

Exact Analytic Formulae of Beam Pointing Based on Achromatic Risley Prisms

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Abstract. The mathematic relationship between the given orientations of two-prism groups and the corresponding pointing position of outgoing beam is the basic prerequisite needed in applications of achromatic risley prisms to steer beam in free space. Based on vectorial law of refraction, this paper finishes the derivation of the analytic formulae of the relationship wanted. Then the test samples obtained by the analytic formulae, the method of paraxial approximation and professional the optical software are compared respectively. The results prove the correctness of the analytic formulae, as well as its necessity and superiority in high-precision pointing solution when beam is steered over wide angular range.

Introduction

Traditionally, beampointing mechanism in photoelectric acquisition, tracking and pointing system is composed of gimbal and fast steering mirror mechanism together, in order to meet the demands of wide scan angular range, fast response frequency and high pointing precision simultaneously. Gimbal mechanism has advantage in wide scan angular range, but is dragged in the performance of dynamic movement for its big volume and inertia [1,2,3,4]. On the contrary, fast steering mirror mechanism does well in fast response frequency and high pointing precision. However, those superiorities can only be kept within really narrow scan angular range [5,6]. Consist of the first and second refraction prism sharing the same spin axis, risley prisms mechanism realizes beams spatial movement by altering orientations of two prisms. With relative wide scan angular range, fast response frequency, high pointing precision and compact simple structure, it can satisfy all those demands mentioned above in a single apparatus [7], thus showing strong comparative advantages and obvious potential for substitution [8] in many fields like photoelectric detection, space optical communication, fiber optic switch and so on. Applying risley prisms, the Geoscience Laser Altimeter System launched in 2003 achieves precise pointing of laser beam [9]. The beamsteering device developed by OPTRA Inc., which can reach pointing deviation under 1 milliradian over steering angular range of 60° , is based on risley prisms [4]. So does the inter satellite laser communication device designed by SIOM of Chinese Academy of Sciences, which reaches scanning accuracy under 50 microradian within horizontal and vertical steering angular range of $\pm 15^\circ$ [10].

The mathematic relationship between the given orientations of two prisms and the corresponding pointing position of outgoing beam is the prerequisite needed in applications of risley prisms to steer beam in free space. [11]. Method of paraxial approximation is adopted in solving optical refraction usually. However, it has obvious deviation of pointing solution when beam is steered in wide angle, which goes against the superiorities of risley prisms. Another method is curve fitting

with discrete sample points. Helped by professional optical software like ZEMAX, a set of sample solutions in very high precision can be obtained. Unfortunately, fitting doesn't work well globally under the influence of highly nonlinear characteristic existing in the relationship wanted.

Vectorial law of refraction can deal with the problem above. Subject to complex mathematic techniques, only recently the analytic formulae of the relationship wanted was derivated by Yang Y and Li Y for different risley prisms structure respectively [11,12]. Nevertheless, those formulae are extracted from structures without achromatic optical apparatus, which means they are available for scenarios using single-wave beam merely. Considering so many applying scenarios of risley prisms in need of multi-wave beam, and increase of solution complexity brought by additional achromatic optical apparatus, it is important and significant to derive exact analytic formulae that can be used in pointing solution of achromatic risley prisms.

Based on vectorial law of refraction, this paper focuses on the process of deriving the analytic formulae of the mathematic relationship wanted. And it illustrates the test samples obtained by the analytic formulae, the method of paraxial approximation and the professional optical software ZEMAX to prove the correctness of the formulae. Then the analysis of advantages of different methods are given to demonstrate the necessity and superiority of the analytic formulae in high-precision pointing solution when beam is steered over wide angular range.

Structure of achromatic risley prisms

Fig.1 shows the achromatic risley prisms mechanism, four prisms rotate around the shared spin axis (axis-Z). Primarily, risley prisms mechanism is composed of the right-angled prisms Π_2 and Π_3 only, but now the trapezoid prisms Π_1 and Π_4 are added for compensating chromatic aberration. So the prisms Π_2 and Π_3 are called primary prisms while the prisms Π_1 and Π_4 are called achromatic prisms.

The orthogonal surfaces of the primary prisms Π_2 and Π_3 are parallel mutually and perpendicular to the spin axis. Π_2 and Π_3 are provided with the same open angle α_1 and refraction index n_1 . The additional achromatic prisms Π_1 and Π_4 are mirrored placed and provided with the same open angle α_2 and refraction index n_2 . Besides, their surfaces close to Π_2 and Π_3 are parallel to the inclined surfaces of the Π_2 and Π_3 respectively.

In movement, the achromatic prism Π_1 and the primary prism Π_2 shared the same rotating angle θ_1 around the spin axis. And the achromatic prism Π_4 and the primary prism Π_3 shared rotating angle θ_2 in a similar state. Angle θ_1 and θ_2 are measured from the positive axis-X by the vertex of the prism Π_2 and Π_3 respectively. Incidence beam is injected reversely along the axis-Z, and outgoing beam are described by azimuth angle Θ measured from the positive axis-X and elevation angle Φ measured from the negative axis-Z.

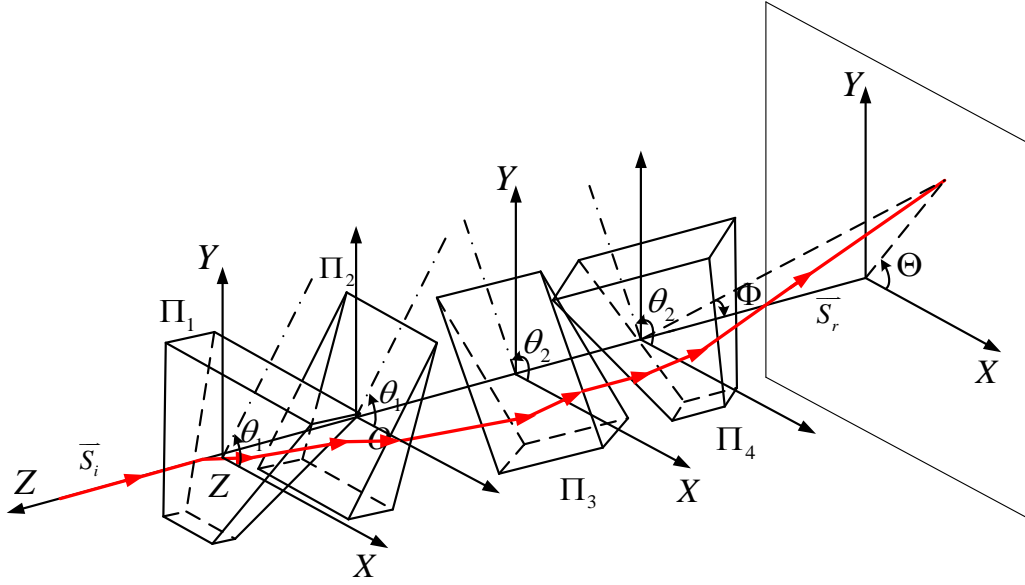


Fig.1 structure of the achromatic risley prisms mechanism

Method of paraxial approximation

Method of paraxial approximation treats prism as wedge with small open angle and supposes beam is refracted to the thicker part of prism regardless of the position of incidence beam and the orientation of prisms. Paralleled mutually on the opposite surfaces, the achromatic prism Π_1 and the primary prism Π_2 are fixed placed and rotate around the axis-Z with shared angle θ_1 . As Fig.2 shows, benefit from some known results [13], the steered angle Ψ_1 of beam passing through the two prisms is represented as

$$\Psi_1 = (n_1 - 1)\alpha_1 - (n_2 - 1)\alpha_2 \quad (1)$$

Similarly, the steered angle Ψ_2 of beam passing through the primary prism Π_3 and achromatic prism Π_4 is given by:

$$\Psi_2 = (n_1 - 1)\alpha_1 - (n_2 - 1)\alpha_2 \quad (2)$$

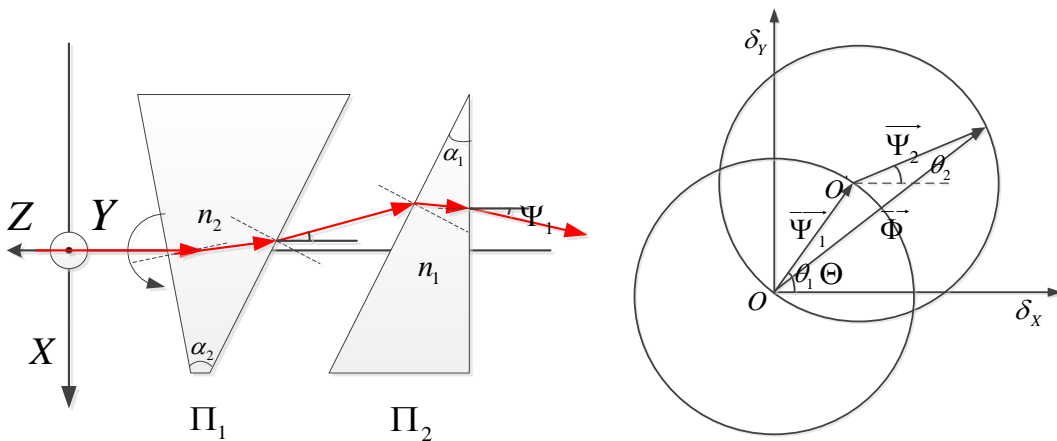


Fig.2 position of the Π_1 and Π_2 Fig.3 central algorithm

Fig.3 shows how to obtain the pointing position of outgoing beam after all the refractions with central algorithm [13]. Point O represents the spin axis (axis-Z) of the mechanism, and coordinate axis- δ_x , δ_y represents the projection of the orientation of beam on the axis-X and axis-Y respectively. Then spatial beam can be described as a plane vector in the coordinate while the vector module represents elevation (angle between outgoing beam and the negative axis-Z) and the angle

between the vector and axis- δ_x represents azimuth (orientation of outgoing beam from the positive axis-X) [13].

Incident from the negative axis-Z, the pointing position of intermediate beam passing through the achromatic prism Π_1 and the primary prism Π_2 is represented as a plane vector $\overrightarrow{\Psi_1}$. Along with rotation of the first prism group, $\overrightarrow{\Psi_1}$ originates from point O and terminates on moving placement marked as O' of a circular trace with radius Ψ_1 of Eq.1. Then based on the intermediate beam, the plane vector $\overrightarrow{\Psi_2}$ describes the pointing position of ultimate beam which passes through the primary prism Π_3 and the achromatic prism Π_4 further. $\overrightarrow{\Psi_2}$ starts from O' and ends in a circular moving trace with radius Ψ_2 of Eq.2 as the second prism group rotating. Finally and globally, the total pointing position of outgoing beam based on the original spin axis-Z is regarded as vector sum $\overrightarrow{\Phi}$ from $\overrightarrow{\Psi_1}$ and $\overrightarrow{\Psi_2}$. Projection of $\overrightarrow{\Phi}$ on the axis- δ_x and δ_y are given by:

$$\Phi_x = \Psi_1 \cos \theta_1 + \Psi_2 \cos \theta_2 \quad (3)$$

$$\Phi_y = \Psi_1 \sin \theta_1 + \Psi_2 \sin \theta_2 \quad (4)$$

From Eq.3 and Eq.4, the elevation angle Φ and the azimuth Θ are given by:

$$\Phi = \sqrt{\Psi_1^2 + \Psi_2^2 + 2\Psi_1\Psi_2 \cos(\theta_1 - \theta_2)} \quad (5)$$

$$\tan \Theta = \frac{\Psi_1 \sin \theta_1 + \Psi_2 \sin \theta_2}{\Psi_1 \cos \theta_1 + \Psi_2 \cos \theta_2} \quad (6)$$

Analytic formulae based on the vectorial law of refraction

Unit vector \vec{s}_i and \vec{n} represent incidence beam and normal vector of refraction surface respectively. The outgoing beam after transmitting from medium of index n_1 to medium of index n_2 is described as \vec{s}_r below. This equation is called vectorial law of refraction.

$$\vec{s}_r = \frac{n_1}{n_2} [\vec{s}_i - (\vec{s}_i \cdot \vec{n}) \cdot \vec{n}] - \sqrt{1 - \frac{n_1^2}{n_2^2} + \frac{n_1^2}{n_2^2} (\vec{s}_i \cdot \vec{n})^2} \cdot \vec{n} \quad (7)$$

Fig.4 shows the refraction process of incidence beam from the negative axis-Z. And the normal vector of each surface is listed in order of refraction as follow:

$$\vec{n}_1 = (\sin(\alpha_2 - \alpha_1) \cos \theta_1, \sin(\alpha_2 - \alpha_1) \sin \theta_1, \cos(\alpha_2 - \alpha_1)) \quad (8)$$

$$\vec{n}_2 = (-\sin \alpha_1 \cos \theta_1, -\sin \alpha_1 \sin \theta_1, \cos \alpha_1) \quad (9)$$

$$\vec{n}_3 = (\sin \alpha_1 \cos \theta_2, \sin \alpha_1 \sin \theta_2, \cos \alpha_1) \quad (10)$$

$$\vec{n}_4 = (-\sin(\alpha_2 - \alpha_1) \cos \theta_2, -\sin(\alpha_2 - \alpha_1) \sin \theta_2, \cos(\alpha_2 - \alpha_1)) \quad (11)$$

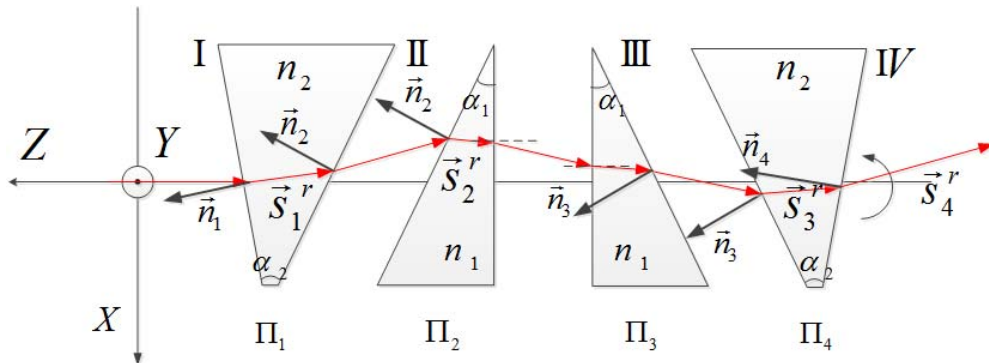


Fig.4 refractions in achromatic risley prisms mechanism

With vectorial law of refraction, pointing position of outgoing beam after each refraction can be solved sequentially.

Refraction I, from the air to the achromatic prism Π_1 . Substitute $\vec{s}_i = [0, 0, -1]$ and Eq.8 to Eq.7, the pointing position of beam after Refraction I is given by:

$$\vec{s}_1^r = \frac{1}{n_2} [a_1 \sin(\alpha_2 - \alpha_1) \cos \theta_1, a_1 \sin(\alpha_2 - \alpha_1) \sin \theta_1, -a_2] \quad (12)$$

Where constants a_1 and a_2 are:

$$a_1 = \cos(\alpha_2 - \alpha_1) - \sqrt{n_2^2 - \sin^2(\alpha_2 - \alpha_1)}$$

$$a_2 = \sin^2(\alpha_2 - \alpha_1) + \sqrt{n_2^2 - \sin^2(\alpha_2 - \alpha_1)} \cos(\alpha_2 - \alpha_1)$$

Refraction II, from the achromatic prism Π_1 to the primary prism Π_2 . Considering parallelism of the right surface of the prism Π_1 and the left surface of the prism Π_2 would not alter transmission direction of beam, the two refractions taking place on those surfaces can be simplified into one from medium of index n_2 to medium of index n_1 directly. Substitute $\vec{s}_i = \vec{s}_1^r$ and Eq.9 to Eq.7, the pointing position of this step is given by:

$$\vec{s}_2^r = \{c_1 \cos \theta_1, c_1 \sin \theta_1, c_2\} \quad (13)$$

Where constants b_1, b_2, b_3, c_1 and c_2 are:

$$b_1 = \sin(\alpha_2 - \alpha_1) \sin \alpha_1 [\cos(\alpha_2 - \alpha_1) - \sqrt{n_2^2 - \sin^2(\alpha_2 - \alpha_1)}]$$

$$+ \cos \alpha_1 [\sin^2(\alpha_2 - \alpha_1) + \sqrt{n_2^2 - \sin^2(\alpha_2 - \alpha_1)} \cos(\alpha_2 - \alpha_1)]$$

$$b_2 = b_1 - \sqrt{1 - n_2^2 + b_1^2}$$

$$b_3 = \frac{\sqrt{1 - n_2^2 + b_1^2}}{n_1} - \sqrt{1 - \frac{1}{n_1^2} + \frac{1}{n_2^2} (1 - n_2^2 + b_1^2)}$$

$$c_1 = \frac{1}{n_1} (a_1 \sin(\alpha_2 - \alpha_1) - b_2 \sin \alpha_1) - b_3 \sin \alpha_1$$

$$c_2 = \frac{b_2}{n_1} \cos \alpha_1 - \frac{a_2}{n_1} + b_3 \cos \alpha_1$$

Refraction III, from the primary prism Π_2 to the achromatic prism Π_4 . Sharing the same index n_1 , the orthogonal surfaces of the prism Π_2 and Π_3 are parallel mutually and perpendicular to the spin axis (axis-Z). So their relative rotation would not alter transmission direction of beam, which means the two refractions taking place there can be neglected. Besides, similar simplification like the refraction II can be applied to the parallelism of the prisms Π_3 and Π_4 . Thus the two original refractions are regarded as one from medium of index n_1 to medium of index n_2 directly. Substitute $\vec{s}_i = \vec{s}_2^r$ and Eq.10 to Eq.7, the pointing position of this step is represented as:

$$\vec{s}_3^r = \left\{ \frac{n_1 c_1}{n_2} \cos \theta_1 - \frac{g(\Delta\theta)}{n_2} \sin \alpha_1 \cos \theta_2, \frac{n_1 c_1}{n_2} \sin \theta_1 - \frac{g(\Delta\theta)}{n_2} \sin \alpha_1 \sin \theta_2, \frac{n_1 c_2}{n_2} - \frac{g(\Delta\theta)}{n_2} \cos \alpha_1 \right\} \quad (14)$$

Where correlation variables $f(\Delta\theta)$ and $g(\Delta\theta)$ are:

$$f(\Delta\theta) = c_1 \sin \alpha_1 \cos \Delta\theta + c_2 \cos \alpha_1, \quad \Delta\theta = \theta_1 - \theta_2$$

$$g(\Delta\theta) = n_1 f(\Delta\theta) + \sqrt{n_2^2 - n_1^2 + n_1^2 f^2(\Delta\theta)}$$

Refraction IV, from the achromatic prism Π_4 to the air. Substitute $\vec{s}_i = \vec{s}_3^r$ and Eq.11 to Eq.7,

the pointing position of the last step is represented as:

$$\begin{aligned}\vec{s}_4' = & \{n_1c_1 \cos \theta_1 - g(\Delta\theta) \sin \alpha_1 \cos \theta_2 + p(\Delta\theta) \sin(\alpha_2 - \alpha_1) \cos \theta_2, \\ & n_1c_1 \sin \theta_1 - g(\Delta\theta) \sin \alpha_1 \sin \theta_2 + p(\Delta\theta) \sin(\alpha_2 - \alpha_1) \sin \theta_2, \\ & n_1c_2 - g(\Delta\theta) \cos \alpha_1 - p(\Delta\theta) \cos(\alpha_2 - \alpha_1)\}\end{aligned}\quad (15)$$

Where correlation variables $h(\Delta\theta)$ and $p(\Delta\theta)$ are:

$$\begin{aligned}h(\Delta\theta) &= -n_1c_1 \sin(\alpha_2 - \alpha_1) \cos \Delta\theta + n_1c_2 \cos(\alpha_2 - \alpha_1) - g(\Delta\theta) \cos \alpha_2 \\ p(\Delta\theta) &= h(\Delta\theta) + \sqrt{1 - n_2^2 + h^2(\Delta\theta)}\end{aligned}$$

Above all, direction cosine (K, L, M) of the pointing position of outgoing beam are:

$$K = \frac{n_1c_1 \cos \theta_1 + g(\Delta\theta) \sin \alpha_1 \cos \theta_2 - p(\Delta\theta) \sin(\alpha_2 - \alpha_1) \cos \theta_2}{q(\Delta\theta) - r(\Delta\theta) \cos \Delta\theta - s(\Delta\theta)} \quad (16.1)$$

$$L = \frac{n_1c_1 \sin \theta_1 + g(\Delta\theta) \sin \alpha_1 \sin \theta_2 - p(\Delta\theta) \sin(\alpha_2 - \alpha_1) \sin \theta_2}{q(\Delta\theta) - r(\Delta\theta) \cos \Delta\theta - s(\Delta\theta)} \quad (16.2)$$

$$M = \frac{n_1c_2 - g(\Delta\theta) \cos \alpha_1 - p(\Delta\theta) \cos(\alpha_2 - \alpha_1)}{q(\Delta\theta) - r(\Delta\theta) \cos \Delta\theta - s(\Delta\theta)} \quad (16.3)$$

Where correlation variables $q(\Delta\theta)$, $r(\Delta\theta)$ and $s(\Delta\theta)$ are:

$$\begin{aligned}q(\Delta\theta) &= n_1^2(c_1^2 + c_2^2) + g^2(\Delta\theta) + p^2(\Delta\theta) + 2g(\Delta\theta)p(\Delta\theta) \cos \alpha_2 \\ r(\Delta\theta) &= 2n_1c_1[g(\Delta\theta) \sin \alpha_1 - p(\Delta\theta) \sin(\alpha_2 - \alpha_1)] \\ s(\Delta\theta) &= 2n_1c_2[g(\Delta\theta) \cos \alpha_1 + p(\Delta\theta) \cos(\alpha_2 - \alpha_1)]\end{aligned}$$

From Eq.16, the elevation angle Φ and the azimuth Θ of the outgoing beam are given by:

$$\begin{aligned}\Phi &= \arccos(-M) \quad (17) \\ \Theta &= \begin{cases} \arctan(\frac{L}{K}); & \text{when } K \geq 0 \text{ and } L \geq 0 \\ \arctan(\frac{L}{K}) + 2\pi; & \text{when } K \geq 0 \text{ and } L < 0 \\ \arctan(\frac{L}{K}) + \pi; & \text{when } K < 0 \end{cases} \quad (18)\end{aligned}$$

Inspection and analysis

As a highly professional and widely applicated commercial optical software, the precision of ZEMAX in beam pointing solution is guaranteed. In order to obtain more accurate beam pointing solution reference over wide steering angular range, this paper utilizes ZEMAX to simulate the process of beam refraction and the results of pointing solution. After building corresponding achromatic risley prisms model in ZEMAX, reference samples are computed by giving several couples of testing orientations θ_1 and θ_2 of two-prism groups. The beam used in ZEMAX simulation is single-wave because this paper focuses on verifying the correctness and superiorities of the analytic formulae in pointing solution rather than optical design problem of achromatic process. For beam in any wavelength and medium of any index, the goal of inspection can be reached only if those parameters are accordant in simulation of ZEMAX and substitution of the analytic formulae.

Pointing deviation of two beams is represented by spatial angle between their directive vectors. In fig.5, horizontal axis represents 13 sample orientations of $\Delta\theta = \theta_1 - \theta_2$ over angular range of $0 \sim 2\pi$ in step of $\pi/6$, and vertical axis represents deviation of beam pointing solution between different methods in corresponding orientations. The blue line illustrates the deviations between the method

of paraxial approximation and ZEMAX, while the red points depict the deviations between the analytic formulae and ZEMAX.

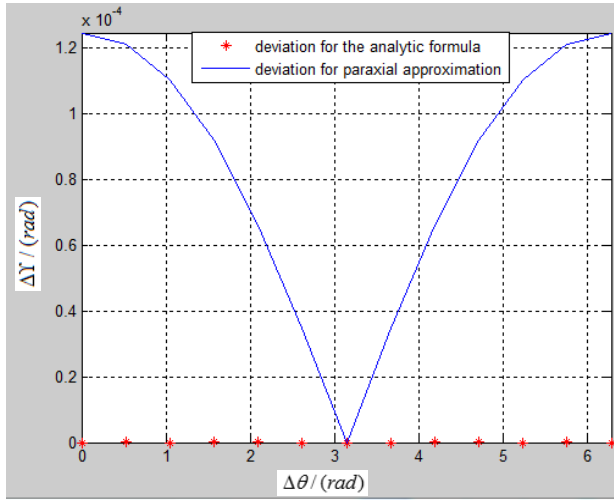


Fig.5 deviation between different methods

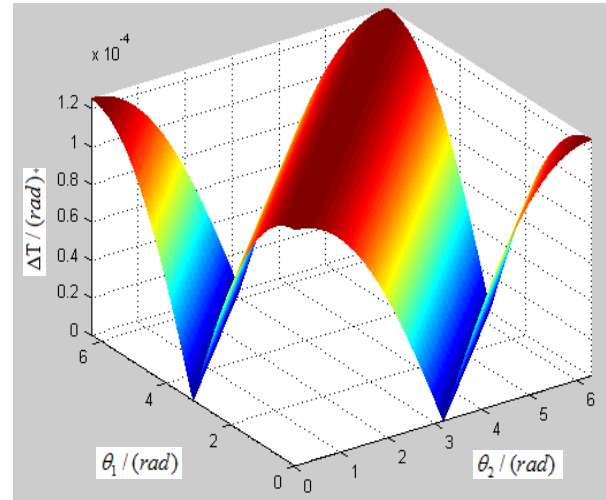


Fig.6 deviation between paraxial and analytic

For the method of paraxial approximation, only if $\Delta\theta = \pi$ can the minimum deviation 0 be reached. And the maximum deviation is on the order of 10^{-4} radian when $\Delta\theta$ near 0 or 2π . In general, the greater absolute value of $\Delta\theta - \pi$ is, the bigger solving deviation is.

For the analytic formulae, the minimum deviation is 0 while others are all on the order of 10^{-8} radian. Actually, should truncation error be eliminated, the others would be 0 as well. That is to say the result obtained by the analytic formulae is in accordance with the solution reference exactly.

Fig. 6 shows numerous deviations of pointing solution between results obtain by paraxial approximation and the analytic formulae when θ_1 and θ_2 alter over angular range of $0 \sim 2\pi$. For paraxial approximation, this figure also coincides with the change rule of solving deviation concluded before. Both the simulation of ZEMAX and Eq.5 tell that the steered angle of beam increases as the absolute value of $\Delta\theta - \pi$ grows, which means this method has obvious deviation of pointing solution when beam is steered in wide angle. So the necessity and superiority of the analytic formulae in high-precision pointing solution over wide steering angular range is proved from the reverse side.

Summary

Of the mathematic relationship between the given orientations of two-prism groups and the corresponding pointing position of outgoing beam, the analytic formulae derived from vectorial law of refraction can achieves great precision in pointing solution over wide steering angular range. Compared with the traditional method of paraxial approximation, which is simple but has obvious solving derivation when beam is steered in wide angle, the analytic formulae is more practical and reliable in scenarios demanding high precision in pointing solution while beam would be steered over wide angular range.

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