Mechanism Design for the Joint Control of Pollution

F. Xue Zhang *, J. Wang

School of Economics and Management, North China University of Technology, 100144 Beijing, China

Abstract. From the perspective of the central government, this paper analyzed that the externality of pollution control would lead to market inefficiency, and studied the implementation in dominant strategy equilibrium and sub-game perfect Nash equilibrium. Study found: Market inefficiency came from private information and inadequate supervision of the mechanism designer, and the sufficient and necessary conditions of implementation in Nash equilibrium have been gotten, also a dynamic mechanism to realize the joint control of regional air pollution was built. At the same time, a mechanism with asymmetric information was posed to implement the goal function in dominant equilibrium.

1 Introduction

At present, the air pollution control is a huge pressure in China. Because of the negative externalities of air pollution, a province alone cannot solve the basic problem. The State Council issued the document no. 37 《air pollution prevention action plan》 (2013) which was put forward to designing a joint control of regional air pollution mechanism for Beijing-Tianjin-Hebei or Yangtze River Delta, and coordinating the central government and the relevant departments of state council to solve the regional prominent environmental problems. However, the mechanism design for the joint control of pollution is a key problem.

Some scholars have used the game theory to study the pollution control problem, but the number of research on the joint control is less. J. Xue ec etl. [1] established a beijing-tianjin-hebei inter-provincial atmospheric pollution control model and discussed the Shapley value of cooperative allocation from the regional pollution control perspective. In the model, if parameters such as the maximum and minimum processing ability of the provinces are real statistical data, then results are more accurate. The game models of pollution control among different players have been constructed. W. Dungumaro ec etl. [2] discussed the positive role of public participations in environmental protection. The interactive strategy in pollution control between government and enterprises has been studied by G. Xue Zhang and M. Chu Zhong [3]. The results showed that the environmental quality could be improved by increasing the reputation cost and political cost. L. Yang and X. Hong Gao [4] studied the game process of environmental degradation between the environmental supervision departments and manufacturers, and considered that the emission standard P and excessive emission fines determined the results of model. The dynamic game model of cumulative emissions with coupling constraints which studied about international climate change agreement was established by O. Bahn and A. Haurie[5]. But some of the above models assumed that the ability of the government department is perfect, this assumption does not accord with the actual situation. Therefore, under the assumption of bounded rationality, the evolutionary game theory has been used by some scholars. They analyzed the evolutionary process of pollution control strategy, and found out replicated dynamic equations and evolutionary stable strategies [6-9].

From the perspective of mechanism design, the implementation of the mechanism has been studied by some scholars in different equilibrium concepts. M. Glachant[10] constructed the incentive model to solve the information asymmetry between government and enterprises. D. Abreu and A. Sen[11] constructed an implementable social choice function in Nash equilibrium with only two players, and the necessary and sufficient conditions for the implementation were presented, also the results have been extended to strict Nash equilibrium and coalition-proof Nash equilibrium. Based on the implementation of the Bayes equilibrium, P. Kang Wu and F. Xue Zhang et al.[12] discussed the condition of implementation for different mechanisms when agents' parameter space was affected by the designer. J. Hovi and I. Areklett[13] considered the condition which affected effective implementation in five different notions of equilibrium – the Nash equilibrium, the subgame perfect equilibrium, the renegotiation proof equilibrium, the coalition proof equilibrium, and the perfect Bayesian equilibrium.

Empirical methods have been used by some scholars, but they mainly focus on the relationship between environmental quality and economic growth. Economic growth mainly influences environmental quality by scale effect, structure effect and technical effect. For scale effect, the interaction between economic growth and environmental quality has been studied by A.K. Tiwari[14]; For structure effect, T. Yuan Lu [15] studied the influence of industrial structure on the quality of environment; For technical effect, the influence of technological progress direction on the environment has been analyzed by Q. Zhi Dong ec etl. [16]. These existing researches have focused on the unilateral influence of industrial structure, technological progress and other factors on environmental pollution, but ignored the impact of environmental pollution on these factors. From the research method, the spatial spillover effect of environmental pollution was studied by using the spatial econometric model (I. Daniefar[17]). H. KaiYuan and H. Jian Li [18] proposed the hypothesis which is the dilemma of strategic emission reduction under the collusion between government and enterprises, and tested the general panel and space econometric model with the fixed effects.

At present, the air pollution control in Beijing-Tianjin -Hebei and the Yangtze River Delta is still in exploration and trial stage, and it is still difficult to realize the regional pollution control. M. Finus[19] believed that cooperation between the two governments was difficult because there was no national institution in the global level could implement cooperation, and the threat of punishing illegal behavior could be necessary to maintain stability, but the model did not take the agent's incomplete information and risk preferences into account. Compared to M. Finus[19], this paper will base on the view of central government, and design the implementable pollution control mechanism of local government in complete and incomplete information , so that achieve the unity of regional pollution control and economic coordination development.

The theoretical model of this paper is based on the E. Maskin's [20] classic article. The optimal reduction of emissions when participants reached and did not reach a reduction agreement has been discussed which proved some of government intervention was called for. In addition, the equilibrium results of mechanism design under complete information and incomplete information were analyzed. In complete information, the preference reversal condition has been led into, and he discussed the implementable mechanism in SPNE. It was proved that each agent told the truth (i.e. reporting the true type) was the only subgame perfect Nash equilibrium. In incomplete information, Vickrey-Clarke-Groves mechanism has been used. He constructed a transfer function containing other agents' utility function and proved that telling the truth was a dominant strategy. In addition, although the pivotal mechanism was feasible in dominant strategy equilibrium, it was not balanced, then the implementable mechanism under the concept of Bayesian equilibrium has been designed; the incentive compatibility conditions and Bayesian monotonicity which were not only necessary but sufficient for a social choice function to be implementable in Bayesian equilibrium has been proved.

Compared with E. Maskin[20]. Firstly, we extended the utility function to the general form, then introduced the variables of economic growth and the cost effect of the economic growth. Secondly, the condition for the contradiction of social optimal and local optimum, and measures for the central government to realize the social efficiency were given. We concluded the Pareto optimal would not be reached without the central government which was consistent with that of Maskin's. Thirdly, the information between local governments which was complete but incomplete in central government has been considered, and the implementable dynamic pollution control mechanism has been designed, so as the implementable conditions of mechanism. In the end, we gave the specific steps on how to achieve the mechanism. Fourthly, when the local governments had private information, we analyzed that the central government could improve the social benefits by pollution control mechanism, and the specific scheme of pollution control mechanism in incomplete information has been given.

The structure of this paper is as follows: the first part is the introduction. The second part discusses the pollution control mechanism of local governments in complete information. The third part discusses the pollution control mechanism design of the central government in incomplete information. The fourth part summarizes the full text, and gives the policy implications of the model.

2 Pollution Control Mechanism Design in Complete Information

2.1 Technology and preference

There are N local governments involved in pollution control, the type of agent i is $\theta_i \in H_i$, and the utility function $U_i(\theta_i, g_i, r_i) = \theta_i V(g_i, r) - r_i - \varphi(g_i) + t_i$.

g_i is the local economic growth rate, r_i is the aggregate local pollution control, and unit pollution control cost is 1, $\varphi(\cdot)$ is the cost function of the local economic growth, t_i is a monetary transfer to community i, $V(\cdot)$, $\varphi(\cdot)$ are two order continuous differentiable function. And,

$$\frac{\partial V}{\partial g_i} > 0, \frac{\partial^2 V}{\partial g_i^2} < 0 \quad ; \quad \frac{\partial V}{\partial r} > 0, \frac{\partial^2 V}{\partial r^2} < 0 \quad ;$$

$\frac{\partial \varphi}{\partial g_i} > 0, \frac{\partial^2 \varphi}{\partial g_i^2} > 0$; $r = \sum_{i=1}^{N} r_i$, $\theta_i \in [a, b]$. If there is no central government, then $\sum_{i=1}^{N} t_i \le 0$, and internal resource endowment equilibrium condition is $\sum_{i=1}^{N} t_i = 0$.

2.2 Social optimal conditions and individual optimality conditions with no supervision of central government

The problem of agent i is:

$$\max_{g_i, r_i} \quad U_i(\theta_i, g_i, r_i) = \theta_i V(g_i, r) - r_i - \varphi(g_i) + t_i \tag{1}$$

The first order necessary condition is:

$$\begin{cases} \frac{\partial U_i}{\partial g_i} = 0 \\ \frac{\partial U_i}{\partial r_i} = 0 \end{cases} \tag{2}$$

That is

$$\begin{cases} \theta_i \frac{\partial V}{\partial g_i} = \varphi'(g_i) \\ \frac{\partial V}{\partial r_i} = \frac{1}{\theta_i} \end{cases} \tag{3}$$

The optimal solution (r_i^*, g_i^*) satisfies the formula (2), (3).The problem of social optimization is: $\max_{g_i, r} \sum_{i=1}^{N} U_i(\theta_i, g_i, r_i)$

$\because \sum_{i=1}^{N} t_i = 0$, There is a no central government regulatory pollution problem.

$\therefore$ The first order necessary condition is:

$$\begin{cases} \sum_{i=1}^{N} \theta_i \frac{\partial V}{\partial g_i} = \sum_{i=1}^{N} \varphi'(g_i) & (4) \\ \frac{\partial V}{\partial r} = \frac{1}{\sum_{i=1}^{N} \theta_i} & (5) \end{cases}$$

The optimal solution (r_i^*, g_i^*) satisfies the formula (4), (5), and $r^* = (r_1^*, r_2^* \cdots r_N^*)$. If there is no central government, agents consult to share the cost of pollution control. For example, the $\theta_i \cdot r^*$ will not achieve the social optimum.

Theorem 2.1 When $\sum_{i=1}^{N} \theta_i \geq 1$, there is a free rider behavior, the agent i choose $r_i = 0$.

Proof: see appendix.

This theorem gives the conditions for the negative externalities of pollution, and when agents' private information meets the condition $\sum_{i=1}^{N} \theta_i \geq 1$, if there is no supervision of central government, local government will not take the initiative to control pollution.

Theorem 2.2 The conditions for central government to achieve social optimum is $\sum_{i=1}^{N} \frac{\partial r_i}{\partial V} = \frac{\partial r}{\partial V}$.

Proof: see appendix.

The economic implication of this theorem is that if the central government want to achieve the social optimum through pollution control, it is necessary to restrict the behavior of local government. Of course, the central government can achieve the unification of individual and society optimal by tax, purchase of pollution emissions and subsidies, etc..

2.3 Mechanism design of pollution control in complete information

This is the case in which all agents observe the preference θ_i , but it is unverifiable to the central government as mechanism-imposing authority. So the complete information mainly refers to the non existence of private information among agents, and discussions of the implementation problem for the central governments in Nash equilibrium concept (NE) and the subgame perfect Nash equilibrium (SPNE).

2.3.1 Implementation of the mechanism in Nash equilibrium

This section gives the necessary and sufficient conditions for the implementation of the mechanism based on the concept of Nash equilibrium. There are N agents, strategic group $((g_1^*, r_1^*), \ldots (g_i^*, r_i^*), \ldots (g_n^*, r_n^*))$ is a Nash equilibrium, (g_i^*, r_i^*) is the optimal strategy for agent i when other participants' strategies were given. That is $U_i(\theta_i, (g_i^*, r_i^*, g_{-i}^*, r_{-i}^*)) \geq U_i(\theta_i, (g_i', r_i', g_{-i}^*, r_{-i}^*))$.

Before we give the necessary and sufficient conditions, we first introduce two concepts: monotonic and non veto power. For any $\theta, \hat{\theta} \in \Theta$,if $(g_i, r_i) = f(\theta_i, \theta_j)$, $(g_i, r_i) \neq f(\hat{\theta}_i, \hat{\theta}_j)$,then exist agent i and (g_i', r_i') ,that make $U_i(\theta_i, g_i, r_i) \geq U_i(\theta_i, g_i', r_i')$,but $U_i(\hat{\theta}_i, g_i', r_i') > U_i(\hat{\theta}_i, g_i, r_i)$,then $f(\theta_i, \theta_j)$ is monotonic. No veto power is defined under type θ , N-1or more agents consider the optimal result is (g_i, r_i) , then no matter how agent i chooses, for all agents j 6= i , $(g_i', r_i') \in (g_i, r_i) = f(\theta_i, \theta_j)$, then we have $U_i(\theta_i, g_i, r_i) \geq U_i(\theta_i, g_i', r_i')$.

θ is a real type, then each agent reporting $\hat{\theta} = \theta$ will be a balanced result of Pareto Efficiency. As long as the participants report $\hat{\theta} \neq \theta$, the equilibrium results are not unique. In order to ensure the mechanism is Nash implementable and the agents truly show their types, the social choice function $f(\theta_i, \theta_j)$ is needed to meet two conditions which are monotonous and non veto. In this paper, the number of local governments is more than three, and they prefer a bigger monetary transfer, so no veto power is automatically satisfied.

Theorem 2.3 If a social choice function $f(\theta_i, \theta_j)$ is implementable in Nash equilibrium, then it is monotonic. If a social choice function that satisfies monotonicity and no veto power is Nash implementable.

Proof: see appendix.

2.3.2 Implementation of the mechanism in Subgame perfect Nash equilibrium

In this section, we relax monotonicity by invoking the Subgame perfect Nash equilibrium, and discuss the implementation problem in complete information and dynamic game.

Preference reversal assumption: For any of the two agents i , j ,and any θ_i , θ_j , there exists two choices:

$$[r_i(\theta_i, \theta_j), r_j(\theta_i, \theta_j), g_i(\theta_i, \theta_j), t_i(\theta_i, \theta_j)],$$
$$[r_i'(\theta_i, \theta_j), r_j'(\theta_i, \theta_j), g_i'(\theta_i, \theta_j), t_i'(\theta_i, \theta_j)]$$

There exists:

$$\theta_i V(r_i, r_j, g_i, t_i) - r_i(\theta_i, \theta_j) - \phi(g_i(\theta_i, \theta_j)) + t_i(\theta_i, \theta_j) > \theta_i V(r_i', r_j', g_i', t_i') - r_i'(\theta_i, \theta_j) - \phi(g_i'(\theta_i, \theta_j)) + t_i'(\theta_i, \theta_j) \tag{6}$$

$$\theta_i' V(r_i', r_j', g_i', t_i') - r_i'(\theta_i, \theta_j) - \phi(g_i'(\theta_i, \theta_j)) + t_i'(\theta_i, \theta_j) > \theta_i' V(r_i, r_j, g_i, t_i) - r_i(\theta_i, \theta_j) - \phi(g_i(\theta_i, \theta_j)) + t_i(\theta_i, \theta_j) \tag{7}$$

The economic implications of above assumptions are as follows: two different local governments have different preferences for two different pollution control models. For example, Beijing is more inclined to change the fuel structure, and focus on the upgrade of oil. While Shandong, Liaoning and other provinces focus more on the industrial structure adjustment, Hebei more focused on capacity transformation in steel or other industrials.

Thus, the agent θ_i more prefers the pollution control model of (r_i, r_j, g_i, t_i) but agent θ_j more prefers (r_i', r_j', g_i', t_i'). Social choice function $f(\theta_i, \theta_j)$ represents the result of the implementable mechanism.

The complete information and dynamic pollution control mechanism is as follows:

Stage 1: The central government gives the pollution control targets.

(1)Agent i announces a type $\hat{\theta}_i$

(2)Agent j chooses to agree $\hat{\theta}_i$, in which case we go to Stage 2, or disagree $\hat{\theta}_i$ by announcing some $\hat{\theta}_i' \neq \hat{\theta}_i$, in which case we go to Stage（3）

(3) Central government requires agent i to choose between $\left[r_i(\hat{\theta}_i, \hat{\theta}_i'), r_j(\hat{\theta}_i, \hat{\theta}_i'), g_i(\hat{\theta}_i, \hat{\theta}_i'), t_i(\hat{\theta}_i, \hat{\theta}_i')\right]$ and $\left[r_i'(\hat{\theta}_i, \hat{\theta}_i'), r_j'(\hat{\theta}_i, \hat{\theta}_i'), g_i'(\hat{\theta}_i, \hat{\theta}_i'), t_i'(\hat{\theta}_i, \hat{\theta}_i')\right]$.If agent i chooses the former, then agent j is fined t^*; if he chooses the latter, agent i is fined t^*. The mechanism stops here.

Stage 2: This is the same as Stage 1. Agent j announces a type $\hat{\theta}_j$. Agent i choose to agree $\hat{\theta}_j$, in which case we go to Stage 3, or disagree $\hat{\theta}_j$ by announcing some $\hat{\theta}_j' \neq \hat{\theta}_j$, then agent j choose between $\left[r_i(\hat{\theta}_j, \hat{\theta}_j'), r_j(\hat{\theta}_j, \hat{\theta}_j'), g_j(\hat{\theta}_j, \hat{\theta}_j'), t_j(\hat{\theta}_j, \hat{\theta}_j')\right]$ and $\left[r_i'(\hat{\theta}_j, \hat{\theta}_j'), r_j'(\hat{\theta}_j, \hat{\theta}_j'), g_j'(\hat{\theta}_j, \hat{\theta}_j'), t_j'(\hat{\theta}_j, \hat{\theta}_j')\right]$.

If agent j chooses the former, then agent i is fined t^*; if he chooses the latter, agent j is fined t^*.

Stage 3: If $\hat{\theta}_i$ and $\hat{\theta}_j$ have been announced by these two agents, the outcome $f(\hat{\theta}_i, \hat{\theta}_j)$ is implemented, the goal of Central government is realized.

The idea of mechanism design is that though mutual supervision of pollution emission in different agents, the non available private information can be obtained from the process of selecting the reverse preference among agents. Then different agents can form a joint control and supervision mechanism of pollution emission.

3 Pollution Control Mechanism Design In Incomplete Information

Type θ_i is the private information, and probability distribution of types is common knowledge. So the central government as a mechanism designer can neither observe nor know the type of agents. In this section, we discuss the implementation mechanism of the central government in the dominant strategy equilibrium.

In dominant strategy, when decide their own optimal strategy, agents don't have to consider others choices. For a θ_i type agent, if $U_i(\theta_i, (g_i^*, r_i^*, g_{-i}, r_{-i})) \geq U_i(\theta_i, (g_i', r_i', g_{-i}, r_{-i}))$, then (g_i^*, r_i^*) is the dominant strategy for all agents.

If the agents' preferences are quasi-linear, the social choice function $f(\theta_i, \theta_j)$ will be implemented, so that it satisfies the Pareto efficiency. That is as follow:

$$\sum_{i=1}^{N} r_i = r^* \tag{8}$$

$$(g_i^*, r^*) = \arg\max \sum_{g_i, r} U_i(\theta_i, g_i, r_i) \tag{9}$$

If agent i did not receive any transfers, he would maximize his own utility, $\max_{g_i, r_i} U_i(\theta_i, g_i, r_i)$, which would not achieve the social optimal reduction r^*. Therefore, we introduce the Groves mechanism, and give a transfer to agent i, the transfer payment equal to the sum of other participants utility .That is to say:

$$t_i(\hat{\theta}) = \sum_{j \neq i} \left(\hat{\theta}_j V\left(g_j^*(\hat{\theta}_j), r^*(\hat{\theta})\right) - r_j(\hat{\theta}_j) - \phi(g_j^*(\hat{\theta}_j))\right) + \tau_i(\hat{\theta}_{-i}) \tag{10}$$

where $\tau_i(\cdot)$ is an arbitrary function of θ_{-i}, A mechanism in which each agent i announces $\hat{\theta}_i$ and the outcome is $\left(\left(g_1(\hat{\theta}), \cdots, g_N(\hat{\theta})\right), \left(r_1(\hat{\theta}), \cdots, r_N(\hat{\theta})\right), \left(t_1(\hat{\theta}), \cdots, t_N(\hat{\theta})\right)\right)$ satisfies（8）、（9）and（10）.

Theorem 3.1 All agents i whose types are θ_i, tell the truth (report $\hat{\theta}_i = \theta_i$) is the dominant strategy.

The transfer payment is designed to be a function of other participants' types, and the economic implication is that the decision making of some local governments brings loss to other local governments, and the loss can be obtained by other governments as income. Thus the pollution problem can be alleviated through the joint control of local governments.

Proof: see appendix.

4 Conclusions And Policy Implications

In this paper, the implementation of mechanism design was applied to the pollution control. Through the construction of general utility function of local government, the implementation in complete and incomplete information was discussed, and the dynamic and dominant equilibrium mechanism was given. The conclusions are as follows:

(1)When $\sum_{i=1}^{N} \theta_i \geq 1$, there was no supervision, and the local government would have no incentive to carry out pollution control because of the negative externality.

(2)In complete information, the condition for central governments to solve the problem of pollution externality was $\sum_{i=1}^{N} \frac{\partial r_i}{\partial V} = \frac{\partial r}{\partial V}$. That is, the total marginal utility of the local government in pollution reduction should be equal to the marginal utility of social total emission reduction.

(3) We designed the dynamic pollution control mechanism, through the local governments directly mutual constrain, we solved the problem of pollution control of local governments in incomplete information.

(4) By introducing the transfer payment as a function of the private information of other local governments, the implementation of mechanism problem in quasi linear utility was solved. The mechanism design in incomplete information was given when the target was the dominant equilibrium.

Through this paper, the following policy recommendations are put forward. Firstly, the central government should restrict the pollution control behavior of local governments and increase the local government's punishment and supervision. Secondly, the private information should be available by local government in the process of selecting the reverse preference, so as to achieve mutual supervision between different participants. Thirdly, by introducing the transfer payment between local governments, However, if it does change this choice (i.e., it is "pivotal"), then he pays the corresponding loss imposed on the rest of society. The core agents' strategy (Such as in the pollution control of Huan-Bo-Hai economic circle, Beijing is the core agent of the project decision-making) can achieve pollution control management.

This paper does not discuss the dynamic mechanism with incomplete information, and the implementation of the mechanism under the sequential equilibrium and the Bayes equilibrium will be the direction of further research.

5 Appendix

Theorem 2.1 Prove: When $r_i = 0$, the individual utility is $\theta_i V(g_i, r(\theta_{-i})) - \varphi(g_i) + t_i$, $r(\theta_{-i})$ represents the sum of the pollution reduction of other agents except agent i. The social optimal is

$$\theta_i V(g_i, r^*) - \theta_i r^* - \varphi(g_i) + t_i,$$

Let the function

$$F(g_i, r) = V(g_i, r_{(\theta_{-i})}) - V(g_i, r) + r.$$

When $r^* = (r_1^*, r_2^* \cdots r_N^*) = 0$,then $F(r^*) = 0$,

$$\frac{\partial F}{\partial r} = V'(g_i, r - r_i) - \frac{\partial V(g_i, r)}{\partial r} + 1,$$

$\because V' \geq 0$, when $r = r^*$, we have

$$\frac{\partial V(g_i, r)}{r} = \frac{1}{\sum_{i=i}^{N} \theta_i}.$$

When $\sum_{i=1}^{N} \theta_i \geq 1$, then $\frac{\partial F}{\partial r} \geq 0$,

$$\therefore \theta_i V(g_i, r(\theta_{-i})) - \varphi(g_i) + t_i \geq \theta_i V(g_i, r^*) - \theta_i r^* - \varphi(g_i) + t_i$$

At this point, agent i will choose $r_i = 0$, which arise free rider behavior.

Theorem 2.2 Prove: If the central government wants to realize the social optimum, it is necessary to ensure that personal optimum r_i is at least as big as the social optimum r_i^* for agent i. The first order condition of two cases is required to be consistent, which g_i^* automatically satisfied. Then $\frac{\partial V}{\partial r} = \frac{1}{\sum_{i=1}^{N} \theta_i}$ remains consistent with $\frac{\partial V}{\partial r_i} = \frac{1}{\theta_i}$. So we have $\sum_{i=1}^{N} \frac{1}{\frac{\partial V}{\partial r_i}} = \frac{1}{\frac{\partial V}{\partial r}}$.

The sum of the unit utility of pollution emission for each participant shall be equal to the unit utility of the social pollution. At this time, the social optimum can still be realized.

Theorem 2.3 Prove: Necessity: Suppose $f(\theta_i, \theta_j)$ is implementable in Nash Equilibrium, and (g_i^*, r_i^*) is a Nash equilibrium in state θ, $f(\theta_i, \theta_j) = (g_i^*, r_i^*)$, but $f(\theta_i, \theta_j) \neq f(\hat{\theta}_i, \hat{\theta}_j)$,then (g_i^*, r_i^*) cannot be a Nash equilibrium in state $\hat{\theta}$ .Therefore, there must exist an agent i such that $U_i(\hat{\theta}_i, g_i', r_i') > U_i(\hat{\theta}_i, g_i^*, r_i^*)$ .But because (g_i^*, r_i^*) is a Nash equilibrium in state θ ; agent i prefer (g_i^*, r_i^*) to (g_i', r_i') . Hence,

$U_i\left(\theta_i, g_i^*, r_i^*\right) \geq U_i\left(\theta_i, g_i^{'}, r_i^{'}\right)$ .So $f\left(\theta_i, \theta_j\right)$ is monotonous.

Sufficiency: $\left(g_i, r_i\right)$ Satisfies condition of $\max_{g_i, r} \sum_{i=1}^{N} U_i(\theta_i, g_i, r_i)$,and $t_i = 0$ .Suppose two types of space θ and $\hat{\theta}$,given that $\sum_{i=1}^{N} r_i^{'}$, $t_i^{'}$ Satisfy $\sum_{i=1}^{N} r_i > \sum_{i=1}^{N} r_i^{'}$,then there must exist $t_i^{'} > 0$,such that $r_i^{'} = r_i$,so $U_i\left(\theta_i, g_i^{'}, r_i^{'}\right) = U_i\left(\theta_i, g_i, r_i\right)$.

Because $\theta_i > \hat{\theta}_i$ and $\sum_{i=1}^{N} r_i > \sum_{i=1}^{N} r_i^{'}$, we have $U_i\left(\hat{\theta}_i, g_i, r_i\right) > U_i\left(\hat{\theta}_i, g_i^{'}, r_i^{'}\right)$ as monotonicity requires.

For any θ_i , $\hat{\theta}_i$ we have $U_i(\theta, g_i, r_i) \geq U_i\left(\theta, g_i^{'}, r_i^{'}\right)$ for all i; Then $\left(g_i, r_i\right)$ is a Nash equilibrium of $f\left(\theta_i, \theta_j\right)$ under the condition of monotonicity.

Theorem 3.1 Prove: When other agents report θ_{-i} , the profit for agent i telling the truth is:

$$\theta_i V\left(g_i^*(\theta_i, \hat{\theta}_{-i}),\ r^*(\theta_i, \hat{\theta}_{-i})\right) - r_i(\theta_i, \hat{\theta}_{-i}) - \varphi\left(g_i^*(\theta_i, \hat{\theta}_{-i})\right) +$$

$$\sum_{j \neq i}\left(\theta_j V\left(g_j^*(\theta_i, \hat{\theta}_{-i}),\ r^*(\theta_i, \hat{\theta}_{-i})\right) - r_j(\theta_i, \hat{\theta}_{-i}) - \varphi\left(g_j^*(\theta_i, \hat{\theta}_{-i})\right)\right) + \tau_i(\hat{\theta}_{-i}),$$

$$= \left(\theta_i + \sum_{j \neq i} \hat{\theta}_j\right) V\left(g_i^*(\theta_i, \hat{\theta}_{-i}), r^*(\theta_i, \hat{\theta}_{-i})\right) - r^*(\theta_i, \hat{\theta}_{-i}) - \sum \varphi(g_i^*(\theta_i, \hat{\theta}_{-i})) + \tau_i(\hat{\theta}_{-i})$$

According to（9）,we have

$$\left(\theta_i + \sum_{j \neq i} \hat{\theta}_j\right) V\left(g_i^*(\theta_i, \hat{\theta}_{-i}), r^*(\theta_i, \hat{\theta}_{-i})\right) - r^*(\theta_i, \hat{\theta}_{-i}) - \sum \varphi(g_i^*(\theta_i, \hat{\theta}_{-i})) + \tau_i(\hat{\theta}_{-i})$$

$$\geq \left(\theta_i + \sum_{j \neq i} \hat{\theta}_j\right) V\left(g_i^{'}, r^{'}\right) - r^{'} - \sum \varphi(g_i^{'}) + \tau_i(\hat{\theta}_{-i}),$$

Particularly, when

$$r^{'} = r^*\left(\hat{\theta}_i, \hat{\theta}_{-i}\right),\quad g_i^{'} = g_i^*\left(\hat{\theta}_i, \hat{\theta}_{-i}\right),$$

Choose $\hat{\theta}_i = \theta_i$ is a dominant strategy.

The proviso is that we have not yet ensured that the transfer functions (10) are feasible. One way of ensuring feasibility is to take

$$\tau_i(\hat{\theta}_{-i}) = -\max_{g_i, r} \sum_{j \neq i}\left(\hat{\theta}_j V\left(g_j,\ r\right) - r_j(\hat{\theta}_j) - \varphi(g_j)\right),$$

Then, community i's transfer becomes

$$t_i\left(\hat{\theta}\right) = \sum_{j \neq i}\left(\hat{\theta}_j V\left(g_j^*(\hat{\theta}_j),\ r^*(\hat{\theta})\right) - r_j(\hat{\theta}_j) - \varphi(g_j^*(\hat{\theta}_j))\right)$$

$$- \max_{g_i, r} \sum_{j \neq i}\left(\hat{\theta}_j V\left(g_j,\ r\right) - r_j(\hat{\theta}_j) - \varphi(g_j)\right).$$

If community i's announcement has no effect on the social choice, the community i pays nothing. However, if it does change this choice, i pays the corresponding loss imposed on the rest of society.

Reference:

[1] J. Xue, L. Wan Xie, M. Chang Li, Inter-Provincial Cooperative Game Model of Beijing,Tianjin and Hebei Province Air Pollution Control , 810-816, **34**(2014)

[2] W. Dungumaro, F. Madulu, Public Participation in Integrate Water Resources Management, 1009－1014, **28**(2003)

[3] G. Xue Zhang, M. Chu Zhong, Research about Government Regulation and the Firm Environment Pollution under Perspective of Game Theory, 31-35, **21**(2011)

[4] L. Yang, X. Hong Gao, A Study on the Game between Environmental Regulators and Manufacturers under Perspective of Economic, 51-55, **21**(2012)

[5] O. Bahn, A. Haurie, A Cost-Effectiveness Differential Game Model for Climate Managements, 1-19, **10**(2015)

[6] Y. Fang Lu, Evolutionary Game Analysis on Environmental Pollution Problem, 148-152, **9** (2007)

[7] Y. Wang, J. Luo, Dynamic Evolution Game Analysis of the Strategic Alliance of Enterprise Pollution Control, 68-71, **17**(2011)

[8] W. Bo Yang, W. Yong Wang, An Analysis of the Non Equilibrium Game of the Central and Local Governments in Urban Pollution Control, 52-56, **3**(2014)

[9] W. Zhang et al., Analysis on Evolutionary Game of Government Regulation Mode and Enterprise Pollution Emissions, 108-113, **11**(2014)

[10] M. Glachant, The Use of Regulatory Mechanism Design in Environmental Policy: A Theoretical Critique, 2-12, (1998)

[11] A. Dilip, S. Arunava, Virtual Implementation in Nash Equilibrium, 997-1021, **59** (2008)

[12] P. Kang Wu, F. Xue Zhang et al., Research on Auction Mechanism of State-Owned Land, 141-148, **22**(2014)

[13] J. Hovi, I. Areklett, Enforcing the Climate Regime: Game Theory and the Marrakesh Accords1International, 1–26, **4**(2004)

[14] A K. Tiwari, Comparative Performance of Renewable and Nonrenewable Energy Source on Economic Growth and CO2 Emissions of Europe and Eurasian countries, 2356-2372, **31**(2011)

[15] T. Yuan Lu, Z. Shun Wu, X. Ma, An Empirical Analysis of the Environmental Effects of Industrial Structure Adjustment in China, 69-72, **2** (2011)

[16] Q. Zhi Dong ec etl., The Direction of Technological Progress, the Scale of Urban Land Use and Environmental Quality, 111-124, **10** (2014)

[17] I. Danaeifar, The Estimation Parameters of Kuznets Spatial Environmental Curve in European Countries, 19-26, **2**(2014)

[18] H. Kai Yuan, H. Jian Li, Emission reduction Strategies Under Plight Collusion, 134-141, **25**(2015)
[19] 19.M.Finus,Game Theory and International Environ mental Cooperation, (2000)
[20] B. Sandeep, E. Maskin, Mechanism Design for the Environment, 305-324, **1** (2002)