

# Research on Safety Assessment Method for Bridge Structure Based on Variable Weight Synthesis Method

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**Abstract.** Variable weight synthesis method is a safety assessment method which is using the varied weight principle, base on analytic Hierarchy Process and combining the theory of Grey relevancy, it reduce influence that secondary cause is not obvious in assessment process, and make the result more correct and dependable. In the paper, variable weight principle and the assessment method were expounded detailedly.

## 1. Variable weight principle and Variable weights method

### 1.1 Variable weight principle

In the literature “The application of variable weight synthesis method in estimate of real estate investment environment”, variable weight synthesis method is the definition of:

- 1) Assume that a m-dimensional vector,  $W^0 = (W_1^0, W_2^0, \dots, W_m^0)$ , if present  $W_j^0 \in (0,1)$  makes  $\sum_{j=1}^m W_j^0 = 1$  correct for arbitrary variables  $j \in \{1, 2, \dots, m\}$ ;
- 2) Given a m-d vector of parts keep weight  $W(X) \Delta [W_1(X), W_2(X), \dots, W_m(X)]$ , for given mapping:  $W : [0,1]^m \rightarrow (0,1]$ , it makes that:

$$(1) \sum_{j=1}^m W_j(X) = 1$$

(2) Assume that  $\alpha_j, \beta_j \in [0,1]$ , and  $\alpha_j \leq \beta_j$ , for arbitrary variables  $j \in \{1, 2, \dots, m\}$ , present  $W_j$  in regard to  $x_j$  in the internal  $[0, \alpha_j]$  and  $[\beta_j, 1]$  respectively decrease and increase progressively.

3) Assume that the vector  $S(X) \Delta [S_1(X), S_2(X), \dots, S_m(X)]$  is a m-d vector of parts variable weight, for the given mapping  $S : [0,1]^m \rightarrow (0, +\infty)^m$ , for arbitrary variables  $j \in \{1, 2, \dots, m\}$ , assume  $\alpha_j, \beta_j \in [0,1]$ , and  $\alpha_j \leq \beta_j$  present that:

(1) For arbitrary variables  $j \in \{1, 2, \dots, m\}$  for fixed weights vector  $W^0 = (W_1^0, W_2^0, \dots, W_m^0)$ ,

$$W_j(X) = \frac{W_j^0 S_j(X)}{\sum_{k=1}^m W_k^0 S_k(X)} \quad \text{in the internal } [0, \alpha_j] \text{ and } [\beta_j, 1] \text{ respectively decrease and increase progressively.}$$

increase progressively.

(2) If  $0 \leq x_i \leq x_k \leq \alpha_i \wedge \alpha_k$ ,  $S_i(x) \geq S_k(x)$ , and  $\beta_i \vee \beta_k \leq x_i \leq x_k \leq 1$ , then

$$\frac{w^0 \cdot S(x)}{\sum_{j=1}^m w_j^0 S_j(x)} \square \left\{ \frac{w_1^0 \cdot S_1(x)}{\sum_{j=1}^m w_j^0 S_j(x)}, \frac{w_2^0 \cdot S_2(x)}{\sum_{j=1}^m w_j^0 S_j(x)}, \dots, \frac{w_m^0 \cdot S_m(x)}{\sum_{j=1}^m w_j^0 S_j(x)} \right\} \quad (1)$$

be called m-d vector of parts variable weight.

## 1.2 Amendment of variable weights method

In the variable weights method, when synthesis assessment the assessed value of hundred-mark system, it will result data with denominator larger, and produce larger round-off error. For convenience of calculations and considering the required precision, to transform hundred-mark system into percentage of system, and then, by adjusting coefficients in the formulas, getting the synthesis assessed value of hundred-mark system. in addition, for some sequences of inspection data, it is impossible to collect all the data, but it is needed to compare the curve of sequences of inspection data, so in this paper, considering base on the grey theory related principle to offset the malconformation and incomprehensive of data. The paper make the improvement.

$$V = \sum_{j=1}^m 100w_j(x_1, \dots, x_m, w_1^{(0)}, \dots, w_m^{(0)})x_j \quad (2)$$

In formula:  $V$  is the assessed value;

$w_j^{(0)}$  is initial weight value of the  $j$ th indicators, and  $\sum_{j=1}^n w_j^{(0)} = 1$  ;

$w_j$  is weight value after variable weight;

$x_j$  is the percentage of assessment value of the  $j$ th indicators.

And

$$w_j(x_1, \dots, x_n, w_1^{(0)}, \dots, w_n^{(0)}) = \frac{w_j^{(0)} \frac{\partial B(x_1, \dots, x_n)}{\partial x_j}}{\sum_{k=1}^n w_k^{(0)} \frac{\partial B(x_1, \dots, x_n)}{\partial x_k}} \quad (3)$$

In formula,  $B(x_1, \dots, x_n)$  is balanced function, according to experience of engineering practice, the balanced function is:

$$B(x_1, \dots, x_n) = \sum_{j=1}^n x_j^\alpha \quad (0 < \alpha \leq 1) \quad (4)$$

simultaneous equations (3) and (4), it can get:

$$w_j(x_1, \dots, x_n, w_1^{(0)}, \dots, w_n^{(0)}) = \frac{w_j^{(0)} x_j^{\alpha-1}}{\sum_{k=1}^n w_k^{(0)} x_k^{\alpha-1}} \quad (5)$$

so

$$V = \sum_{j=1}^m \frac{100w_j^{(0)} x_j^\alpha}{\sum_{k=1}^n w_k^{(0)} x_k^{\alpha-1}} \quad (6)$$

$\alpha$  : balance coefficient, according to experience of engineering practice,  $\alpha = 0.2$ , it can meet most of engineering situation.

The inspection data of bridge can be divided into three categories: the first kind is non-numeric data with describing the state of bridge components only or dividing grade simply; the second is a single numerical; and the third is a data series. it is needed that applies dimensionless method to the third kind data, compare and analyze the two curves of data series. Introduce grey correlativity analysis and calculate non-uniform variation coefficient to get the assessment result, the formula as follows.

$$V = \sum_{j=1}^m \frac{100w_j^{(0)}x_j^\alpha}{\sum_{k=1}^n w_k^{(0)}x_k^{\alpha-1}} r(x_0, x_i) \quad (7)$$

And  $r(x_0, x_i)$  calculate according to the formula as below[5][8]:

$$r(X_0, X_i) = \frac{1}{n-1} \sum_{k=1}^{n-1} \left[ 1 + \left| \frac{a^{(1)}(x_0(k+1))}{x_0(k+1)} - \frac{a^{(1)}(x_i(k+1))}{x_i(k+1)} \right| \right]^{-1} \quad (8)$$

## 2. Advantage of variable weight synthesis method

Compared with constant weight synthesis method, variable weight synthesis method can make the influence of components with damage badly more prominent. In structure, maybe individual component damage can be lead to the bridge operational be limited, even impact to the safety of the bridge. Compared with the result of constant weight synthesis method and variable weight synthesis method as below.

Table1 assess table of a bridge

indicator	weight	Technical status	explain
pier and abutment	0.31	80	better
girder	0.13	80	better
pylon	0.14	75	normal
stay cable	0.37	80	better
subsidiary facilities	0.04	5	failure badly

Using the data in table1, assessment the health of bridge with variable weight synthesis method and constant weight synthesis method, the result as below.

1)constant weight synthesis method:

$$BCI_2 = 80 \times 0.31 + 80 \times 0.13 + 74 \times 0.14 + 80 \times 0.37 + 5 \times 0.04 = 75.51 > 66$$

The conclusion is “qualified”;

2)variable weight synthesis method( $\alpha = 0.2$ ):

$$\sum_{k=1}^m w_k^{(0)} x_k^{\alpha-1} = \sum_{k=1}^9 w_k^{(0)} x_k^{-0.8} = 0.31 \times 0.80^{-0.8} + 0.13 \times 0.80^{-0.8} + 0.14 \times 0.75^{-0.8} + 0.37 \times 0.80^{-0.8} + 0.04 \times 0.05^{-0.8} = 1.5840$$

score  $V_2$ :

$$V_2 = \sum_{j=1}^m \frac{100w_j^{(0)}x_j^{(\alpha-1)}}{\sum_{k=1}^n w_k^{(0)}x_k^{(\alpha)}} = \frac{100}{1.584} (0.2965 + 0.1243 + 0.1322 + 0.3539 + 0.0220) = 58.64$$

The conclusion is “disqualified”.

The different conclusion with the different value of  $\alpha$  is show in table2. The value of  $\alpha$  reflect tolerance of local damage, with the increase of  $\alpha$ , tolerance increase.

Table2 consultable table of equal coefficient

equal coefficient	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1.0$
Score of girder	49.52	58.64	68.59	74.19	75.51

By the above examples, it can be got that when the subsidiary facilities damage badly, the result of constant weight synthesis method is “qualified” while variable weight synthesis method is “disqualified”. It is show that variable weight synthesis method can well reflect the local defect and considering the malconformation into assessment. It is more tally with the actual situation.

### 3. Application example of method

The inspection data of line shape control about abridge is in table 3.

Table3 level evaluation of reference point

coordinate (m)	designed elevation (X <sub>0</sub> )	measured elevation (X <sub>i</sub> )	Error (m)	assessed value (D <sub>i</sub> )
0	86.550	86.538	0.012	90
20	86.844	86.830	0.014	90
40	87.079	87.056	0.023	80
60	87.247	87.235	0.012	90
80	87.432	87.419	0.013	90
100	87.534	87.520	0.014	90
120	87.541	87.517	0.024	80
140	87.531	87.508	0.023	80
160	87.371	87.357	0.014	90
180	87.168	87.153	0.015	90
200	86.993	86.977	0.016	90
220	86.774	86.752	0.022	80
240	86.537	86.526	0.011	90

According to formula (5), calculate variation of weight show as fig.1 as below.

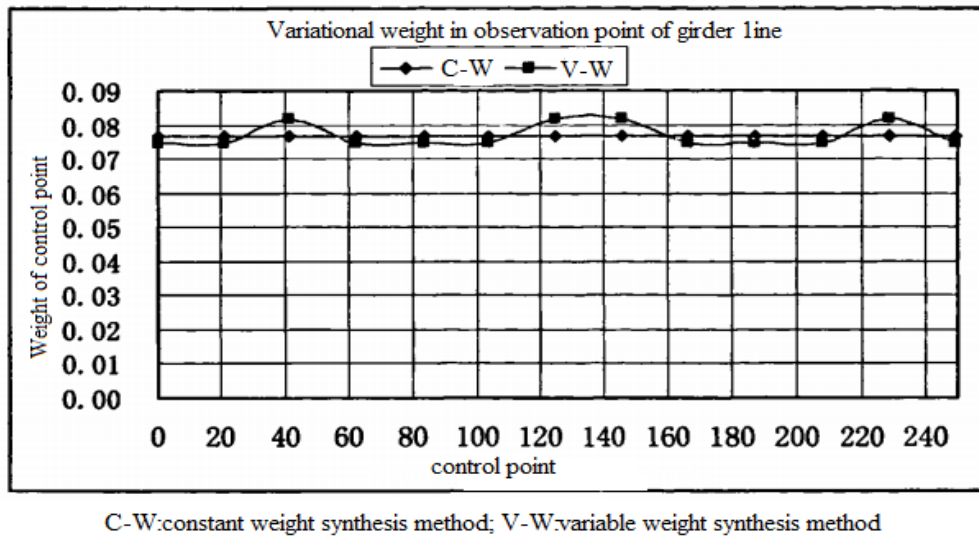


Fig.1 Variational weight in observation point of girder line

According to formula(7)and(8)calculate as below:

$$V = \sum_{j=1}^m \frac{100w_j^{(0)}x_j^\alpha}{\sum_{k=1}^n w_k^{(0)}x_k^{\alpha-1}} r(x_0, x_i) = 84.14$$

$x_k = D_i / 100$ ,  $D_i$ : assessed value.  $w^{(0)} = 1/n$  ( $n=13$ ) initial weight value, See table3-1.

$$r(X_0, X_i) = \frac{1}{n-1} \sum_{k=1}^{n-1} \left[ 1 + \left| \frac{a^{(1)}(x_0(k+1))}{x_0(k+1)} - \frac{a^{(1)}(x_i(k+1))}{x_i(k+1)} \right| \right]^{-1} = 0.968$$

$X_0 = [X_0(1), X_0(2), \dots, X_0(13)]$  designed elevation;

$X_i = [X_i(1), X_i(2), \dots, X_i(13)]$  measured elevation.

Assessment score of girder alignment is 84.14, contrast table4, the bridge can be classified grade 1.

Table4 The reference standard of bridge technique condition evaluation

Score	Grade	Explain
$80 < V \leq 100$	Grade 1	Well, only daily maintenance
$60 < V \leq 80$	Grade 2	Normal, daily maintenance and minor repair
$40 < V \leq 60$	Grade 3	Weak, repair and load test
$20 < V \leq 40$	Grade 4	Bad, Stop running and strengthen
$0 < V \leq 20$	Grade 5	Danger, considering dismantle and rebuild

Note: the evaluation standard is made according to "Evaluation procedure for bearing capacity of highway bridge".

#### 4. Conclusion and prospect

The article explains the variable weight synthesis method, and apply the method into the process of bridge assessment. Though it has the rationality of the method, but it still exist some problem. First, although the assessment apply new method, but most of the index still reference codes and specifications and previous research results, it needs to do a more in-depth discussion. Second, the initial weight value used in the paper, it is still the average weight which has certain blindness, it

needs to do a more in-depth discussion, too.

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