Study of Failure Criterion for Brittle Rocks Based on Ellipse Criterion

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Abstract. When rock-like materials are tested under triaxial compression[1] ,two kinds of failure may happen,that is,ductile and brittle failures.The essential reason for rock failure is the relative displacement among mineral particles making up rock.Generally, rock failure can be either tensile or shear,which depends on the predominant mechanism when failure happen.Two distinct deformation mechanisms of shearing and volume dilatation are quantitatively analyzed in rock from the fundamental thermodynamics[2]. Their competition is deduced to intrinsically dominate the strength and failure behaviors of brittle rock. Both the intrinsic shear and normal strengths give rise to the critical mechanical energies to activate destabilization of amorphous structures, under pure shearing and volume dilatation, respectively, and can be determined in terms of elastic constants.

1. Introduction

The strength of the material and damage theory is the study of materials under various stress yield or damage rule[3].The mechanical failure, especially the catastrophic brittle fracture, of materials often causes great loss to human life and property.Nevertheless, any materials will inevitably fail eventually in an objective sense. Therefore, it is critical to understand the failure mechanisms and behaviors in an attempt to predict and intentionally avoid the failure of materials and components.

2. Elliptical failure criteria

Here, the ellipse criterion and its basic formulas are briefly introduced to aid in the understanding of readers[4].Considering the uniaxial loading condition, the nominal stress on any plane of a sample can be resolved into a shear stress τ_n long the plane and a normal stress σ_n perpendicular to the plane, as illustrated in Figure 1(a). Correspondingly, two parameters can be introduced to manifest the material's inherent failure resistances.The critical stress for activating pure shear (mode II) failure is termed as the intrinsic shear strength τ_0 (Figure 1(b)). In parallel, the intrinsic normal strength σ_0 depicts the critical stress resistant to pure normal (mode I) failure (Figure 1(c)).
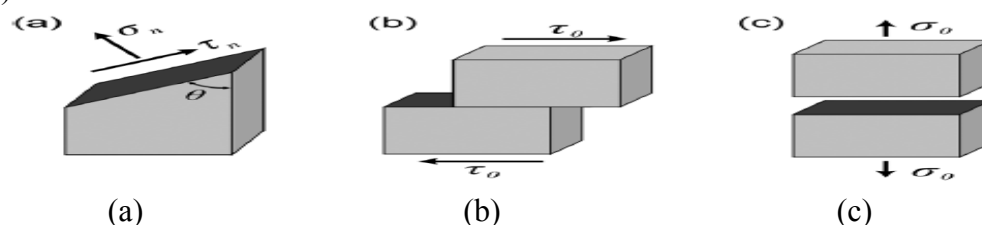

(a)   (b)   (c)

FIG. 1. Schematic illustrations of (a) the resolved shear stress τ_n and normal stress σ_n on any plane of a sample subjected to uniaxial loading as well as(b) the intrinsic shear strength τ_0 and (c) normal strength σ_0 of materials under pure shear (mode II) and normal (mode I) failure.

According to the ellipse criterion ,rock failure can be either tensile or shear .Under compression, the sample tends to fail in the shear mode and the compressive strength σ_c and failure angle θ_c can be expressed as

$$\sigma_c = 2\tau_0 \sqrt{1 + \alpha^2 / 2} \tag{1}$$

And

$$\theta_c = arc \cot \sqrt{1 + \alpha^2} \tag{2}$$

Respectively.In comparison, the failure mode under tension depends on the value of parameter a. From the mechanics viewpoint, macroscopic shear failure occurs when $\alpha \le \sqrt{2}/2$, while normal failure prevails when $\alpha > \sqrt{2}/2$.

Material under compression shear failure occurs, the critical shear strain will tend to a constant value,thus the shear strength can be expressed as a linear function of shear modulus G following:

$$\tau_0 = G\gamma_c \tag{3}$$

Where G is Shear modulus , γ_c is the critical shear strain, τ_0 is shear strength.

From the thermodynamic equilibrium perspective, the critical normal strength σ_0 can be quantified by matching the energetic driving force and resistance for volume dilatation according to $U_V(\sigma_0) = W_{V, i, e.,}$,

$$(1-2\nu)\sigma_0^2 / 6E = K \chi_c^2 / 2 \tag{4}$$

Where E is the yong's modulus ,K is bulk modulus, χ_c is the maximum volume elastic strain ,ν is the poisson's ratio.

Meanwhile, the elastic constants are intrinsically associated with each other following $E = 2G(1+\nu) = 3K(1-2\nu)$ in rock,and other isotropic materials.Therefore, σ_0 can be deduced as a function of G and ν by combining this relation with Eq. (4) as

$$\sigma_0 = 2G\chi_c(1+\nu)/(1-2\nu) \tag{5}$$

Accordingly, the failure mode factor a ($\alpha = \tau_0/\sigma_0$) can be determined just in terms of Poisson's ratio ν by combining Eqs. (3) and (5) as

$$\alpha = \gamma_c(1-2\nu)/2\chi_c(1+\nu) \approx 2(1-2\nu)/(1+\nu) \tag{6}$$

Furthermore, both compressive and tensile strengths and failure angles of rock can be predicted using the elastic constants.Under compression, the strength σ_c and failure angle θ_c can be obtained by combining Eqs. (6) and (3) with Eqs. (1) and (2) as

$$\sigma_c = 2G\gamma_c \sqrt{(1+\nu)^2 + 2(1-2\nu)^2} / (1+\nu) \tag{7}$$

And

$$\theta_c = arc \cot(\sqrt{(1+\nu)^2 + 4(1-2\nu)^2} / (1+\nu)) \tag{8}$$

3. Experimental verification about the elliptical criteria

3.1 Test material

The slate used in the test is from the tunnel of Huaitong (Huaihua- Tongdao) highway 24 blocks[5] . After rock sample collection, it need to be cut and polished into a standard sample (length to diameter ratio of 2:1), rock sample surface is smooth, the surface of the upper and lower parallel are controlled in 0.5 mm, surface flatness is controlled in 0.1 mm.A total of 37 samples were used in the test, 28 of which were successful.

3.2 Test method and equipment

The scheme about the uniaxial test is to load on the samples[6] (see FIG. 2).Test had eight groups.Uniaxial experimental schematic diagram is shown in figure 2.Sample with four side strain gauge, the horizontal and vertical strain measurement.Test in YE - 2000 type pressure testing machine, the biggest test is 2 MN, precision grade is 1 class, loading rate which control loading is 0.1 kN/s .
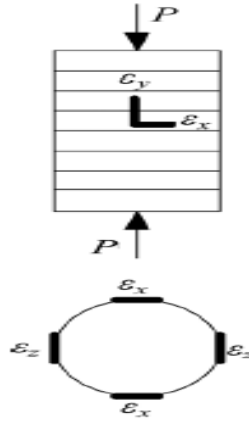

FIG.2. Uniaxial test schematic diagram

Through the experiment of rocks, the uniaxial compressive strength and elastic parameters are shown in Table1.

Table.1 The related parameters in the rock

test	shear modulus G (experiment value 10^3MPa)	poisson ratio v (experiment value)	compressive strength σ_c
1	12.840	0.269	70.5
2	12.032	0.277	70.6
3	13.625	0.286	78.7
4	15.225	0.312	76.9
5	15.872	0.321	80.2
6	16.528	0.329	83.4
7	18.193	0.352	89.8
8	18.672	0.378	92.5

According to equation (7) and data measured in the laboratory of can to obtain the critical shear strain valuesγ_c by simple calculation, the value as the known parameters, so it can be measured by brittle rock G and the elastic parameters of the uniaxial compressive strength.

Calculated according to the elliptical damage criterion of brittle rock compressive strength compared with the experimental value of the following:

Table.2 The table of compressive strength

test	shear modulus G (experiment value) 10^3MPa	poisson ratio ν （experimental value)	the critical shear strain γ_c （calculation value 10^{-3}）	compressive strength σ_c	
				Calculated value(MPa)	experiment value (MPa)
1	12.840	0.269	2.510		70.5
2	12.032	0.277	2.510	67.4	70.6
3	13.625	0.286	2.510	75.6	78.7
4	15.225	0.312	2.510	82.4	76.9
5	15.872	0.321	2.510	85.3	80.2
6	16.528	0.329	2.510	88.3	83.4
7	18.193	0.352	2.510	95.6	89.8
8	18.672	0.378	2.510	96.6	92.5

Results showed that the application of elastic parameters of rock failure criterion calculated by elliptical uniaxial compressive strength is relatively close to the experimental value, which illustrates the application of elliptic criterion on the rock damage has certain feasibility and deserves further study.

4. Summary

Brittle deformation of rock material is a relative displacement between the mineral particles of the rock, Therefor it making the original heritability between each particle's ability to reduce or lost.The mechanical failure of rock is essentially an energy critical process originating from the destabilization of amorphous structures induced by input mechanical energy. The energy barrier can be surmounted by two distinct deformation mechanisms of shearing and volume dilatation. The strength and failure behaviors are intrinsically dominated by the competition between them. The failure of rock under both tensile and compressive states can be reasonably described by incorporating thermodynamic analysis and a well-developed ellipse criterion. Both intrinsic shear and normal strengths give rise to he critical mechanical energies for structural destabilization, under pure shear (mode II) and normal (mode I) conditions respectively, and can be determined in terms of elastic constants. As a consequence, the strength and failure behaviors of rock can be predicted precisely in a non-destructive manner just according to G and .Quantitative relations have been established systematically and verified through experimental results.

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