Then, for $b \in Z_+^d, u \in Z^d$, we have

$$\langle \Psi_{ab+m}(\cdot), \hat{\Psi}_{ab+r}(\cdot - w) \rangle = d_{0,w} d_{m,r} I_s, \quad m, r \in 0, 1, 2, \mathbf{L}, a-1.$$

Proof. Since we set R the following partition: $R = \bigcup_{u \in Z} \left(\frac{1}{a}[0, 2p] + 2up\right)$ and we have

$$\begin{aligned} (2p) \langle \Psi_{ab+m}(\cdot), \Psi_{ab+r}(\cdot - w) \rangle &= \int_R \hat{\Psi}_{ab+m}(w) \Psi_{ab+r}(w)^* \cdot e^{in \cdot w} dw \\ &= \int_{R^d} P^{(m)}\left(\frac{1}{a}w\right) \hat{\Psi}_b\left(\frac{1}{a}w\right) \cdot \hat{\Psi}_b\left(\frac{1}{a}w\right)^* P^{(r)}\left(\frac{1}{a}w\right)^* \cdot \exp\{in \cdot w\} dw \\ &= \sum_{u \in Z} \int P^{(m)}\left(\frac{1}{a}w\right) \hat{\Psi}_b\left(\frac{1}{a}w\right) \cdot \hat{\Psi}_b\left(\frac{1}{a}w\right)^* P^{(r)}\left(\frac{1}{a}w\right)^* \cdot e^{in \cdot w} dw \\ &= \int_{\frac{1}{a}[0, 2p]} P^{(m)}\left(\frac{1}{a}w\right) \left[\sum_{u \in Z} \hat{\Psi}_b\left(\frac{1}{a}w+2up\right) \cdot \hat{\Psi}_b\left(\frac{1}{a}w+2up\right)^* \right] P^{(r)}\left(\frac{1}{a}w\right)^* \cdot e^{in \cdot w} dw \\ &= \int_{[0, 2p]} d_{m,r} \cdot I_s \exp\{in \cdot w\} dw = d_{0,n} \cdot d_{m,r} \cdot I_s. \end{aligned}$$

Corollary 1 Suppose $\{\Psi_u(x), u \in Z_+^d\}$ and $\{\hat{\Psi}_u(x), u \in Z_+^d\}$ are two-direction vector-valued wavelet packets with respect to biorthogonal two-direction vector-valued function $\Psi_0(x)$ and $\hat{\Psi}_0(x)$, respectively. Then, for $v \in Z_+^d$, we have

$$\langle \Psi_v(\cdot), \hat{\Psi}_v(\cdot - w) \rangle = d_{0,w} I_s, \quad w \in Z.$$

Theorem 2. For any $u, v \in Z_+^d$, we have

$$\langle \Psi_v(\cdot), \Psi_u(\cdot - w) \rangle = d_{u,v} d_{0,w} I_s, \quad w \in Z. \quad (15)$$

Proof. For $u = v$, we get (15) by Lemma 1. when $u \neq v$, $u, v \in \Lambda_0$, assuming $u = au_1 + r_1, v = av_1 + m_1$, where $r_1, m_1 \in 0, 1, 2, \mathbf{L}, a-1$. If $v_1 = u_1$, then $r_1 \neq m_1$. We denote

$$\begin{aligned} (2p)^d \langle \Psi_u(\cdot), \hat{\Psi}_v(\cdot - w) \rangle &= \hat{\Psi}_{au_1+r_1}(w) \hat{\Psi}_{av_1+m_1}(w)^* \cdot \exp\{iu \cdot w\} dw \\ &= \int_{R^d} P^{(r_1)}(x) \hat{\Psi}_{u_1}(x) \hat{\Psi}_{v_1}(x)^* P^{(m_1)}(x)^* \cdot e^{iw \cdot x} dw \end{aligned}$$

$$\begin{aligned}
&= \int_{[0,2p]} P^{(r_1)}(x) \left\{ \sum_{w \in Z} \hat{\Psi}_{b_1}(x+2wp) \cdot \hat{\Psi}_{u_1}(x+2wp)^* \right\} P^{(m_1)}(x)^* \cdot e^{i w \cdot x} dx \\
&= \int_{[0,2p]} \sum_{s \in \{0,1,2,\mathbf{L},a-1\}} P^{(r_1)}[Q^{-T}(x+2pw)] \cdot P^{(m_1)}[Q^{-T}(x+2pw)]^* \cdot \exp\{i w \cdot w\} dw \\
&= \int_{[0,2p]} d_{r_1, m_1} I_s \cdot \exp\{i w \cdot w\} dw = O.
\end{aligned}$$

If $a_1 \neq b_1$, Put $u_1 = au_2 + r_2$, $v_1 = av_2 + m_2$, where $u_2, v_2 \in Z_+$, and $n_2, m_2 \in \{0,1,2,\mathbf{L},a-1\}$. If $u_2 \neq v_2$, we order $u_2 = au_3 + r_3$, $v_2 = av_3 + m_3$, where $u_3, v_3 \in Z_+$, $r_3, m_3 \in \{0,1,2,\mathbf{L},a-1\}$. After taking finite steps (denoted by k), we obtain $u_k, v_k \in \{0,1,2,\mathbf{L},a-1\}$. If $u_k = v_k$, so we have (15) follows. If $u_k \neq v_k$, then we have

$$\langle \Psi_{u_k}(\cdot), \Psi_{v_k}(\cdot - w) \rangle = O \Leftrightarrow \sum_{w \in Z} \hat{\Psi}_{u_k}(x+2np)^* \hat{\Psi}_{v_k}(x+2np) = O, w \in Z.$$

Furthermore, we obtain

$$\begin{aligned}
(2p) \langle \Psi_u(\cdot), \hat{\Psi}_v(\cdot - w) \rangle &= \int_{R^d} \hat{\Psi}_u(w) \hat{\Psi}_{v_1}(w)^* e^{i w \cdot w} dw = \int_R \hat{\Psi}_{au_1+r_1}(h) \hat{\Psi}_{av_1+m_1}(h)^* \cdot \exp\{i w \cdot w\} dw \\
&= \int_R P^{(r_1)}(x) \hat{\Psi}_{u_1}(x) \hat{\Psi}_{v_1}(x)^* P^{(m_1)}(x)^* \cdot e^{i w \cdot w} dw \\
&= \int_{R^d} P^{(r_1)}(x) P^{(r_2)}((1/a)^2 w) \hat{\Psi}_{a_2}((1/a)^2 w) \cdot \hat{\Psi}_{v_2}((1/a)^2 w)^* P^{(m_2)}((1/a)^2 w)^* P^{(m_1)}(x)^* \cdot e^{i w \cdot w} dw \\
&= \mathbf{L L L L L L L L} \\
&= \int_{(1/a)^k([0,2p])} \left\{ \prod_{i=1}^k P^{(r_i)}((1/a)^i h) \right\} \cdot O \cdot \left\{ \prod_{i=1}^k P^{(m_i)}((1/a)^i h) \right\}^* \cdot \exp\{i w \cdot w\} dw = O.
\end{aligned}$$

Therefore, for any $u, v \in Z_+$, result (15) follows.

Conclusion

We present an approach for constructing biorthogonal two-direction vector-valued wavelet and discuss their properties by virtue of time-frequency analysis method, matrix theory and functional analysis method. Two new biorthogonal bases of $L^2(R, C^s)$ are obtained.

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