

Influence of Modulation Rate on Energy Efficiency for Relay-aided Transmission: Amplify-and-forward Protocol

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Abstract. This paper describes a general relay system on the condition of a given bit-error-rate(BER). The message bits are transmitted with the assistance of a relay node using one-way amplify-and-forward(AF) relaying protocols, and the effect of modulation rate on energy efficiency (EE) is studied. First, we calculate the transmission power consumption, then the total power consumption can be obtained when circuit power consumption is taken into account. Finally, the expression of EE is derived. The theoretical analysis shows that the energy efficiency is the quasiconcave function of the modulation rate. Through the simulation results we know that when the modulation rate increases, the energy efficiency first increases and then decreases.

Introduction

In wireless communication system, the transmission of information is affected by multipath fading, shadow fading, the influence of path loss and some other factors. The multi-antenna communication technology plays important role against channel fading^[1]. However, in practice, the installation of multiple antennas to improve the system performance is not realistic. In order to solve the problem, relay coordination communication plays an important role. Its core idea is that we deploy a relay node between the transmitter and the receiver to transmit the broadcast signal^[2-3].

The energy-efficient communication has become more and more important as a new design goal for various systems^[4]. Energy efficiency (EE) can be evaluated by the number of message bits transmitted with per-Joule total energy consumption. In practical systems, not only the power needed to transmit information bits but also the circuit energy consumptions should be considered. However, some papers has studied the EE of communication systems only consider transmission energy. For instance, approximate expressions for transmission energy per message bit were derived for flat-fading channels in wide band regime^[5].

In fact, we should consider the circuit energy to optimize the EE of communication systems. For instance, modulation schemes were optimized in [6] for communication system, in which the circuit power was assumed to remain fix and independent of the bit transmission rate. In [7], the paper studies the optimum energy efficiency and the corresponding spectral efficiency for flat-fading channels with rate-dependent circuit power in a more general form.

In wireless networks, the use of relaying provides higher link reliability when the source-destination link suffers severe fading. Among different cooperative strategies, the amplify-and-forward and the decode-and-forward cooperative techniques are often employed in cooperative networks. In [8], power allocation schemes are proposed for the total transmitted power in a relaying system based on bit-error-rate(BER) analysis. Considering amplify-and-forward and decode-and-forward for relaying systems and direct transmission, it perform a fair comparison with respect to their power consumption.

In this paper, we aim at maximizing the EE subject to a target BER at the destination for AF relay system. We describe the total energy consumption as the sum of amplifier's power consumption and circuit power consumption. We formulate the EE function and the quasiconcavity of EE is analyzed. Finally, the theoretical analysis is corroborated by numerical experiments.

The rest of this paper is organized as follows. The system model is described in the next Section. The EE analysis is made in Section III. And simulation results are shown in Section IV to explain our theoretical analysis. Some conclusions are made in Section V.

Description of system and protocols

We consider the scenario where a source node S and a destination node D, intend to transmit message bits with the assistance of a relay node R, as shown in figure 1. The noise power spectral density of single sideband is equal to N_0 Watts per Hertz. The channel between every two of the three nodes is flat-fading with bandwidth B Hz, and remain unchanged during the bits transmission. The transmit time per symbol is T_s . The channel amplitude gain between node S and the relay and that between node D and the relay equal to h_{sr} and h_{rd} , respectively.

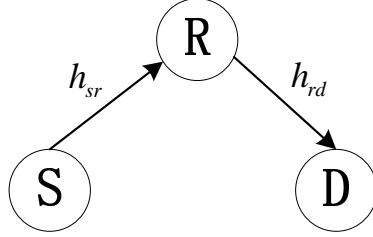


Fig.1 Three-point relay model

The protocols carry out the data transmission in two time slots. In the first slot, node S transmit modulated symbols. The relay receives and processes these symbols, then broadcast these symbols to node D in the second slot. We will describe the protocol and study their EE performance in the next sections.

We assume that the total energy consumed by the RF transmission energy consumption and the total circuit energy consumption. Define the efficiency of the power amplifier for nodes S and R is equal to ε , the circuit power consumption per node are the same.

For flat-fading channels, the receive SNR γ and BER for MQAM transmission can be approximated as [9],

$$p_e = c_1 \exp\left(\frac{-c_2 \gamma}{2^{c_3 \theta} - c_4}\right) \quad (1)$$

where c_1, c_2, c_3, c_4 is equal to 0.2, 1.6, 1, 1 relatively. θ is the bits number per symbol that node S transmit to node R.

In order to achieve a given target BER P_e^{tar} at the destination, the required target SNR γ^{tar} can be calculated

$$\gamma^{tar}(\theta) = \frac{2^{c_3 \theta} - c_4}{c_2} \ln \frac{c_1}{p_e^{tar}} \quad (2)$$

Energy efficiency analysis for AF relaying.

A. Description of the AF protocol

The one-way AF relaying is carried out as follows. In the first slot, node S transmit symbols to relay, the symbols received at the relay is

$$y_{sr} = \sqrt{P_s} h_{sr} x_s + n_{sr} \quad (3)$$

where x_s denotes the unit-variance modulated symbol transmitted by node S. n_{sr} is the additive white Gaussian noise with zero-mean symmetric complex Gaussian distribution of variance N_0B . P_s denote the average symbol power used by node S. In the second slot, the relay amplifies and broadcasts the received symbols. The symbol broadcasted by the relay node is

$$x_r = \beta y_{sr} \quad (4)$$

where β is the amplification coefficient. Assume the average symbol power used by the relay is equal to P_r . Thus, β can be expressed by

$$\beta = \sqrt{\frac{P_r}{P_s |h_{sr}|^2 + N_0B}} \quad (5)$$

At node D, the received symbol is

$$\begin{aligned} y_{rd} &= h_{rd}x_r + n_{rd} \\ &= \sqrt{P_s} \beta h_{sr} h_{rd} x_s + \beta h_{rd} n_{sr} + n_{rd} \end{aligned} \quad (6)$$

when the node D receive the symbols and decode the message bits. Then the signal-to-noise ratio at the node D is equal to

$$\gamma(P_s, P_r) = \frac{P_s g_{sr} P_r g_{rd}}{P_s g_{sr} + P_r g_{rd} + 1} \quad (7)$$

where $g_{sr} = \frac{|h_{sr}|^2}{N_0B}$, $g_{rd} = \frac{|h_{rd}|^2}{N_0B}$ represent the channel power gains normalized by the noise power.

B. Derivation of the energy efficiency

The link performance function of energy efficiency of $\eta(\theta)$ is derived when assures bit error rate is lower than P_e^{tar} . It can be defined as the number of message bits transmitted with per-Joule total energy consumption:

$$\eta(\theta) = \frac{\theta}{[P_c(\theta) + P_{RF}(\theta)]T_s} = \frac{\theta B}{P_c(\theta) + P_{RF}(\theta)} \quad (8)$$

where B is the channel bandwidth, $T_s = 1/B$ is the transfer time per symbol, $P_c(\theta)$ represents the total circuit power, $P_{RF}(\theta)$ represent the minimum total transmission power consumption assures the bit error rate is lower than γ^{tar} . We will derive them one by one.

Circuit power consumption includes the parts of signal processing and circuit module, such as AD converter, DA converter, frequency synthesizers, mixer, filter and amplifier^[10]. Based on literature [11], the total circuit power consumption can be divided into static circuit part and dynamic part of the associated with the transmission rate, namely:

$$P_{cir}(\theta) = P_{c0} + k\theta B \quad (9)$$

where P_{c0} and k is related to the concrete realization circuit of fixed parameters.

First show circuit power per symbol for the first slot.

$$P_{c,1} = 2P_{cir}(\theta)$$

Then show circuit power per symbol for the second slot.

$$P_{c,2} = 2P_{cir}(\theta)$$

The total circuit power consumption is

$$P_c(\theta) = 4P_{cir}(\theta) = 4(P_{c0} + k\theta B) \quad (10)$$

We derive the minimum transmission energy consumption to satisfy the BER requirement. To achieve this goal, we formulate the following problem:

$$\begin{aligned} P_{RF}(\theta) = \min_{P_s, P_r} & \frac{P_s + P_r}{\varepsilon} \\ \text{s.t. } & \gamma(P_s, P_r) \geq \gamma^{tar}(\theta) \end{aligned} \quad (11)$$

By using the KKT conditions for the problem, it is shown in Appendix A that the optimum source power P_s and relay power P_r are respectively:

$$P_{s,opt} = \frac{\gamma^{tar}}{g_{sr}} + \sqrt{\frac{\gamma^{tar}(\gamma^{tar} + 1)}{g_{sr}g_{rd}}}, \quad P_{r,opt} = \frac{\gamma^{tar}}{g_{rd}} + \sqrt{\frac{\gamma^{tar}(\gamma^{tar} + 1)}{g_{sr}g_{rd}}} \quad (12)$$

The total transmission energy can be derived as follow:

$$P_{RF}(\theta) = \left[\left(\frac{1}{g_{sr}} + \frac{1}{g_{rd}} \right) \gamma^{tar} + 2\sqrt{\frac{\gamma^{tar}(\gamma^{tar} + 1)}{g_{sr}g_{rd}}} \right] \frac{1}{\varepsilon} \quad (13)$$

If $\gamma^{tar}(\theta) \gg 1$, then the transmission power is derived by

$$P_{RF}(\theta) = \left(\frac{1}{\sqrt{g_{sr}}} + \frac{1}{\sqrt{g_{rd}}} \right)^2 \gamma^{tar} \frac{1}{\varepsilon} \quad (14)$$

Then the energy efficiency can be formulated as

$$\eta(\theta) = \frac{\theta}{E(\theta)} = \frac{\theta B}{\left(\frac{1}{\sqrt{g_{sr}}} + \frac{1}{\sqrt{g_{rd}}} \right)^2 \gamma^{tar} \frac{1}{\varepsilon} + 4(P_{c0} + k\theta B)} \quad (15)$$

C. Analysis of the energy efficiency

To facilitate the EE-rate (modulation rate) tradeoff analysis, we first formulate the EE-rate function $\eta(\theta)$, defined as the EE corresponding to a given rate θ . Denote the optimum EE as η^* , i.e.

$$\eta^* = \max_{\theta \geq 0} \eta(\theta) \quad (16)$$

Consider the maximization of $\eta(\theta)$ by optimizing the θ . We all know $\eta(\theta)$ is a strictly quasiconcave function of $\theta > 0$.

Proof: See the Appendix.

According to the strict quasiconcavity of $\eta(\theta)$,

$$\min\{\eta(\theta_1), \eta(\theta_2)\} < \eta(\theta) \quad (17)$$

holds $\forall \theta \in (\theta_1, \theta_2)$. The strict quasiconcavity is a key feature for $\eta(\theta)$. To explain above description, we first derive the derivative of $\eta(\theta)$ with respect to θ as follows:

$$\eta'(\theta) = \frac{B}{f^2(\theta)} [f(\theta) - \theta f'(\theta)] \quad (18)$$

$$\text{where } f(\theta) = \left(\frac{1}{\sqrt{g_{sr}}} + \frac{1}{\sqrt{g_{rd}}} \right)^2 \gamma^{tar}(\theta) \frac{1}{\varepsilon} + 4(P_{c0} + k\theta B)$$

There exists a unique θ^* , $\eta(\theta)$ is strictly increasing with $\theta \in (0, \theta^*)$ and strictly decreasing with $\theta \in (\theta^*, +\infty)$.

Proof: See the Appendix.

Simulation results

In this section, we evaluate the EE of relay system and simulations result will show to explain our theoretical analysis.

A. Linear relay transmission link

We consider that these nodes are located on a straight line, as shown in figure 2. The distance between nodes S and D is $d=10\text{km}$, and between nodes S and R is d_1 . The noise power spectral density $N_0 = -170\text{dBm} / \text{Hz}$, $B = 1\text{MHz}$.

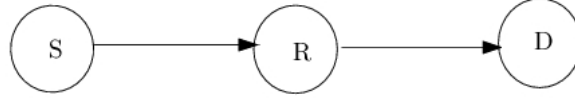


Fig.2 Linear relay transmission link

The channel power gain is evaluated according to

$$|h_{sr}|^2 = G_0 d_1^{-3}, |h_{rd}|^2 = G_0 (d - d_1)^{-3} \quad (19)$$

where $G_0 = -70\text{dB}$.

From the figure 3, it shows when θ changes from 1 to 10, the energy efficiency $\eta(\theta)$ is first increased and then decreased. At the same time, the energy efficiency is larger if the relay node close to source node and lower if the relay node far away source node.

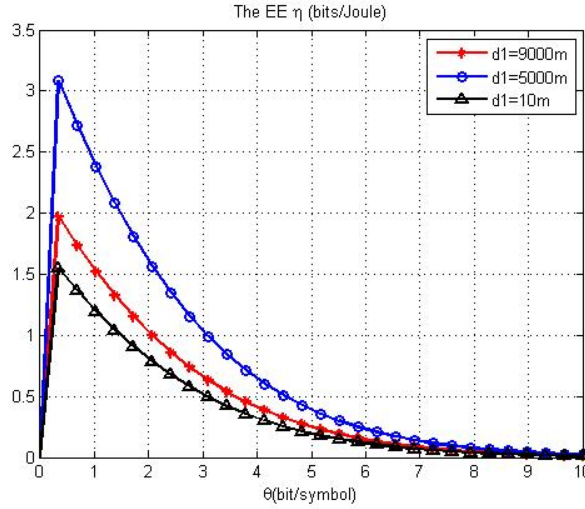


Fig.3 The trade-off between EE and modulation rate

B. Random relay transmission link

In order to carry out the energy efficiency analysis for the more general relay scenarios, we consider the situation of the source, the relay and the terminal is not in a straight line. First, we make the following assumptions:

- 1) The link of the three nodes is considered as a triangle, any two links can be regarded as the two sides of the triangle;
- 2) The source node and the destination node do not move, as one side of the triangle is fixed $d_{sd} = 10$;
- 3) To satisfied the conditions of the triangle $d_{sr} + d_{rd} > d_{sd}$, we assumed $d_{sr} + d_{rd} = 11$;

When a symbol is transmitted, we change the location of node R, it can be found that the change of R is the focus of node S and D, as shown in figure 4.

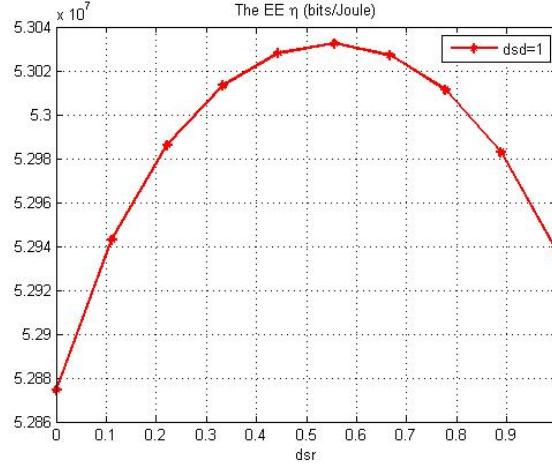


Fig.4 The trade-off between EE and relay location

Under these assumptions, we can see that the energy efficiency performance curve is symmetrical. The optimum location of relay is the midpoint of the source node and the destination node. It shows $d_{sr} = 5.5, d_{sr} = d_{rd}$.

Conclusion

We have considered the optimum EE and corresponding rate for a communication link over a flat-fading channel. The maximum EE and the modulation rate for the one-way AF relaying protocol has been derived. According to the general or special relay scene, some simulations are performed to demonstrate the facts. Finally, it is shown that when the modulation rate increases, the energy efficiency first increases and then decreases.

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Appendix

A. the derivation of the optimum power

Another expression is

$$\min\left(\frac{x}{g_{sr}} + \frac{y}{g_{rd}}\right)$$

$$\text{s.t. } xy \geq \gamma^{tar} (x + y + 1)$$

where $x = P_s g_{sr}, y = P_r g_{rd}$. The minimum total transmit power required for $\gamma = \gamma^{tar}$ can be derived from KKT conditions as:

$$L(x, y, \lambda) = \frac{x}{g_{sr}} + \frac{y}{g_{rd}} - \lambda[xy - \gamma^{tar} (x + y + 1)]$$

$$\frac{\partial L}{\partial x} = 0 \Rightarrow y = \gamma^{tar} + \frac{1}{\lambda g_{sr}}$$

$$\frac{\partial L}{\partial y} = 0 \Rightarrow x = \gamma^{tar} + \frac{1}{\lambda g_{rd}}$$

From the equation,

$$xy - \gamma^{tar}(x+y+1) = 0 \Rightarrow \lambda^* = \frac{1}{\sqrt{\gamma^{tar}(\gamma^{tar}+1)g_{sr}g_{rd}}}$$

$$x^* = \gamma^{tar} + \frac{1}{\lambda^* g_{rd}}$$

$$y^* = \gamma^{tar} + \frac{1}{\lambda^* g_{sr}}$$

$$\text{then } P_{s,opt} = \frac{x^*}{g_{sr}}, P_{r,opt} = \frac{y^*}{g_{rd}}$$

B. Proof of the quasiconcavity of $\eta(\theta)$

According to [11], $\eta(\theta)$ is strictly quasiconcave if its domain and its sublevel sets

$$S_\alpha = \{\theta \in \text{dom}\eta \mid \eta(\theta) \geq \alpha\}$$

for $\forall \alpha \in R$ are concave.

Note that $\forall \theta \geq 0$, $\eta(\theta) > 0$. We now prove the strict concavity of S_α when $\alpha > 0$. Hence,

$$S_\alpha = \{\theta \geq 0 \mid g(\alpha, \theta) = B\theta - \alpha P(\theta) \geq 0\}$$

Assume θ_1 and θ_2 are any two points on the contour of S_α . Obviously $\theta_1 > 0, \theta_2 > 0, \forall \theta \in (\theta_1, \theta_2)$, $g(\alpha, \theta) > \min\{g(\alpha, \theta_1), g(\alpha, \theta_2)\} \geq 0$

follows from the strict concavity of $g(\alpha, \theta)$ with respect to θ . Thus, S_α is a strictly concave set.

Therefore, $\eta(\theta)$ is strictly quasiconcave of θ .

C. Proof of the property of $\eta(\theta)$

To prove the first sentence, suppose there exist θ_1 and θ_2 satisfying $\theta_1 < \theta_2$ and $\eta(\theta_1) = \eta(\theta_2) = \eta^*$. $\forall \theta \in (\theta_1, \theta_2), \eta^* = \min\{\eta(\theta_1), \eta(\theta_2)\} < \eta(\theta)$, leading to a contradiction with $\eta^* \geq \eta(\theta)$. Therefore, there must exist a unique θ^* satisfying $\eta(\theta^*) = \eta^*$. Moreover, θ^* must satisfy

$$\forall \theta \geq 0, \eta'(\theta^*)(\theta - \theta^*) \leq 0$$

As said earlier, $\theta^* > 0$ must hold, thus $\eta'(\theta^*) = 0$ must hold to satisfy the condition. This proves the first claim.

We now prove the second claim.

For any θ_1 and θ_2 satisfying $0 < \theta_1 < \theta_2 < \theta^*$,

$$\eta(\theta_1) = \min\{\eta(\theta_1), \eta(\theta^*)\} < \eta(\theta_2)$$

This means that $\eta(\theta)$ is strictly increasing with $\forall \theta \in (0, \theta^*)$. Therefore, $\eta'(\theta) > 0$ must hold $\forall \theta \in (0, \theta^*)$.

For any θ_1 and θ_2 satisfying $\theta^* \leq \theta_1 < \theta_2$,

$$\eta(\theta_2) = \min\{\eta(\theta_2), \eta(\theta^*)\} < \eta(\theta_1)$$

This means that $\eta(\theta)$ is strictly decreasing with $\forall \theta \in (\theta^*, +\infty)$. Therefore, $\eta'(\theta) < 0$ must hold $\forall \theta \in (\theta^*, +\infty)$. This proves the second claim.

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