

An Optimal Preventive Maintenance Scheduling Model

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Abstract. A generalized model for determining minimum-cost preventive maintenance schedules of a maintained reliability system is presented. Through balancing the cost of system failure during operation against the cost of planned maintenance, an optimal policy is established which minimizes the expected cost rate. A simple equation to determine the optimal period between maintenances for a system with the known failure distribution is acquired, and the calculation for the system with Gamma failure distribution is given.

Introduction

Preventive maintenance (PM) of large complex systems such as computers, radars and airplanes is of great importance in reliability theory. PM includes inspection of a system, and repair or replacement of its parts so as to decrease the possibility of system failure during operation. On the other hand, the cost of maintenance increases with the frequency of PM. In order to make the best use of a system, it is necessary to find an optimal inspection strategy by taking into account the cost of the system failure and the cost of planned maintenance.

There are some literatures concerning various PM strategies. PM models with minimal repairs have been studied by Barlow and Hunter [1]. The authors discussed two age-replacement policies: one is useful in maintaining simple systems (with one component) and the other for large and complex systems (with multi-components). Their models have been generalized by Bechelt and Fisher [2] to the case of two types of failure distributions. Park [3] proposed a policy based on the number of minimal repairs performed. Most models are employed to determine optimal PM policy by minimizing the expected total cost during the system life cycle. In this paper, the optimal period between PM is found by minimizing the average cost per unit operation time, i. e. cost rate of the system.

Minimum average cost rate criterion

Suppose that the failure of a system during operation is immediately identified and $f_T(t)$ is the probability density function of the system failure time. Let us call the cost in consequence of failure C_F and the cost of PM C_M , where C_F and C_M are constants. If the period between the

present PM and the next is τ , we divide the time interval $[0, \tau]$ into N small time intervals: $[0, t_1], \dots, [t_{i-1}, t_i], \dots, [t_{N-1}, t_N]$. The cost per unit operation time of the system, i.e. cost rate is give by

$$\xi_\tau = \sum_{i=1}^N \frac{C_F}{t_i} \mathbf{I}\{t_{i-1} < T \leq t_i\} + \frac{C_M}{\tau} \mathbf{I}\{T > \tau\} \quad (1)$$

where $\mathbf{I}\{A\}$ is an indicator function of event A and T the system failure time..

The Eq. 1 means that if failure has occurred at time interval $[t_{i-1}, t_i]$, then the cost rate becomes equal to C_F/t_i , and if after period τ when PM is executed, the cost rate will be equal to C_M/τ . The cost rate ξ_τ is a random variable and its expected value, i.e. average cost rate is

$$E\xi_\tau = \sum_{i=1}^N \frac{C_F}{t_i} \mathbf{P}\{t_{i-1} < T \leq t_i\} + \frac{C_M}{\tau} \mathbf{P}\{T > \tau\} \quad (2)$$

The small time interval length $\Delta t = t_i - t_{i-1}$ decreases with the increase of the number N . We have

$$\mathbf{P}\{t_{i-1} < T \leq t_i\} \approx f_T(t) \Delta t \quad (3)$$

So when $N \rightarrow \infty$, Eq. 2 becomes

$$E\xi_\tau = \int_0^\tau \frac{C_F}{t} f_T(t) dt + \frac{C_M}{\tau} \int_\tau^\infty f_T(t) dt \quad (4)$$

The optimal period τ^* can be obtained by setting the partial derivative of $E\xi_\tau$ with respect to τ zero and solving the equation, i.e.

$$\frac{dE\xi_\tau}{d\tau} = \frac{C_F - C_M}{\tau} f_T(\tau) - \frac{C_M}{\tau^2} \int_\tau^\infty f_T(t) dt = 0 \quad (5)$$

Numerical example

Assume a two parameter Gamma distribution for the system failure. Gamma distribution is a natural extension of the exponential distribution. If the shocks to a system occur independently at the same rate λ and the system will occur failure after α shocks, then the system failure time T is the Gamma distribution. The probability density function is

$$f_T(t) = \frac{\lambda^\alpha t^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda t} \quad t > 0 \quad (6)$$

Where λ is the scale parameter; α the shape parameter, and $\Gamma(\alpha)$ the Gamma function, i.e.

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \quad (7)$$

Assume $\alpha = 2$, Eq. 6 becomes

$$f_T(t) = \frac{\lambda^2 t}{\Gamma(2)} e^{-\lambda t} = \lambda^2 t e^{-\lambda t} \quad (8)$$

Substituting $f_T(t)$ into Eq. 5 yields

$$(C_F - C_M)\lambda^2 e^{-\lambda\tau} - \frac{C_M}{\tau} \int_{\tau}^{\infty} \lambda^2 t e^{-\lambda t} dt = 0 \quad (9)$$

where $\int_{\tau}^{\infty} \lambda^2 t e^{-\lambda t} dt = \lambda\tau e^{-\lambda\tau} - e^{-\lambda\tau}$. Eq. 9 can be rewritten as

$$\left(\frac{C_F}{C_M} - 1\right)\lambda\tau^2 - \lambda\tau - 1 = 0 \quad (10)$$

So the optimal period between PM is

$$\tau^* = \frac{C_M + \sqrt{C_M(4C_F - 3C_M)}}{2\lambda(C_F - C_M)} \quad (11)$$

Substituting τ^* into Eq. 4, it gives the corresponding average cost rate $E\xi_{\tau^*}$. The results of calculations are shown in Table 1.

Table 1 Optimal period between PM and average cost rate for ($\alpha = 2; C_M = 10$)

C_F	$\lambda = 0.1$		$\lambda = 0.2$		$\lambda = 0.3$		$\lambda = 0.4$		$\lambda = 0.5$	
	τ^*	$E\xi_{\tau^*}$								
20	16.18	1.92	8.09	3.85	5.39	5.77	4.05	7.70	3.24	9.62
30	10.00	2.63	5.00	5.26	3.33	7.90	2.50	10.53	2.00	13.16
40	7.68	3.21	3.84	6.42	2.56	9.64	1.92	12.85	1.54	16.06
50	6.40	3.71	3.20	7.43	2.13	11.14	1.60	14.86	1.28	18.57
60	5.58	4.16	2.79	8.33	1.86	12.49	1.40	16.66	1.12	20.82
70	5.00	4.57	2.50	9.15	1.67	13.72	1.25	18.30	1.00	22.07
80	4.56	4.95	2.20	9.91	1.52	14.86	1.14	19.81	0.91	24.77
90	4.22	5.31	2.11	10.62	1.41	15.92	1.05	21.23	0.84	26.54
100	3.93	5.64	1.97	11.28	1.31	16.93	0.98	22.57	0.79	28.21
110	3.70	5.96	1.85	11.92	1.23	17.88	0.93	23.84	0.74	29.80

The following interpretations can be given to the data in Table 1.

- (1) By holding the cost of PM (C_M) constant, a system with high cost of failure (C_F) will require frequent maintenance. This will increase the average cost rate of operating the system. For example, for a system with $\lambda = 0.1$, it requires PM of 16.18 time units during the system operation at the average cost rate of 1.92, given $C_F = 20$. However, for the case where $C_F = 100$, PM of 3.93 time units at the average cost rate of 5.64 is incurred.
- (2) The period between PM (τ^*) decreases for a system with decreasing value of λ . That is, the smaller the amount of failure a system generates during its operation, the less frequent is the maintenance it may need. For example, for $\lambda = 0.1$ (i.e. one failure per 10 time units), τ^* has

a value of 5.58; $E\xi_{\tau^*} = 4.16$, and for $\lambda = 0.5$ (i.e. five failures per 10 time units), τ^* has a value of 1.12; $E\xi_{\tau^*} = 20.82$.

Conclusions

This paper has established a minimum average cost rate criterion for determining economic PM schedules of maintained reliability systems. Compared with existing models, the minimum average cost rate criterion is natural and the equation for calculating the optimal PM period is simple. The application of the approach to a system with Gamma type failure distribution is presented. Gamma distribution is selected over other distributions because of its versatility in describing cumulative system shocks. The computational results are shown that a system with higher failure rate and cost would require more frequent maintenance.

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