Theoretical Calculation of Stiffness of Composite Laminated Beam with T-Section

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Abstract—Based on the constitutive equation and deformation equation, the total stiffness matrix of laminated composite beam with T-section are built up by using the relation of 3D stress-strain transformation. Then calculation methods of the axial and bending stiffness of laminated composite beam with T-section are derived. Axial and bending stiffness of laminated composite beam with T-section are calculated under the uniformly distributed load or concentrated force with various boundary condition. A comparison between analytical calculation results and ANSYS calculation results shows good agreement. The results show that this method is reliable. And easy to apply in the initial design of composite hull. The research makes foundation for theoretically calculate laminated composite beam with arbitrary cross-section.

Keywords— laminated composite beam; T-section; stiffness; 3D stress-strain transformation

I. INTRODUCTION

References [1, 2] are related to analysis of composite laminate beam. But the analysis methods are too complex to be applied in engineering design of composite laminate structure. For the former, the calculation can be degraded from the theories of laminated plates. The stiffener of various shapes (L-type, T-type and hat shaped, etc) is used to stiffen the stiffness and strength of composite laminate beams. It is difficult to determine the stiffness characteristics and structural response through resolution. Most scholars adopt the theory of composite laminate beams or approximate equations [3-5] to calculate the mechanical response. Now the finite element software is widely applied in design and calculation [6-8] of composite laminate beams. But for modeling and calculation of large composite models, the calculation periods of finite element is too long. Also, it needs a lot of design parameters, making it difficult to perform flexible parameterized study. The theoretic calculation method is more convenient and flexible for preliminary design of composite structures.

II. THE TOTAL STIFFNESS MATRICES OF COMPOSITE LAMINATE BEAM WITH T-SECTION

In Fig. 1-Fig. 2, we establish a local coordinate system (x_1, y_1, z_1) and (x_2, y_2, z_2) on the middle surface of the web laminate and base laminate. Assuming that (1-2-3) is the coordinate direction of fiber material that is laid inside (x'_2, y'_2) . The angle-ply β is defined as follows: the positive

direction of x'_2 will coincide with the fiber direction after counterclockwise rotating β around axis z'_2 .

Fig. 1. Sketch of beam with T-section.

Fig. 2. Sketch of web laminate section.

The following equations can be obtained:

$$\begin{aligned} [\sigma]_{1-2-3}^k &= [T_\sigma(\theta)]_{x_2} [T_\sigma(\beta)]_{z'_2} [\sigma]_{x_2-y_2-z_2}^k \\ [\varepsilon]_{1-2-3}^k &= [T_\varepsilon(\theta)]_{x_2} [T_\varepsilon(\beta)]_{z'_2} [\varepsilon]_{1-2-3}^k \end{aligned} \quad (1)$$

where,

$$[\sigma]_{1-2-3}^k = [Q]_{1-2-3}^k [\varepsilon]_{1-2-3}^k \quad (2)$$

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$[Q]^k$ is called as reduced stiffness matrices of k^{th} layer and the subscripts 1 and 2 refer to the properties along the fiber and transverse to the fiber respectively. Substituting (2) into (1) results in:

where,

$$\left[\hat{Q}_{x_2-y_2} \right]^k = \left[T_\sigma(\beta) \right]_{z'_2}^{-1} \left[T_\sigma(\theta) \right]_{x_2}^{-1} [Q]^k \left[T_\varepsilon(\theta) \right]_{x_2} \left[T_\varepsilon(\beta) \right]_{z'_2} = \begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} & \hat{Q}_{16} \\ \hat{Q}_{12} & \hat{Q}_{22} & \hat{Q}_{26} \\ \hat{Q}_{16} & \hat{Q}_{26} & \hat{Q}_{66} \end{bmatrix}^k \quad (4)$$

$$\begin{aligned} \hat{Q}_{11} &= Q_{11} \cos^4 \beta + 2(Q_{12} \cos^2 \theta + 2 \cos^2 \theta Q_{66}) \sin^2 \beta \cos^2 \beta + Q_{22} \sin^4 \beta \\ \hat{Q}_{12} &= \sin^2 \beta \cos^2 \beta (Q_{11} + \cos^4 \theta Q_{22} - 4 \cos^2 \theta Q_{66}) + (\cos^4 \beta + \sin^4 \beta) \cos^2 \theta Q_{12} \\ \hat{Q}_{22} &= Q_{11} \sin^4 \beta + 2 \sin^2 \beta \cos^2 \beta (\cos^2 \theta Q_{12} + 2 \cos^2 \theta Q_{66}) + Q_{22} \cos^4 \beta \cos^2 \theta \\ \hat{Q}_{16} &= \sin \beta \cos^3 \beta (Q_{11} - \cos^2 \theta Q_{12} - 2 \cos^2 \theta Q_{66}) + \sin^3 \beta \cos \beta (Q_{12} \cos^2 \theta - Q_{22} \cos^4 \theta + 2 \cos^2 \theta Q_{66}) \\ \hat{Q}_{26} &= \sin^3 \beta \cos \beta (Q_{11} - \cos^2 \theta Q_{12} - 2 \cos^2 \theta Q_{66}) + \sin \beta \cos^3 \beta (\cos^2 \theta Q_{12} - \cos^4 \theta Q_{22} + 2 \cos^2 \theta Q_{66}) \\ \hat{Q}_{66} &= \sin^2 \beta \cos^2 \beta (Q_{11} + \cos^4 \theta Q_{22} - 2 \cos^2 \theta Q_{12} - 2 \cos^2 \theta Q_{66}) + (\cos^4 \beta + \sin^4 \beta) Q_{66} \end{aligned}$$

The reference axis for the T-section is initially selected as the mid-height of the web laminate. The stiffness matrices of web laminate respect to own mid-plane can be obtained.

Mx ; other components can be ignored. (6) can be converted into:

$$\begin{aligned} [\bar{A}]_w &= \int_{-\frac{h}{2}}^{\frac{h}{2}} [A'] dz_2 = h[A'] \\ [\bar{B}]_w &= \int_{-\frac{h}{2}}^{\frac{h}{2}} ([B'] - z_2 [A']) dz_2 = [B']h \\ [\bar{D}]_w &= \int_{-\frac{h}{2}}^{\frac{h}{2}} ([D'] - 2z_2 [B'] + z_2^2 [A']) dz_2 = [D']h + \frac{h^3}{12} [A'] \end{aligned} \quad (5)$$

The total stiffness matrices of the laminate beam can be obtained by multiplying the width of each laminate.

III. AXIAL AND BENDING STIFFNESS OF LAMINATED COMPOSITE BEAM WITH T-SECTION

The following equations can be obtained:

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}_{6 \times 6}^{-1} \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (6)$$

ε_x^0 , ε_y^0 and γ_{xy}^0 are the mid-plane strains. κ_x , κ_y and κ_{xy} are the mid-plane curvatures. N_x , N_y and N_{xy} are the resultant forces per unit width of the laminate. M_x , M_{xy} , and M_x are the resultant moments per unit width of the laminate.

Currently, most laminated composite beams with T-section are narrow beams. As for them, axial distortion caused by axial strain is due to poisson ratio and can be ignored. Therefore, only two components need to be taken into account i.e. N_x and

$$\begin{Bmatrix} \bar{\varepsilon}_x^0 \\ \bar{k}_x \end{Bmatrix} = \begin{bmatrix} \bar{a}_{11} & \bar{b}_{11} \\ \bar{b}_{11} & \bar{d}_{11} \end{bmatrix} \begin{Bmatrix} \bar{N}_x \\ \bar{M}_x \end{Bmatrix} \quad (7)$$

Where $\bar{a}_{11}$, $\bar{b}_{11}$, $\bar{d}_{11}$ is the total flexibility matrix coefficient of the laminated composite beam with T-section. According to the definition of neutral axis, the load component $\bar{N}_x$ will not cause the change of curvature $\bar{k}_x$, and the load moment $\bar{M}_x$ will not cause the change of $\bar{\varepsilon}_x^0$. When the moment $\bar{M}_x$ acting on the axial direction is only considered, (7) can be expressed as following:

$$\begin{cases} \bar{\varepsilon}_x^0 = \bar{b}_{11} \bar{M}_x \\ \bar{k}_x = \bar{d}_{11} \bar{M}_x \end{cases} \quad (8)$$

When the structure is subject to the axial deformation, $\bar{k}_x = 0$. According to the definition of the axial stiffness, the axial stiffness of the laminated composite beam with T-section is calculated:

$$(EA)_{beam} = \frac{\bar{d}_{11}}{\bar{a}_{11} \bar{d}_{11} - \bar{b}_{11}^2} \quad (9)$$

Similarly, when the structure is subject to pure bending, $\bar{\varepsilon}_x^0 = 0$. According to the definition of the bending stiffness, the bending stiffness of the laminated composite beam with T-section is calculated as following:

$$(EI)_{beam} = \frac{\bar{a}_{11}}{\bar{a}_{11}\bar{d}_{11} - \bar{b}_{11}^2} \quad (31)$$

IV. NUMERICAL CALCULATION

The length of beam is 0.3m. There are 10 layers in base laminate, the length is 50.8mm. There are 8 layers in web laminate, the height is 25.4mm. The thickness of each layer is 0.125mm. The single-layer laminate is laid with epoxy carbon fiber composite materials. The parameters are as follows: $E_1=121\text{GPa}$, $E_2=8.6\text{GPa}$, $G_{12}=4.7\text{GPa}$, $\mu_{12}=0.27$. The layup of laminated composite T-section are as following: ①base laminate: 45°/ -45°/ -45°/ 45°/ 0°/ 0°/ 45°/ -45°/ -45°/ 45°; web

laminate: 45°/ -45°/ -45°/ 45°/ 45°/ -45°/ -45°/ 45°. ②base laminate: 90°/ 0°/ 90°/ 0°/ 90°/ 90°/ 0°/ 90°/ 0°/ 90°; web laminate: 45°/ -45°/ -45°/ 45°/ 45°/ -45°/ -45°/ 45°. ③base laminate: 90°/ 0°/ 90°/ 0°/ 90°/ 90°/ 0°/ 90°/ 0°/ 90°; web laminate: 90°/ 0°/ 90°/ 0°/ 90°/ 90°/ 0°/ 90°/ 0°/ 90°. The results calculation are as seen in Fig. 3-Fig. 5 and Table I. The theoretical calculation results matches the finite element calculation, which means that the model applied, formula derived and programs written in this paper are correct and reliable. Furthermore, the layup of fiber is effect on the stiffness of laminated composite T-section. The 90°/ 0° layup is better than the 45°/ -45° layup. The axial stiffness of laminated composite T-section is better than the bending stiffness of laminated composite T-section.

TABLE I. DEFORMATION OF COMPOSITE BEAM WITH T-SECTION

layup	The max bending deformation						Axial deformation		
	Concentrated force $F=300\text{N}$ (Simply supported)			Evenly-distributed load $q=1000\text{N/m}$ (clamped end)			Axial tensile force $N_c=1000\text{N}$		
	Present Method (mm)	FEM Result (mm)	Difference	Present Method (mm)	FEM Result (mm)	Difference	Present Method (mm)	FEM Result (mm)	Difference
①	2.063	2.044	1%	0.258	0.254	1.5%	0.103	0.102	1%
②	1.447	1.431	1.2%	0.185	0.181	1.6%	0.072	0.071	1.4%
③	1.230	1.226	1.7%	0.164	0.161	2.1%	0.052	0.051	1.9%

Fig. 3. Axial displacement

Fig. 5. Deformation due to uniform load.

Fig. 4. Deformation due to concentrated force.

V. CONCLUSION

Based on the constitutive equation, deformation equation and parallel axis theorem of stiffness matrices, the total stiffness matrices of composite laminate beam with T-section was built up by using the relation of 3D stress-strain transformation. Then the analytical calculation equations of the axial and bending stiffness of composite laminate beam with T-section were derived. The axial and bending stiffness of composite laminate beam with T-section were calculated under the uniformly distributed load or concentrated force with various boundary conditions. A comparison between analytical calculation results and ANSYS calculation results showed good agreement. Thus, the methods and the calculation equations derived in this paper are reliable. This paper lays a basis for initial design and optimization of the composite laminate beam with T-section and provides theoretical support for theoretical calculation of composite laminate beams with

arbitrary shape section. The work in the paper has high engineering value in initial design of composite structures.

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