Study on Numerical Computation Method in Advanced Mathematics

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Abstract. Advanced mathematics is a required course in colleges and universities of China. It represents the basis for all natural sciences. Advanced mathematics is strongly abstract and has a very complex logicality. As such, some students in colleges and universities at present feel bored and even hate to study advanced mathematics so that it influences the smooth progress of course teaching and students' performance. This paper presents a numerical computation method related to advanced mathematics. Its feature is that it can solve the approximate solution of the question, which can make the question more figurative and visualized. All of the colleges and universities hope to improve students' initiative and enthusiasm in learning advanced mathematics and improve the teaching quality of the course with the help of this special advanced mathematics studying method.

Introduction

Numerical computation method is a highly scientific teaching method of advanced mathematics. It can solve the mathematical model in the process of solving advanced mathematics questions and is quite helpful for improving students' mathematical calculation ability. It requires students to fully and flexibly utilize the learnt knowledge of advanced mathematics to break through difficult questions in the process of solving advanced mathematics questions and apply numerical computation method into students' own idea in question-solving.

The Primary Task of Numerical Computation Method

In the universities, the primary task of numerical computation method is to study on the approximate solution of mathematical model. It can visualize, objectify, abstract, specify and simplify some mathematical problems that can not or is difficult to be solved accurately. Numerical computation method mainly includes three parts: functional approximation, numerical solution of differential equations and algebraic equation solution. Advanced mathematics is highly abstract and rigorously logical. For example, when a point of a function satisfies its existence, there must be rather abstract logical reasoning to prove that it indeed exists, but its exact value can not be solved. In this reasoning, the abstract and logic of advanced mathematics is often difficult for students to understand, so it is not surprising that many students can not grasp the theorem and conclusion in question covered in advanced mathematics. At this point, if we can utilized numerical computation method to find out the approximate point existed in the difficulties of the question and use the way of intuitive chart to convince students that it actually exists from image thinking and let them understand the theorem implied in the question so that it can bring a sense of accomplishment for students to learn advanced mathematics so as to promote thei enthusiasm, in this process, it can also enhance their logical thinking ability to lay a solid foundation for future study of advanced mathematics. These above mentioned set of idea is consistent with the proposed advanced mathematics learning requirements under current information technology background and also correspond to the "four principles" proposed by Harvard calculus alliance at the end of last century. Namely, all the concept teaching in advanced mathematics must be presented in intuitive ways such as text, numerical, graphic and algebraic so that students are convinced that they do exist so as to reduce the difficulty of learning advanced mathematics, so introducing numerical computation methods into modern advanced mathematics curriculum is necessary [1].

The Relation Analysis of Numerical Computation Method and Advanced Mathematics

Looking from advanced mathematics curriculum system in colleges and universities what it concerns is accurate solution. For example, in function f (x) expression, if in accordance with the relevant rules involved in advanced mathematics, r its calculation should include derivative, differentiation, integration, and calculation of maximum and minimum values, and the ordinary differential equation such as f (x) = 0 or y = f (x, y) will derive from it. But if it is in practical calculation process, you will find that the function f (x) expression may not be so complicated, because the information about f (x) that people are able to master is very limited. At this point, if we only solve problems by conventional methods in advanced mathematics, it would seem to be inadequate. Yet if at this time we adopt the numerical computation method, by integrating theoretical approaches such as numerical approximation, algebraic equations into advanced mathematics teaching of problem-solving process, it will make some mathematical problem to be solved scientifically and efficiently so that advanced mathematics and numerical computation methods form an inevitable interrelation.

The Relevant Applied Research in Numerical Computation Method in Advanced Mathematics

In advanced mathematics study, problems facing students are various. It hereby illustrates, with examples, the specific application of numerical computation method in advanced mathematics from three aspects: functional approximation, the numerical solution of differential equations and algebraic equations

The Application of Numerical Computation Method in Function Approximation Expression.

Function approximate expression involves three aspects: calculation of function value, calculation of function derivative value and differential value, calculation of definite integral value at range of function. Their common goal is to give approximation expression of the function f (x):

The Best Square Approximation based on Function f(x)

By numerical computation method, we do not need to understand the function f (x) value to fitting f (x) in a certain interval [a, b]. Considering that this theory is too complicated, you do not need to deliberately build any function space concept, instead you just introduce a new algebraic structure through the function set such as using the interpolation method to calculate numerical value of function f (x) calculations, and it is the most important core algorithms in numerical computation method.

Overview of Interpolation Method and Analysis of Practical Case

The critical part of interpolation method is the interrelated parts of Lagrange interpolation and advanced mathematics knowledge. The interrelated part covers two points: first, two points on the plane can determine a straight line and its equation can be used as linear approximation of a function f (x) ; second, it can also use Rolle Theorem to achieve derivation process for error expression in approximation function.

It thus can be seen that interpolation method is approximation expression of function square. It can solve the calculation process of function values. And it reflects the error in the calculation process of function value by considering the error of counter point function in interpolation method. The following is to prove the effectiveness of interpolation method higher by solving a advanced mathematics question.

The basic idea of interpolation method is to present the derivative value and function value existed in number of points of function in the figure, in other words, it is several groups of experimental data. It hereby needs to do a polynomial approximation function to represent the given function or data to describe the complete function relationship. There are many types of interpolation method, such as Newton interpolation, Lagrange interpolation, Hermite interpolation or spline interpolation. Here Lagrange interpolation is set as an example, assuming a set of functions f (x), and being able to give a set of data.

Suppose there is a polynomial Ln n (x) with the number not more than n, which need to satisfy the following functional expression:

Ln(x)=f(xj),in which j=0,1,…,n.

After analysis, it can be known that the interpolation polynomial of Lagrange should be expressed as:

$$\text{Ln(x)=} = \sum_{i=0}^{n} l_i(x) f(x_i)$$

$$l_i(x) = \prod_{j=0、j\neq 1} \frac{x - x_j}{x_i - x_j}$$

Here Lagrange interpolation method is used to configure a polynomial for f (x) as follows:

$$P_1(x) = l_0(x) f(a) + l_1(x) f(b) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b) = 0$$

Above expression can be f (x), so, to derive its error should be:

$$|R_2(x)| = |P_1(x) - f(x)| \leq \frac{1}{2!}，\ \max|\mathrm{f(x)}||(x-a)(b-x)| \leq \frac{1}{8} \max|f''(x)|(b-a)^2$$

As the above formula, if f (x) = sinx function has sinπ = sin0 = 0 in interval {0, π }, then you can build a interpolation function as following:

$$P_1(x) = -\frac{x-\pi}{\pi} \sin 0 + \frac{x}{\pi} \sin \pi = 0$$

So the error estimation o formula is:

$$|\sin x| \leq \frac{1}{8} \pi^2 > 1$$

The above formula is a relatively easy functional expression to be understood for students, so utilizing interpolation method can simplify some relatively complex issues in advanced mathematics calculus so as to let the students fully understand and apply in achieving problem solving process [2].

The Application of Numerical Computation Method in Numerical Solution of Differential Equations.

The relationship between numerical differentiation and advanced mathematics is quite close. Its basic idea is to achieve an approximate relationship between the derivative through approximation relationship existed between the function and its interpolation polynomial. In advanced mathematics, it is called derivative operations, or called numerical differential formula. In order to let students fully and comprehensively understand numerical differential calculation method, it is necessary to explain that numerical methods for differential equation are sensitive to independent variable value. To illustrate with derivative concept, when its node independent variable value has a rather large difference, numerical differentiation also will have a large error changes. It proves that there is a big limitation of numerical differentiation in the calculation method.

And compared to numerical differentiation, numerical integration is relatively more complex. It also found through the study that the use of numerical integration concept to establish formulas for deduction, the relevant conclusions can be deduced in accordance with numerical computation methods. This paper also uses an example to reveal the effectiveness of numerical computation method for numerical integration.

Compared with elementary functions, it needs to use derivation law to solve derivative, with the aid of derivation law, most function calculations can have result. However, when it comes to calculating function definite integral or indefinite integral, we can not adopt the simple elementary function, because it can not be used to calculate the integral value question. Therefore, in this case we should choose numerical integration formula to calculate an approximate value of definite value integral so as to help students analyze and solve problems more easily. This paper recommends a numerical integration formula called trapezoidal formula to solve the question, the equation is as following:

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + 2\sum_{k=1}^{n-1} f(x_k) + f(b) \right]$$

In the above formula, its step length should be h = (ba) / n, if we use solving method of specific definite integral, it can be more conducive to the students' knowledge and understanding of definite integral, so this expansion can be accepted by more students. From the perspective of form essence, teachers may consider to use this formula to replace the limit in calculus in the process of teaching numerical integration and completely remove limitation symbol and take accordant value for all the small trapezoid to get approximation value of integral definition and improve students' comprehension for definite value integral and after-taken limit. The following takes a question as an example:

Question: definite value integration $I = \int_0^1 e^{-x}dx$ solve.

The calculation formula can be based on integral from above question :

$$I = \int_0^1 e^{-x}dx = -e^{-x}\Big|_0^1 = e^0 - e^{-1} = 0.6321$$

If here we use trapezoid formula to calculate, so the calculation results are different under different step length. Due to the increasingly high level of encryption of cellular, numerical integration has also been close to the real value of the integral. So it can be concluded based on Table 2 and the above question example. Students can more easily understand the true meaning of definite value integral through numerical computation methods, and have an initial understanding of the relevant questions of after-taken limit [3].

The Application of Numerical Computation Method in Algebraic Equation.

While solving the root of algebraic equations, most of algebraic equations are unable to achieve the exact solution of questions, although teachers can give proof of the existence of algebraic equation solution, yet they are unable to give exact solutions, which can not be explain the question by facts. Therefore in the solving process of root of some algebraic equations, so teachers can not make students fully understand the process of solving and the correctness of the result. This paper simplifies the solution of algebraic equation root through numerical computation method so as to achieve better teaching results.

Example question: to prove the only root existed in equation of x+sinx+1=0 at the interval of （-π/2, π/2）

With regard to this question, although the conventional continuous function media theorem can prove the existing root in the specified range of the equation, but it can not prove what exactly the root is. In addition, the equation of the question is the typical transcendental equation, so it can not directly solve the root of the question. This paper utilizes the iterative method in numerical computation method to solve its approximate solution of the equation. First, it needs to construct iteration scheme the equation x + sinx + 1 = 0, the following formula is obtained:

$$x_{k+1} = \varphi(x_k) = x_k - (x_k + \sin x_k + 1)/(1 + \cos x_k)$$

If here when X_0=0, then when $|X_{k+1}-X_k| < 10^{-8}$, iteration will be finished, so it can obtain the data value for each iteration of the above formula. After four times of iterative calculation for algebraic equation, the function value 5.80-11 has infinitely close to 0, at this point x = -0.510974 should be the solution to the equation. Based on this data, we can find approximate solutions, in order to make the proof algebraic equations more specific and visualized and explain the existence of the solution, so that students can more easily understand the theorem of iterative method for solving algebraic equation. Numerical computation method again succeeds in visualizing the abstract and letting students to learn and understand the algebraic equations knowledge of advanced mathematics intuitively.

It can be learned through deduction process of the question, in advanced mathematics, numerical computation methods not only care whether the algorithm converges, but also care about the convergence speed and acceleration method, so the convergence speed is also important factors in determining the number of iterations. The slower convergence is, the more iteration times will be, which at the same time leads to increase of calculation volume, on the contrary, the calculation efficiency will be improved. Students can make judgment about the effectiveness of the method for solving advanced mathematics questions by experiencing the specific link and the difference between numerical computation method and question of advanced mathematics [4].

Summary

Mathematics is the fundamental tool for solving natural science and it is the intersection of many disciplines. Especially for advanced mathematics, the inherent abstractness of it may not be easily understood by learners. Numerical computation method is to visualize the abstract questions in advanced mathematics so that students can more easily understand and master the relevant knowledge of mathematics and advanced mathematics can be more visualized. Meanwhile, numerical computation method is a good way to make advanced mathematics no longer boring. Through introducing some simple elementary numerical computation methods to solve advanced mathematics questions, it realized the simplification of the teaching for student and also finds the most reasonable and excellent learning methods for students for their future study of advanced mathematics.

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