

Nonlinear Observer Design for One-Sided Lipschitz Generalized System

Yunan Zhao^{1, a} and Junguo Lu^{1, b}

¹School of Electronics, Information and Electrical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China;

^a284521832@sjtu.edu.cn, ^bjglu@sjtu.edu.cn

Keywords: Generalized system, one-sided Lipschitz condition, observer design, LMI, LME.

Abstract. State estimation of systems satisfying some special nonlinearities have been important topics in nonlinear system theory. In this paper, we discuss the problem of observer design for one-sided Lipschitz nonlinear generalized systems based on generalized quadratic stability and Lyapunov function, by using the linear matrix inequality (LMI) and linear matrix equality (LME) approach, we propose some sufficient conditions for the generalized quadratic stability of one-sided Lipschitz generalized systems, which ensure that the observer error dynamics is generalized quadratically stable. Simulation results on one example are given to illustrate the effectiveness and advantages of the proposed design.

Introduction

During the last decades, enormous efforts are put into the study of nonlinear system [1-2], and the observer design problem for nonlinear systems satisfying a Lipschitz condition has motivated plenty of interests in nonlinear system theory. In [3], Thau presented a sufficient condition which ensures the asymptotical stability of observer error dynamics. Raghavan and Hedrick in [4] derived a constructive design by iteratively solving a series of Riccati equations. What's more, a lot of researchers studied observer design for Lipschitz systems with different methods.

It is interesting that the Lipschitz constants of such functions are often region based, with the operating region enlarged, the Lipschitz constants usually increase dramatically. However, most of the existing approaches can only deal with the situation when Lipschitz constants are small, in other words, these techniques can not find solutions with large Lipschitz constants. For the purpose of overcoming this drawback, Lipschitz continuity is generalised to a less restrictive condition known as one-sided Lipschitz continuity, which has been extensively applied to the stability analysis and numerical analysis of ordinary differential equations. Compared with the Lipschitz constant, the one-sided Lipschitz constant is much more suited for estimating the influence of nonlinear part because it possesses inherent advantages with respect to conservativeness. Inspired by the advances of the one-sided Lipschitz constant, various methods are considered and applied to design observers of nonlinear systems, more results on this problem can be found in [5-6].

In this paper, we extend the nonlinear observer design method on generalized systems. We present some sufficient conditions which ensure that the observer error dynamics is generalized quadratic stability stable. A simulative example is included to illustrate the effectiveness and advantages of the proposed methods. The remainder of the paper is organized as follows: Section 2 states the problem we deal with and introduce some basic definitions. Section 3, which contains the main results, presents LMI/LME-based observer design approaches for one-sided Lipschitz generalized systems. Then section 4 provides an illustrative example. In the end section 5 gives the conclusion of this paper.

Problem statement

In this section, we consider the following nonlinear generalised system

$$\begin{cases} E\dot{x}(t) = Ax(t) + \phi(x, u) \\ y(t) = Cx(t) \end{cases}, \quad (1)$$

where $E \in \mathbb{R}^{n \times n}$ is a singular matrix known as the state matrix, $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the measured output, $A \in \mathbb{R}^{n \times n}$, $C \in \mathbb{R}^{n \times p}$ are const matrices. The nonlinear function $\phi(x, u)$ is continuous with respect to both x and u .

Definition1^[7]. The nonlinear function $\phi(x, u)$ is said to be 'one-sided Lipschitz' in a region D if there exists $\rho \in \mathbb{R}$ such that $\forall x_1, x_2 \in D$,

$$\langle \phi(x_1, u) - \phi(x_2, u), x_1 - x_2 \rangle \leq \rho \|x_1 - x_2\|^2.$$

Definition2^[7]. The nonlinear function $\phi(x, u)$ is said to be 'quadratically inner-bounded' in a region \tilde{D} if there exists $\beta, \gamma \in \mathbb{R}$ such that $\forall x_1, x_2 \in \tilde{D}$,

$$\|\phi(x_1, u) - \phi(x_2, u)\|^2 \leq \beta \|x_1 - x_2\|^2 + \gamma \langle \phi(x_1, u) - \phi(x_2, u), x_1 - x_2 \rangle.$$

Definition3^[8]. System (1) is said to be 'generalized quadratically stable' if there exists a matrix P such that

$$E^T P = P^T E \geq 0,$$

$$[Ax(t) + \phi(x, u)]^T P x(t) + x^T(t) P^T [Ax(t) + \phi(x, u)] < 0.$$

For system (1), we consider a full-order observer of the following form

$$E \hat{\dot{x}}(t) = A \hat{x}(t) + \phi(\hat{x}, u) + L(y - C \hat{x}), \quad (2)$$

where L is a gain matrix for the observer. Denote $e(t) = x(t) - \hat{x}(t)$, $\phi = \phi(x, u)$, $\hat{\phi} = \phi(\hat{x}, u)$. Then the observer error dynamics is given by

$$E \dot{e}(t) = (A - LC)e(t) + \phi - \hat{\phi}, \quad (3)$$

assuming that ϕ is one-sided Lipschitz and quadratically inner-bounded. Let $\tilde{\phi} = \phi - \hat{\phi}$, with definition 2 and definition 3, we obtain

$$e^T(t) \tilde{\phi} \leq \rho e^T(t) e(t), \quad (4)$$

$$\tilde{\phi}^T \tilde{\phi} \leq \beta e^T(t) e(t) + \gamma e(t) \tilde{\phi}. \quad (5)$$

Our design goal is to find an observer gain matrix L for system (1), with ϕ satisfying condition (4) and (5), such that the observer error dynamics (3) is generalized quadratically stable.

Main Results

Theorem1. Assume that ϕ in system (1) satisfies the conditions (4) and (5), the state observer for system (1) holds the form of (2), then the observer error dynamics (3) is generalized quadratically stable, if there exists an invertible matrix P , a matrix R and 2 scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$, such that the following LMI and LME are feasible:

$$E^T P = P^T E \geq 0, \quad (6)$$

$$\begin{bmatrix} A^T P + P^T A - C^T R - R^T C + \varepsilon_1 \rho I + \varepsilon_2 \beta I & P^T - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2 \gamma I}{2} \\ P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2 \gamma I}{2} & -\varepsilon_2 I \end{bmatrix} < 0. \quad (7)$$

If (6) and (7) are feasible, then the gain matrix is given by $L = P^{-T} R^T$.

Proof. Suppose there exists an invertible matrix P satisfying $E^T P = P^T E \geq 0$, select Lyapunov function $V(t) = e^T(t) E^T P e(t)$, then

$$\begin{aligned} \dot{V}(t) &= e^T(t) (A - LC)^T P e(t) + \tilde{\phi}^T P e(t) + e^T(t) P^T (A - LC) e(t) + e^T(t) P^T \tilde{\phi} \\ &= \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix}^T \begin{bmatrix} A^T P + P^T A - C^T L^T P - P^T LC & P^T \\ P & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix}. \end{aligned} \quad (8)$$

For any positive scalars $\varepsilon_1, \varepsilon_2$, with condition (4) and (5), we obtain

$$\varepsilon_1 \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix}^T \begin{bmatrix} \rho I & -\frac{I}{2} \\ -\frac{I}{2} & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix} + \varepsilon_2 \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix}^T \begin{bmatrix} \beta I & \frac{\gamma I}{2} \\ \frac{\gamma I}{2} & -I \end{bmatrix} \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix} \geq 0, \quad (7)$$

Thus, adding the left terms of (7) to (6), we obtain

$$\dot{V}(t) \leq \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix}^T \begin{bmatrix} A^T P + P^T A - C^T L^T P - P^T LC + \varepsilon_1 \rho I + \varepsilon_2 \beta I & P^T - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2 \gamma I}{2} \\ P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2 \gamma I}{2} & -\varepsilon_2 I \end{bmatrix} \begin{bmatrix} e(t) \\ \tilde{\phi} \end{bmatrix}. \quad (8)$$

Let $R = L^T P$, then it follows from (8) that $\dot{V}(t) < 0$ provided that the following LMI holds a feasible solution:

$$\begin{bmatrix} A^T P + P^T A - C^T R - R^T C + \varepsilon_1 \rho I + \varepsilon_2 \beta I & P^T - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2 \gamma I}{2} \\ P - \frac{\varepsilon_1 I}{2} + \frac{\varepsilon_2 \gamma I}{2} & -\varepsilon_2 I \end{bmatrix} < 0.$$

This ends the proof.

Numerical example

Design a nonlinear observer (2) for the generalized system (1) with

$$A = \begin{bmatrix} 1.26 & -48.6 & 0 & 0 \\ -10 & 0 & -1 & 1.33 \\ 0 & 1.6 & 0 & -21.6 \\ 0 & 0 & -1.95 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \phi(x, u) = \begin{bmatrix} \sin(x_1) \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

It is easy to find that $\rho = 1, \beta = 1, \gamma = 0$. Theorem 1 can be applied to design a full-order observer, which gives $\varepsilon_1 = 11244.1, \varepsilon_2 = 15623.852$,

$$P = \begin{bmatrix} 9745.9 & -314.9 & 0 & 0 \\ -314.9 & 9732.49 & 0 & 0 \\ -2.3 & 3.8 & 197.9 & 1107.8 \\ -2.5 & 13.4 & 12984.0 & 3355.5 \end{bmatrix},$$

$$R = \begin{bmatrix} 36475.1 & 0 & 318.9 & -371.7 \\ -571579.3 & 36352.4 & -9436.2 & 14646.9 \end{bmatrix}.$$

Then we get the observer gain matrix

$$L = P^{-T} R^T = \begin{bmatrix} 3.7464 & -58.5853 \\ 0.1213 & 1.8345 \\ -0.4299 & 16.1692 \\ 0.0311 & -0.9733 \end{bmatrix}.$$

We use Matlab/simulink tool for simulation, figure 1 displays the state of the observer error dynamics for system (1), which shows all the state e_1, e_2, e_3, e_4 are generalized quadratically stable. The simulation results verify the effectiveness of the proposed design.

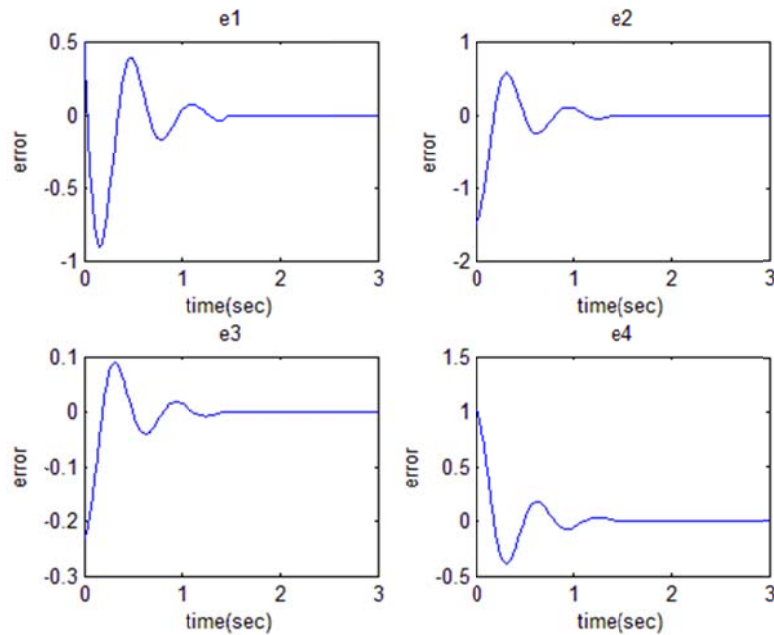


Fig. 1 the observer error dynamics for system (1)

Conclusions

This paper deal with the problem of observer design for one-sided Lipschitz nonlinear generalized systems based on generalized quadratic stability and Lyapunov function. Sufficient conditions are established in Theorem 1 to solve the proposed problem, these conditions are expressed in terms of LMI and LME, which can be easily solved through efficient numerical software. A simulative example is included to illustrate the effectiveness and advantages of the proposed methods.

Acknowledgements

This work was partially supported by the National Natural Science Foundation of China under Grants 60974002 and 61374030.

References

- [1] Barbata A, Zasadzinski M, et al. Exponential Observer for a Class of One-Sided Lipschitz Stochastic Nonlinear Systems. *IEEE Transactions on Automatic Control*, 2015, 60(1): 259 - 264.
- [2] Zhang W, Xie H, et al. Improved results on generalised robust H_∞ filtering for Lipschitz descriptor non-linear systems with uncertainties. *Control Theory and Applications, IET*. 2015, 9(14): 2107-2114.
- [3] Thau F E. Observing the state of non-linear dynamic systems[J]. *International Journal of Control*, 1973, 17(3): 471-479.
- [4] Raghavan S, Hedrick J K. Observer design for a class of nonlinear systems[J]. *International Journal of Control*, 1994, 59(2): 515-528.
- [5] Hu G D. Observers for one-sided Lipschitz non-linear systems[J]. *IMA Journal of Mathematical Control and Information*, 2006, 23(4): 395-401.
- [6] Abbaszadeh M, Marquez H J. Nonlinear observer design for one-sided Lipschitz systems[C]. *American Control Conference (ACC)*, 2010. IEEE, 2010: 5284-5289.
- [7] Zhang W, Su H S, Liang Y, et al. Non-linear observer design for one-sided Lipschitz systems: an linear matrix inequality approach[J]. *Control Theory and Applications, IET*, 2012, 6(9): 1297-1303.
- [8] Lu G, Ho D W C. Generalized quadratic stability for continuous-time singular systems with nonlinear perturbation[J]. *IEEE Transactions on Automatic Control*, 2006, 51(5): 818-823.