Estimation of Water Production Function of Tobacco K326 by Jensen Model

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Abstract. Improper irrigation period or excessive irrigation will reduce the yield and quality of tobacco crop. This paper adopted the Jensen multiplication model method to stimulate the water production function of tobacco under different irrigation conditions. According to the data, a water production functions of flue-cured tobacco K326 were established in root period, vigorous period and mature period. It was shown that the water sensitive index \(\lambda\) was the largest in the vigorous period, which suggested that in this period the amount of irrigation water had the most important influence on the yield of the tobacco and the vigorous period was the crucial stage of the growth. From the point of the water sensitive index \(\lambda\), the water supply should be ensured in the vigorous period of the tobacco.

Introduction

The flue-cured tobacco could only be grown smoothly under the condition of suitable soil moisture, but soil moisture seldom satisfied the need of tobacco growth, drought often became the limiting factors for the tobacco growth and development. In this case, suitable irrigation would be the main measure for field management. According to the report\(^{[1-5]}\), timely and moderate irrigation could promote the growth of tobacco plant, improve the yield and quality, but improper irrigation period or excessive irrigation would reduce the yield and quality of flue-cured tobacco.

A sensitive index of yield for crop in different stages to ensure proper irrigation for different stages depended on the relationship of evaporation (ET) and yield (Y) under different irrigation conditions. The so-called water production function of crop could be summarized as static model and dynamic model\(^{[6]}\). Static model had two major categories of water production function of crop, one was the water production function for the whole growth period, another was the water production function for different stages, the latter contained Blank, Stewart, Singh, Hiller, Sudar and D-G function etc. as addition functions and Jensen, Minhas, Rao, Hanks and HK function etc\(^{[7]}\) as multiplication function.

Linear water production function based on the total evapotranspiration for the whole growth period appeared in the late 18th century. So far, numerous studies of water production functions had been made, most of which were used for wheat, corn, cotton, onion about the relationship between ET and yield, the function under salinity and water, fertilizer stress, or to build the net production efficiency function\(^{[8]}\). Jensen model was one of the water production functions for different stages which used relative evapotranspiration for each growth period as arguments, then corresponding multiplication formula formed the final function\(^{[9]}\).

Case Study

Experiments were carried out in lysimeters situated in Nanjing Vegetables (Flower) Scientific Institute to establish the irrigation system for tobacco K326 during the whole growth periods. The rainy seasons
started from late June and ended in July every year with average annual rainfall days of 117d and average annual rainfall of 1106.5mm. The annual average temperature was 15.7°C, the maximum average humidity was 81%, the maximum wind speed was 19.8m/s and the frost-free period was 237d. Irrigation was effectuated 13 times for 3 growth periods: root elongation period, vigorous period, maturity. Irrigation schedules were shown in Table 1.

**Table 1. Irrigation schedules for tobacco K326 during the whole growth periods**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Root extending period</th>
<th>Early vigorous period</th>
<th>Late vigorous period</th>
<th>Maturity</th>
<th>Total irrigation rate (m³/plant)</th>
<th>Irrigation rate (m³/acre)</th>
<th>Irrigation rate (m³/hm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.08</td>
<td>80</td>
<td>1200</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.12</td>
<td>120</td>
<td>1800</td>
</tr>
<tr>
<td>3</td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.08</td>
<td>0.18</td>
<td>180</td>
<td>2700</td>
</tr>
<tr>
<td>4</td>
<td>0.04</td>
<td>0.08</td>
<td>0.04</td>
<td>0.04</td>
<td>0.20</td>
<td>200</td>
<td>3000</td>
</tr>
<tr>
<td>5</td>
<td>0.04</td>
<td>0.04</td>
<td>0.08</td>
<td>0.08</td>
<td>0.24</td>
<td>240</td>
<td>3600</td>
</tr>
<tr>
<td>6</td>
<td>0.04</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.28</td>
<td>280</td>
<td>4200</td>
</tr>
</tbody>
</table>

The maximum evapotranspiration (ET) and yields (Y) for each growth period of K326 were shown in Table 2.

**Table 2. ET and Y of tobacco K326 under different conditions of irrigation (m³/hm²)**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Root extending period</th>
<th>Early vigorous period</th>
<th>Late vigorous period</th>
<th>Maturity</th>
<th>Total ET (m³/hm²)</th>
<th>Total Y (kg/hm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>302</td>
<td>413</td>
<td>356</td>
<td>386</td>
<td>1457</td>
<td>2420</td>
</tr>
<tr>
<td>2</td>
<td>314</td>
<td>373</td>
<td>342</td>
<td>700</td>
<td>1729</td>
<td>2807</td>
</tr>
<tr>
<td>3</td>
<td>283</td>
<td>624</td>
<td>375</td>
<td>1097</td>
<td>2379</td>
<td>2886</td>
</tr>
<tr>
<td>4</td>
<td>597</td>
<td>1068</td>
<td>407</td>
<td>807</td>
<td>2879</td>
<td>3173</td>
</tr>
<tr>
<td>5</td>
<td>612</td>
<td>570</td>
<td>738</td>
<td>1318</td>
<td>3238</td>
<td>2307</td>
</tr>
<tr>
<td>6</td>
<td>533</td>
<td>1013</td>
<td>821</td>
<td>1192</td>
<td>3559</td>
<td>2675</td>
</tr>
</tbody>
</table>

In consideration of the advantages and versatility of Jensen model, it was used in this study for water production function of the flue-cured tobacco K326.

\[
\frac{Y_a}{Y_m} = \prod_{i=1}^{n} \left( \frac{ET_a}{ET_m} \right)^{\lambda_i}, \quad (1)
\]

Where \( \lambda_i \) is Sensitive Index of yield for hydropenia in different stages; \( i \) is number of stage division, \( i = 1, 2, ..., n \).

When \( T = \ln \left( \frac{Y_a}{Y_m} \right), \quad Xi = \ln \left( \frac{ET_a}{ET_m} \right) \),

Equation 1 could be simplified into a linear equation:

\[
T = \sum_{i=1}^{n} \lambda_i X_i \quad (2)
\]

If the amount of treatments was \( m \), \( J \) groups of \( X_{ij} \) could be got, ( \( j = 1, 2, 3...m \); \( i = 1, 2, ..., n \) )

The objective function could be found with least square method.
\[
\begin{align*}
\text{min } f &= \sum_{j=1}^{m} \left( T_j - \sum_{i=1}^{n} \lambda_i \cdot X_{ij} \right)^2 \\
\text{When } \frac{\partial f}{\partial \lambda_i} &= 0 \\
\frac{\partial f}{\partial \lambda_i} &= -2 \sum_{j=1}^{m} \left( T_j - \sum_{i=1}^{n} \lambda_i X_{ij} \right) \cdot X_{ij} = 0 \\
L_{ik} &= \sum_{j=1}^{m} X_{ij} \cdot X_{kj} \quad (k=1, 2, \ldots, n) \\
L_{it} &= \sum_{j=1}^{m} X_{ij} \cdot T_j \quad (i=1, 2, \ldots, n) \\
\begin{bmatrix}
L_{1t} \\
L_{2t} \\
\vdots \\
L_{nt}
\end{bmatrix} &= \\
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_n
\end{bmatrix} \begin{bmatrix}
L_{11} & L_{12} & L_{13} & \cdots & L_{1n} \\
L_{21} & L_{22} & L_{23} & \cdots & L_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
L_{n1} & L_{n2} & L_{n3} & \cdots & L_{nn}
\end{bmatrix}
\end{align*}
\]

The correlation coefficient \( R \):
\[
R = \left[ \frac{\sum_{i=1}^{n} \lambda_i \cdot L_{i,n+1}}{L_{n+1,n+1}} \right]^2
\]

Results and Analysis

According to the data, water production function of the flue-cured tobacco K326 for the root extending period, the vigorous period, maturity could be made as:

\[
Y_a = \left( \frac{ET_1}{ET_{m1}} \right)^{0.073} \cdot \left( \frac{ET_2}{ET_{m2}} \right)^{0.521} \cdot \left( \frac{ET_3}{ET_{m3}} \right)^{0.143}
\]

\[R^2 = 0.769\]

It was shown that the water sensitive index \( \lambda \) was the largest in the vigorous period, which meant that in this period the amount of irrigation water had the most important influence on the yield of the tobacco and the vigorous period was the most important stage of the growth and development. From the point of the water sensitive index \( \lambda \), the water supply should be ensured in the vigorous period of the tobacco.

According to total yields of each irrigation treatment in different growth periods, irrigation amount did not have an apparent positive relationship with ET in the root elongation period, the reason might be that there were the smaller water demand and slow growth trend in the root elongation period of the flue-cured tobacco K326. Ensuring the irrigation amount in vigorous period, tobacco plants of treatment 4 grew well with more ET in the early vigorous period while less ET in the late. Treatment 6 followed with a higher plant transpiration than those of others. Because treatment 1 and treatment 2 were given smaller amount of irrigation, they transpired less with a better efficiency of water use. Flue-cured tobacco went into the reproductive growth stage in maturity, for all of the treatments, the life activity of the plants was weakened with a decreasing plant transpiration compared with the period
of rapid growth. So the water sensitive index $\lambda$ from Jensen model for 3 periods of the flue-cured tobacco K326 followed that the vigorous period $\rightarrow$ the maturity period $\rightarrow$ the root elongation period.

**Conclusions**

Jensen multiplication model method was adopted to stimulate the water production function of the flue-cured tobacco K326 under different irrigation conditions in root elongation period, vigorous period and mature period. Irrigation amount did not have an apparent positive relationship with ET in the root elongation period and the maturity period, it was shown that the water sensitive index $\lambda$ was the largest in the vigorous period, which suggested that in this period the amount of irrigation water had the most important influence on the yield of the tobacco and the vigorous period was the crucial stage of the growth. From the point of the water sensitive index $\lambda$, the water supply should be ensured in the vigorous period of the tobacco.

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**References**


