Study on the Grindability Ranking of Granite Using Fuzzy Analytical Hierarchy Process and TOPSIS Approaches

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Abstract. The ranking system of granite grindability is very important for high-efficiency grinding key technology. A new method by the combination of Fuzzy Analytic Hierarchy Process (FAHP) method with TOPSIS methods is developed to establish the dependence function and fuzzy relationship. The grindability of ten types of granite was evaluated and classified by this method. With the fuzzy ranking system established and the grindability classification, it is very convenient to evaluate the grindability and select a suitable diamond tool and proper grinding parameters for a new granite type by only the petrographic analysis and mechanical properties testing.

Introduction

The granite is a natural rock material that consist of several minerals, so the grindability of different types of granite is different. The ranking system of granite grindability is very important for high-efficiency grinding key technology. Many experts have been studied the sawability of granites and tried to conceive the ranking method for granites. Reza Mikaeil proposed changes of motor power while cutting stone to stone sawing performance evaluation [1,2]. Saffet Yagiz proposed to evaluate the brittleness of stone sawing of stone, and the use of fuzzy inference system and nonlinear regression to establish a mathematical model of sawing force [3]. Bulent Tiryaki used the specific cutting energy index to evaluate the machinability of the stone, and using artificial neural network prediction model [4]. The aim of this paper is developing a new method by the combination of Fuzzy Analytic Hierarchy Process (FAHP) method with TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) methods.

Applied theoretical concept

Theory of Triangular Fuzzy. In this study, the FAHP is used. Let $X = \{x_1, x_2, x_3, \dots, x_n\}$ be an object set, and $G = \{g_1, g_2, g_3, \dots, g_n\}$ be a goal set, each object is taken, and extent analysis for each goal performed respectively. Therefore, m extent analysis values for each object can be obtained with $M_{gi}^1, M_{gi}^2, \dots, M_{gi}^m (i=1, 2, \dots, n)$, where $M_{gi}^j (j=1, 2, \dots, m)$ all are triangular fuzzy numbers(TFN). The steps of extent analysis can be given as in the following:

Step 1. The value of fuzzy synthetic extent with respect to the i object is defined as

$$S_i = \sum_j M_{gi}^j \otimes [\sum_{i=1}^n \sum_{j=1}^m M_{gi}^j]^{-1} \quad (1)$$

To obtain $\sum_{j=1}^m M_{gi}^j$, the fuzzy addition operation of m extent analysis values for a particular

matrix is performed as follows $\sum_j M_{gi}^j = (\sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{j=1}^m u_j)$, $\sum_{i=1}^n \sum_{j=1}^m M_{gi}^j = (\sum_{i=1}^n l_i, \sum_{i=1}^n m_i, \sum_{i=1}^n u_i)$,

$$[\sum_{i=1}^n \sum_{j=1}^m M_{gi}^j]^{-1} = (\frac{1}{\sum_{i=1}^n u_i}, \frac{1}{\sum_{i=1}^n m_i}, \frac{1}{\sum_{i=1}^n l_i}).$$

Step 2. As $M_1(l_1, m_1, u_1)$ and $M_2(l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility of $M_1 \geq M_2$ is defined as

$$V(M_1 \geq M_2) = \sup_{x \geq y} [\min(m_{M_1}(x), m_{M_2}(y))] \quad (2)$$

and can be expressed as follows

$$V(M_1 \geq M_2) = m(d) = \begin{cases} 1, & m_1 \geq m_2 \\ \frac{l_2 - u_1}{(m_1 - u_1) - (m_2 - l_2)}, & m_1 \leq m_2, \quad u_1 \geq l_2 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Step 3. The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy $M_i (i=1, 2, \dots, k)$ numbers can be defined by

$$V(M \geq M_1, M_2, \dots, M_k) = \min V(M \geq M_i), \quad i = 1, 2, \dots, k \quad (4)$$

Assume that $d'(A_i) = \min V(S_i \geq S_k) \quad (k = 1, 2, \dots, m, \quad k \neq i)$, then the weight vector is given by

$$W' = (d'(A_1), d'(A_2), \dots, d'(A_m))^T \quad (5)$$

Where $A_i (i = 1, 2, \dots, n)$ are i elements.

Step 4. Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \dots, d(A_m))^T \quad (6)$$

where W is a non-fuzzy number.

TOPSIS method. TOPSIS is one of the useful multi-attribute decision making techniques to manage real-world problems. In this paper, TOPSIS method is used for determining the final ranking of the sawability of granites. TOPSIS method is performed in the following steps:

Step 1. Decision matrix is normalized via Eq. (7)

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{i=1}^n f_{ij}^2}} \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, m \quad (7)$$

Step 2. Weighted normalized decision matrix is formed

$$v_{ij} = W_j \times r_{ij} \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, m \quad (8)$$

Step 3. Positive Ideal Solution and Negative Ideal Solution are determined

$$A^+ = \{v_1^+, v_2^+, v_3^+, \dots, v_m^+\}^T, \quad v_j^+ = \max_i \{v_{ij}\}, \quad j = 1, 2, \dots, m \quad (9)$$

$$A^- = \{v_1^-, v_2^-, v_3^-, \dots, v_m^-\}^T, \quad v_j^- = \min_i \{v_{ij}\}, \quad j = 1, 2, \dots, m \quad (10)$$

Step 4. The distance of each alternative from A^+ and A^- are calculated

$$D_i^+ = \sqrt{\sum_{j=1}^m (v_j^+ - v_{ij})^2} \quad D_i^- = \sqrt{\sum_{j=1}^m (v_j^- - v_{ij})^2} \quad (11)$$

Step 5. The closeness coefficient of each alternative is calculated

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-} \quad 0 \leq C_i \leq 1 \quad (12)$$

Step6. By comparing C_i values, the ranking of alternatives are determined.

Application of FAHP-TOPSIS Method to Multi-criteria Comparison of Grindability

Granite Materials and Parameters. Test workpieces are selected typical granite materials. The SiO_2 content, quartz content, Shore hardness, density, compressive strength, flexural strength and abrasion resistance are as the most granite important characteristics that affect grindability. In order to get the universal research conclusion, Ten kinds of granite that widely used are chosen to experiment. these parameters are shown in Table 1.

Table 1 Material parameters of granites

Granite type	SiO_2 [%]	Quartz content [%]	Shore hardness [HSD]	Density [g·cm ⁻³]	Compressive strength [MPa]	Flexural strength [MPa]	Abrasion resistance [g·cm ⁻²]
Wulian Red	67.55	24.3	85	2.68	92.89	8.542	0.6
Cherry Blossom Red	75.25	45.06	104	2.58	149.9	14.1	2.572
Shidao Red	64.29	44.19	110	2.7	234.37	13.63	2.68
Qilu Red	69.01	29.7	87	2.661	162	13.97	3.822
Liubu Red	75.64	43.36	60.4	2.61	203.97	20.89	1.607
Laoshan Red	71.88	29.76	99.7	2.59	208.29	18.06	3.828
Lu Grey	67.25	47.11	104	2.58	149.9	14.1	2.735
Marshal Red	70.22	25.2	90	2.65	147.88	12.87	3.76
China Grey	70.19	38.41	115	2.65	214.40	15.00	3.036
Wulian Flower	62.28	22	85	2.65	90.77	8.35	5.473

Determination of Criteria Weights. The fuzzy judgment matrix is established about SiO_2 content(C1), quartz content(C2), Shore hardness(C3), density(C4), compressive strength(C5), flexural strength(C6) and abrasion resistance(C7) using pair-wise comparison. According to the grinding process goal of granite, the weights for the parameters of granites are analyzed. A comprehensive triangular fuzzy pair-wise comparison matrix is built as in Table 2.

Table 2 Triangular fuzzy pair-wise comparison matrix

	C1	C2	C3	C4	C5	C6	C7
C1	(1, 1, 1)	(1/5, 1/4, 1/3)	(1, 2, 3)	(3, 4, 5)	(1/4, 1/3, 1/2)	(1/3, 1/2, 1)	(2, 3, 4)
C2	(3, 4, 5)	(1, 1, 1)	(4, 5, 6)	(6, 7, 8)	(1, 2, 3)	(2, 3, 4)	(5, 6, 7)
C3	(1/3, 1/2, 1)	(1/6, 1/5, 1/4)	(1, 1, 1)	(2, 3, 4)	(1/5, 1/4, 1/3)	(1/4, 1/3, 1/2)	(1, 2, 3)
C4	(1/5, 1/4, 1/3)	(1/8, 1/7, 1/6)	(1/4, 1/3, 1/2)	(1, 1, 1)	(1/7, 1/6, 1/5)	(1/6, 1/5, 1/4)	(1/3, 1/2, 1)
C5	(2, 3, 4)	(1/3, 1/2, 1)	(3, 4, 5)	(5, 6, 7)	(1, 1, 1)	(1, 2, 3)	(4, 5, 6)
C6	(1, 2, 3)	(1/4, 1/3, 1/2)	(2, 3, 4)	(4, 5, 6)	(1/3, 1/2, 1)	(1, 1, 1)	(3, 4, 5)
C7	(1/4, 1/3, 1/2)	(1/7, 1/6, 1/5)	(1/3, 1/2, 1)	(1, 2, 3)	(1/6, 1/5, 1/4)	(1/5, 1/4, 1/3)	(1, 1, 1)

Table 3 Results of priority weights and standardized weights

	C1	C2	C3	C4	C5	C6	C7
Priority weights	0.1301	1	0	0	0.7427	0.4533	0
Standardized weights	0.0559	0.4299	0	0	0.3193	0.1949	0

The triangular fuzzy synthesis values S_i are calculated by using Eq. (1), The fuzzy values are compared by using Eq. (3), and the values of V are obtained. Then, priority weights are calculated by using Eq. (4). After normalizing the priority weights the standardized weights are extracted, the results of priority weights and standardized weights are shown in table 3.

Ranking the grindability of granite. The weights of C3, C4 and C7 are zero from table 3, it means that these parameters are nonobviously to affect the grindability of granite. The greatest significance parameters of C1, C2, C5 and C6 are selected to rank the grindability of granite. Decision matrix is normalized via Eq. (7) and weighted normalized decision matrix is formed by using Eq. (8). Positive and negative ideal solutions are determined by taking the maximum and minimum values for each criterion via Eqs. (9) and (10):

$$A^+ = \{0.0192, 0.1735, 0.1376, 0.0894\}, \quad A^- = \{0.0158, 0.0810, 0.0533, 0.0357\}.$$

Then, the distance of each method from PIS (positive ideal solution) and NIS (negative ideal solution) with respect to each criterion are calculated, with the help of Eq. (11). Then, closeness coefficient of each granite is calculated by using Eq. (12) and the ranking of the granites are determined according to these values. The grindability ranking of granites are also shown in table 4.

Table 4 Rankings of the grindability of granites according to C_i values

Granite type	D^+	D^-	C_i	Rank
Wulian Red	0.1092	0.0454	0.2936	8
Cherry Blossom Red	0.0580	0.0951	0.6213	5
Shidao Red	0.0330	0.1196	0.7838	2
Qilu Red	0.0824	0.0560	0.4046	7
Liubu Red	0.0226	0.1162	0.8375	1
Laoshan Red	0.0668	0.0855	0.5614	6
Lu Grey	0.0575	0.1019	0.6391	4
Marshal Red	0.1013	0.0405	0.2857	9
China Grey	0.0424	0.0987	0.6993	3
Wulian Flower	0.1362	0.0001	0.0005	10

Conclusions

(1) The grindability is affected by the SiO_2 content, quartz content, compressive strength and flexural strength of the granite.

(2) The criteria of grindability is affected by the above-mentioned factors in a different trend. So, the rights of the factors are distributed in different ways for evaluating the grindability using different criteria.

(3) This new ranking method of granite grindability by means of fuzzy mathematics is reasonable and acceptable. For evaluating the grindability of a new granite type, only the petrographic analysis and mechanical property testing instead of a number of grinding tests are needed to obtain the information about the grindability prediction.

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