Prediction of Slope Deformation Time Series Based on Quasi-Newton

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Abstract: In mining production, the slope's stability is one of the decisive factors of safe production of mine. Security issues and economic losses caused by slope's instability were immeasurable. Predict the slope's deformation according to the monitoring data of slope's displacement is an effective and feasible way. In order to predict the landslide accurately, ensure mine's safety and reduce the loss of property and casualties, set up mathematical model ARMA(n,n) based on time series method, determined the order of the model by improved Pandit-Wu method, determined parameters by Quasi-Newton method of engineering optimization algorithms, at last calculate the dynamic predictive values of a mine's slope displacement monitoring data. The results showed that, the method can improve the accuracy of prediction, and provided a reliable method for all kinds of slope landslides' forecast and early warning.

1 Introduction

As a key technology and problem of mine safety, the study of slope stability is the research direction of geotechnical engineering and some related discipline. Slope angle increasing is one of the methods to lower the cost and improve the mine performance. It is estimated that[1], slope angle increase one degree for large high and steep slope can save stripping rock fee to millions even hundreds millions. Alone with the deeping and steeping of the slope, the difficulty to maintain stability of the slope is getting large and large, therefore increasing the slope angle is one of the technical problem of mine production[2]. For accelerating the development of slope prevention, so as to ensure the safety of mining and improve the economic returns, prediction of slope stability matters hugely.

Among several methods for prediction of landslide, methods majored internal factors bring little effect but the process is complex. Earthquake、strong rainfall and explosion and so on are related to the weather and engineering activity, so it is proposed[3] that the prediction combined weather and monitor is the future research direction for slope deformation. Another researcher promote that the prediction of slope deformation should combined with the monitoring data[4]. Both of these measures are based on the monitoring data, because the prediction through monitoring data is one of the important links for landslide prevention.

As one of the methods of statistics , time series was widely used in the production of engineering[5].Slope deformation always has the characteristic of monotone increasing during the landslide formation process, so the monitoring data of slope deformation could be treat as the non-stationary time series[6].Time series used in prediction of landslide is more precise compared with Gray Dimension、Neural Network and regression coefficient method[7]. The precise of time series is closely related to order and parameter selection. Parameter selection can be classified as rough and accurately estimate. Moment estimation、invertible function and Mayne method are applied to rough estimate; steepest descent method, maximum likelihood method 、Gauss-Newton iterative algorithm and damped least square method[8] are applied to accurately estimate. However, process complex、more iterative times and low efficiency are the disadvantages for these methods.

As one of the most effective methods among engineering optimizational methods, quasi-Newton can achieve the objectives when comes to some parameter problems because of its good convergence[9]. It is a feasible and effective method for solution of parameter when predicting the slope deformation based on the mathematical model of time series[10].

Take the particularity of slope into consideration, then set up the ARMA(n.n) model. Orders were determined by the improved Pandit-Wu and Parameters were selected by Quasi-Newton to make the prediction of one mine's slop deformation; the results show that the prediction accuracy is high.

2 Mathematic model of slope deformation

2.1 Establish the target function

Deformation of slope is often arbitrary, and this kind of random data cannot fit directly by any certain function. Time series model can be applied to describe the law of all historical data and the relationship of data. Therefore, it is viable to describe the dependence relationship of monitoring data of slope deformation. There are many time series economic metric models[11], For a stationary、zero-mean time series $\{x_i\}, i = 1, 2, \cdots, t$,there must be a fitted random difference equation as follows:

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \cdots + \varphi_n x_{t-n} + a_t - \theta_1 a_{t-1} \ldots + \theta_2 a_{t-2} - \theta_m a_{t-m}$$
$$= \sum_{k=1}^{n} \varphi_k x_{t-k} + a_t - \sum_{j=1}^{m} \theta_j a_{t-j} \qquad (1)$$

In the equation: x_t is the factor of time t for time series $\{x_t\}$, φ is the Autoregressive, θ is the Moving Average, $\{a\}$ is the residual error sequence.When the moving average set as zero, the equation is the AR（n）model:

$$x_t = \sum_{i=1}^{n} \varphi_i x_{t-i} + a_t \qquad (2)$$

when the autoregressive stetted as zero, the equation is the MA（m）model:

$$x_t = a_t + \sum_{j=1}^{m} \theta_j a_{t-j} \qquad (3)$$

$\hat{x}_{t+l}$ is the prediction of series, a_{t+l} is the error in future, l is the step length and $l \geq 1$。x_{t+l} is scattered randomly as an unknown quantity when $x_t, x_{t-1}, x_{t-2}, \cdots,$ are given.

Conditional expectation own properties as follows:

(1)conditional expectation of the future series is the prediction of future series:

$$E\left(x_{t+l} \middle| x_t, x_{t-1}, x_{t-2} \cdots\right) = \hat{x}_{t+l} \qquad (4)$$

(2)conditional expectation of the future series is zero:

$$E\left(a_{t+l} \middle| x_t, x_{t-1}, x_{t-2} \cdots\right) = 0 \qquad (5)$$

Then, equation (1)transformed to:

$$\hat{x}_{t+l} = \sum_{k=1}^{n} \varphi_k x_{t+l-k} - \sum_{j=1}^{m} \theta_j a_{t+l-j} \qquad (6)$$

On the basis of $E\left(a_{t+l} \middle| x_t, x_{t-1}, x_{t-2} \cdots\right) = 0$ establish the equivalent relationship as follows:

$$E\left(a_{t+l} \middle| x_t, x_{t-1}, x_{t-2} \cdots\right) = \hat{x}_{t+l} - x_{t+l} = 0 \qquad (7)$$

on the basis of statistical method and the equation (7), set φ、θ as variable, set the objective function as follows:

$$f(\phi, \theta) = \sum_{i=t}^{1} (\hat{x}_{i+l} - x_{i+l})^2 \qquad (8)$$

The equation (8)is the primary time series model.

2.2 Dynamic modelling

Massive amounts of data shows that, the precision of prediction is related with the modelling data's time-validity. Redundant and old data can impact the computational efficiency and precision of prediction. This is important to considerate when set the prediction model. In order to avoid this, it is important to model dynamically, so as to improve the computational efficiency.

Topology of the original monitoring sequence $\{x_i\}, i = 1, 2, \cdots, t$ as follows:

$$x(t-p,t) = \{x_{t-p}, x_{t-p+1}, \cdots, x_{t-1}, x_t\} \tag{9}$$

When p=6 ,we call

$$M_7 = \{x(1,7), x(2,8), \cdots x(t-6,t)\} \tag{10}$$

as the dynamic series. It is certain to make sure the timeliness of prediction based on each item of equation (10).

2.3 Model recognition

Model recognition is the first step for a time series, Box-Jenkins、Pandit-Wu and long order autoregressive are common methods for model recognition.

Box-Jenkins is based on the truncation、trailing of self-correlation and partial autocorrelation function to decide which kind model is the best one. It can be used for model recognition of AR、MA、ARMA. Pandit-Wu is based on Box-Jenkins, and be proposed by experiment and further development. It is on the basis of the concept as follows. Any stationary series can be expressed by an ARMA（n,n-1）model. It's thoughts can be summarized as follow: fitting high-order ARMA（n,n1) with the increasing order of the model gradually until the residual error is not increase any more. It is better than the long order autoregressive because the latter one almost ignore the influence of MA. Because long order autoregressive is designed to approach the precision by a high-order AR.

Due to each item of MA(m) is the error, it can be treat as correction term if minus it from each matched prediction item. Therefore, this can be used to fit an accurate predicted value that closer to measured value. On the basis of this reason，set the slope deformation time series predict model as ARMA(n,n), thus the objective function changed as：

$$f(\phi,\theta) = \sum_{i=t}^{1} (\hat{x}_{i+l} - x_{i+l})^2 = \sum_{i=t}^{1} (\sum_{k=1}^{n} (\varphi_k x_{i+l-k} - \theta_k a_{i+l-k}) - x_{i+l})^2 \tag{11}$$

2.4 Order determination

The step after recognition is order determination. Conventional methods of order determination contains: Plot of variance of residuals、order determination method based on self-correlation and partial autocorrelation function,F-check and principle ordered method and so on. All of these methods are calculate some parameters then plot or lookup table according these parameters, and all above are complex.

Improve the Pandit-Wu order determined method, which can be called a priori approach based on the measured value and residuals. The system basic thoughts is analyze the proportion of various deformations' contribution to the totally slope deformation. Topology of the original monitoring sequence is as follows:

$$x(t-p,t) = \{x_{t-p}, x_{t-p+1}, \cdots, x_{t-1}, x_t\} \tag{12}$$

The residuals square series is:

$$\sigma_\alpha^2(n) = Var\{x_{t-p}, x_{t-p+1}, \cdots, x_{t+1-n}\} n = 1, 2, \cdots, p \tag{13}$$

The significance level is setted as 0.3, and n=1, when meet the condition :

$$(\sigma_\alpha^2(n) \le \sigma_\alpha^2(n+1)) \,||\, (\frac{\sigma_\alpha^2(n) - \sigma_\alpha^2(n+1)}{\sigma_\alpha^2(n)} \ge 0.3) \tag{14}$$

The n will be auto-incrementing, to get to the next cycle, until the end of the cycle.

2.5 Parameter estimation

Parameter estimation is the step after the primary model's order determination of the simple data. Parameter estimation of time series can be expressed as the non-constrained optimization problems like the function (11). The solution is to calculate the parameters by engineering optimization methods. The non-constrained optimization problems can be classified into two groups: one is derivative method, another one is direct solution. Due to the faster speed, we choose the derivative method[9]. The Newton algorithm has faster convergence speed among kinds of derivative methods such as: steepest descent method、conjugate gradient method、quasi-Newton method, however it need to ensure the Hesen matrix is invertible. The quasi-Newton(DFP or BFGS) is an effective method, it has the advantages of small number of convergence and does not require the Hesen matrix is invertible.

Transform the Newton iterative formula to the quasi-Newton:

$$x^{(\beta-1)} = x^{(\beta)} + \lambda_\beta d^{(\beta)} \quad (15)$$

In this formula, x is the argument vector; λ_k is the step size in search; $d^{(\beta)}$ is the search direction:

$$d^{(\beta)} = -[\nabla^2 f(x^{(\beta)})]^{-1} \nabla f(x^{(\beta)}) \quad (16)$$

The method of determination of step size in search is:

$$\min_{\lambda} f(x^{(\beta)} + \lambda d^{(\beta)}) \quad (17)$$

Here: $\Delta g^{(\beta)} = \nabla f(x^{(\beta-1)}) - \nabla f(x^{(\beta)})$ (18)

$$\Delta x^{(\beta)} = x^{(\beta-1)} - x^{(\beta)} \quad (19)$$

For DFP, the replacement of Hesen matrix in $d^{(\beta)}$ is:

$$H_{\beta+1} = H_\beta + \frac{\Delta x^{(\beta)} (\Delta x^{(\beta)})^{\mathrm{T}}}{(\Delta g^{(\beta)})^T \Delta x^{(\beta)}} - \frac{H_\beta \Delta g^{(\beta)} (\Delta g^{(\beta)})^{\mathrm{T}} H_\beta}{(\Delta g^{(\beta)})^T H_\beta \Delta g^{(\beta)}} \quad (20)$$

For BFGS, the replacement of Hesen matrix in $d^{(\beta)}$ is:

$$H_{\beta+1} = H_\beta + \frac{1}{(\Delta x^{(\beta)})^T \Delta g^{(\beta)}} [1 + \frac{(\Delta g^{(\beta)})^T H_\beta \Delta g^{(\beta)}}{(\Delta x^{(\beta)})^T \Delta g^{(\beta)}} \Delta x^{(\beta)} (\Delta x^{(\beta)})^{\mathrm{T}}$$
$$-H_\beta \Delta g^{(\beta)} (\Delta x^{(\beta)})^T - \Delta x^{(\beta)} (\Delta g^{(\beta)})^T H_\beta] \quad (21)$$

Choose one of these into formula (11) to estimate the parameters φ and θ, then we can get the predictive value.

3 Engineering case

3.1 Example verification

Take the case in literature[7] as the sample, make the dynamic prediction of 40 monitoring data from 2005.4 to 2008.7 by the method of this paper. Compare the result of predict and the data in book(compared result is in Fig .1) .

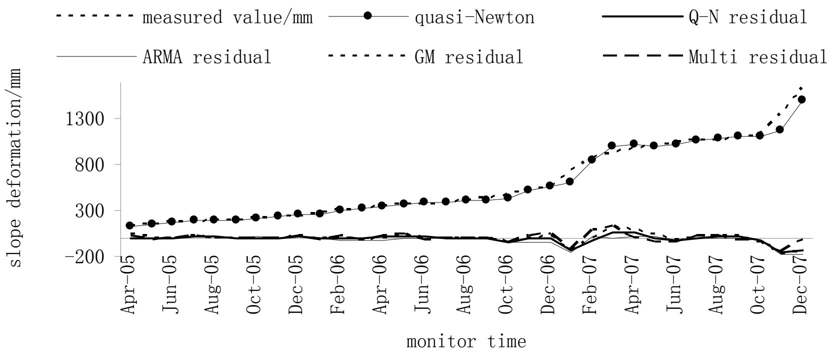

Fig.1 Comparison of prediction and measured value and residuals

As it shows in Fig.1,the degree of fitting of prediction and measured value is high, the residual of quasi-Newton is less than other methods and it trends to be stability and low. During its calculation , the iterative times keeps in three, the orders of magnitude in extreme value during parameters estimation keeps within -20, the error of fitting is low. The prediction accuracy of quasi-Newton is 96%, make comparisons (shows in table 1)of prediction accuracy with ARAM(n,n-1)、GM and polynomial regression, the accuracy is 95.4%、94.73% and 90.34%, it improved 0.5%. On the other hand, the error residual keeps in a low lever than these methods.

table1 comparison of average residuals and prediction accuracy

	quasi-Newton	ARMA	GM	PR
average residual/mm	25.00388	27.8717	33.12976	44.82685
prediction accuracy	96.09%	95.49%	94.91%	90.34%

3.2Application example

One mine is located in a middle-lower and erosion mountains, due to open-pit mining, towering peaks of mountains have become into a circle valley which is more than 300 meters deep. There is no surface water in stope and the groundwater is recharged from the precipitations. Rock and rock mass structure types are different, various size and levels structural plane have developed, efflorescence altered effect is strong, and the geological conditions of the slope engineering is complex. The old exploring data mentions that there were debris flow、crumble and roof collapse and some ill geological phenomena like these in the mining areas, and there always be some small-scale collapse、toppling and fall and bench destruction. Take these into consideration, set three GPS deformation monitoring point 910 、940 and 960 on the working bench at the north and east of the mine. The monitoring was setted as a week.

The monitor point 910 is located in medium-coarse grained granite, point 940 is located in medium-fine granite, both of these are stubborn block mosaic texture, the altered type of these is quartz alum lithification and grand opening serictie alteration belt; the monitor point 964 is locatied in quartz alum lithification and grand opening belt, it belongs to less-hard block mosaic texture. Development of opening lithification effect the strength of rocks, the major characteristic is the low rock strength, free-flowing softener and collapse. What is worse, the mine is located in subtropical monsoon climate , March -June is the rain season, July- September is the typhoon season, it is easy to collapse. Here fitting the monitoring slope deformation data of 13-14 with the method of this paper(Fig .2)

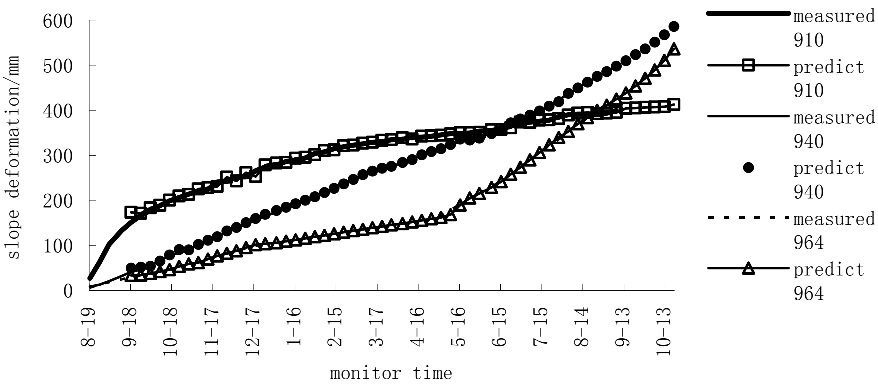

Fig.2 comparison of prediction and measured value of different monitoring points

As it shows in Fig.2, it is fitted well with the method of this paper, the prediction accuracy can reach to 98% and higher for three monitoring points. The results shows that, the deformation of these three points is trend to stability; compared with monitoring point 910, the other two moves faster than it, especially the point 964. It need to take the weather prediction into consideration at the same time when predict the slope deformation, so as to take measured to ensure the safety of personal and property around the mining engineering.

4 Conclusions

It plays a decisive role to choose a effective and right prediction method for slope deformation to ensure a high prediction accuracy, because only under the guidance of the accurate prediction, it could make targeted measures and proper response to reduce the risk of the collapse of the open-mine's high slope across the hidden danger region. Otherwise, it could result in heavy economic losses and baneful social impact. This paper adopt the prediction ARMA(n,n) model based on quasi-Newton and make the slope deformation prediction according to the monitoring data, we can get conclusions from the results as follows:

(1) It is verified ,through the instance , that the improved Pandit-Wu method to determine the order and estimate the parameters by quasi-Newton for ARMA(n,n) model has several advantage, such as fast convergent rate、less iteration time and high calculate efficient and convenience to be popularized and applied.

(2) For the instance in this paper, the prediction accuracy can keep above 96%, it is the highest one among those methods mentioned above. The accuracy can reach up to 98% apply to one mine, so the method of this paper has practicability.

Above all, combined the prediction method based on the slope deformation monitoring data and the weather prediction can make a accurate prediction for slope stability. This paper provide a efficient and reliable method for mining and geological disaster forcast.

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