

Dynamic analysis of nonlocal-gradient elastic nano-beams resting on an elastic foundation

Jianshe Peng^{1, a}, Liu Yang^{1, b}, Fan Lin²

¹School of Mechanical Engineering, Cheng Du University, Cheng Du, 610106, China

²School of Mechanical Engineering and Automation, Xi Hua University, Cheng Du, 610039, China

^apengjianshe2005@163.com, ^b184312526@QQ.com

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Abstract. Dynamic analysis of nonlocal-gradient elastic nano-beams resting on an elastic foundation is investigated in this paper. The nonlocal-gradient elastic beam model, which has two independent gradient coefficients, based on the classical nonlocal elasticity theory and strain gradient theory, can be interpreted the size effect. By employing Galerkin method, the nonlinear partial differential governing equation is decoupled into a set of nonlinear ordinary differential equations which are then solved using Runge-Kutta method. Influence of elastic foundation coefficients and two independent gradient coefficients on the dynamic response of the nano-beam are investigated.

Introduction

Nano-beams are important building blocks in nano-electro-mechanical-systems (NEMS). The size scales associated with nanotechnology are too small to call the applicability of classical scale independent continuum models into question for NEMS structures because they cannot capture the small scale effect observed in both experiments and molecular dynamics simulations [1]. Size-dependent continuum theories have thus attracted increasing attention in modelling nano-structures and devices. Among these, nonlocal theory developed by Eringen [2, 3] has gained wide acceptance, which introduces the small scale effect through a spatial integral constitutive relation and allows for the stress at a point to be dependent on strains at all points in the elastic body. Gradient theory, another size-dependent continuum theory, was introduced by Aifantis and his coworkers [4] in the beginning of 1980s. A mixed nonlocal-gradient elasticity beam model with two independent gradient coefficients based on the classical nonlocal elasticity theory and strain gradient theory is proposed by Jun Shen and Xian Fang Li [5] to study wave propagation in single- and double- walled carbon nanotubes. Abu-Salih and Elata [6] analyzed the electromechanical buckling of a pre-stressed layer bonded to an elastic foundation. The effect of stiffening and softening elastic foundations on the post buckling behavior of the system is discussed.

In this paper, the governing equation of electrostatically actuated nano-beams resting on an elastic foundation has been obtained based on the nonlocal-gradient elastic theory, and then solved by Galerkin theory and Runge Kutta method. Numerical results show that the deflection response and frequency response of nano-beams are all affected by elastic foundation coefficients and two independent gradient coefficients.

Theoretical formulation

Linear foundation term is added to the equation of motion given in Ref. [5,7]. Hence, the governing equation of a nano-beam is

$$EI(1-l_2^2 \frac{\partial^2}{\partial \bar{x}^2}) \frac{\partial^4 \bar{w}}{\partial \bar{x}^4} + rA(1-l_1^2 \frac{\partial^2}{\partial \bar{x}^2}) \frac{\partial^2 \bar{w}}{\partial \bar{t}^2} + k\bar{w} = (1-l_1^2 \frac{\partial^2}{\partial \bar{x}^2})q, \quad (1)$$

where \bar{w} is the resulting transverse displacement of the nano-beam, \bar{x} is the coordinate value along the axial direction, \bar{t} is the time, l_1 and l_2 are the independent gradient coefficients, respectively, E

is the Young's modulus, I is the second moment of inertia of the beam, r is the mass density, A is the cross-sectional area, L is the length, and q is the load, respectively. k is the linear coefficient of the foundation.

Ignoring the fringing field effect, the electrostatic force per unit length of the beam is [8]

$$q(\bar{t}) = \frac{e_0 V^2 b}{2(g - \bar{w})^2}, \quad (2)$$

where e_0 is the vacuum dielectric constant, V is the voltage, b is the width, g is the initial gap between the substrate and the beam.

It can be derived from equations (1) and (2) that the governing equation of the nano-beam is

$$EI(1-l_2^2 \frac{\partial^2}{\partial \bar{x}^2}) \frac{\partial^4 \bar{w}}{\partial \bar{x}^4} + rA(1-l_1^2 \frac{\partial^2}{\partial \bar{x}^2}) \frac{\partial^2 \bar{w}}{\partial \bar{t}^2} + k\bar{w} = (1-l_1^2 \frac{\partial^2}{\partial \bar{x}^2}) \frac{e_0 V^2 b}{2(g - \bar{w})^2}. \quad (3)$$

The dimensionless quantities are introduced to facilitate the following theoretical formulations

$$x = \frac{\bar{x}}{L}, t = \bar{w} \bar{t}, \Omega = \frac{w}{w_L}, w_L = \frac{1}{L^2} \sqrt{\frac{EI}{rA}}, w = \frac{\bar{w}}{g}, a_1 = \frac{l_1}{L}, a_2 = \frac{l_2}{L}, a_3 = \frac{kL^4}{EI}, b = \frac{e_0 V^2 b L^4}{2EIg^3}. \quad (4)$$

Eq. (3) can then be rewritten in dimensionless form as

$$\begin{aligned} & a_2^2 (1-w)^4 \frac{\partial^6 w}{\partial x^6} - (1-w)^4 \frac{\partial^4 w}{\partial x^4} + \Omega^2 a_1^2 (1-w)^4 \frac{\partial^4 w}{\partial t^2 \partial x^2} - \Omega^2 (1-w)^4 \frac{\partial^2 w}{\partial t^2} + a_3 (1-w)^4 w \\ & = b [a_1^2 (\frac{\partial w}{\partial x})^2 - a_1^2 (1-w) \frac{\partial^2 w}{\partial x^2} - (1-w)^2]. \end{aligned} \quad (5)$$

Solution method

The deflection of the nano-beam $w(x, t)$, can be separated into temporal and spatial by function $u(t)$ and $q(x)$, respectively, in the form of a series of products

$$w(x, t) = u_i(t) q_i(x), \quad (6)$$

where $u_i(t)$ is the i th generalized coordinate and $q_i(x)$ is the i th linear un-damped mode shape of the straight nano-beam.

The mode shape for the clamped-clamped beam takes the form of

$$q_i(x) = 1 - \cos 2ipx. \quad (7)$$

Substituting Eq. (6) into Eq. (5) then applying Galerkin method gives

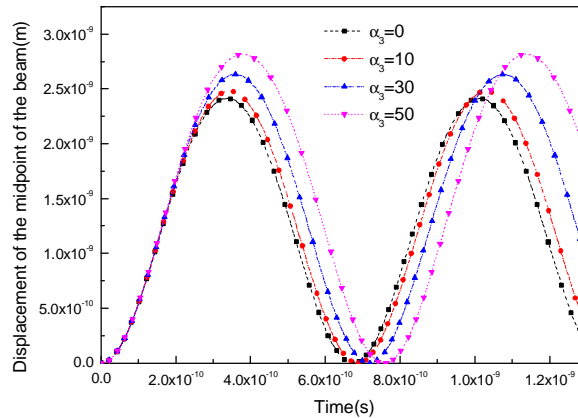


Fig.1 Mid-span deflection for various a_3 values ($a_1 = 0.2, a_2 = 0.1$, Voltage=5V)

$$\begin{aligned}
& u_i^{(2)} \Omega^2 \left[a_1^2 \int_0^1 q_i q_i^{(2)} dx - 4u_i a_1^2 \int_0^1 q_i^2 q_i^{(2)} dx + 6u_i^2 a_1^2 \int_0^1 q_i^3 q_i^{(2)} dx - 4u_i^3 a_1^2 \int_0^1 q_i^4 q_i^{(2)} dx \right. \\
& \left. + u_i^4 a_1^2 \int_0^1 q_i^5 q_i^{(2)} dx - \int_0^1 q_i^2 dx + 4u_i \int_0^1 q_i^3 dx - 6u_i^2 \int_0^1 q_i^4 dx + 4u_i^3 \int_0^1 q_i^5 dx - u_i^4 \int_0^1 q_i^6 dx \right] \\
& + u_i \left[a_2^2 \int_0^1 q_i q_i^{(6)} dx - \int_0^1 q_i q_i^{(4)} dx + b a_1^2 \int_0^1 q_i q_i^{(2)} dx - 2b \int_0^1 q_i^2 dx + a_3 \int_0^1 q_i^2 dx \right] \\
& + u_i^2 \left[-4a_2^2 \int_0^1 q_i^2 q_i^{(6)} dx + 4 \int_0^1 q_i^2 q_i^{(4)} dx - b a_1^2 \int_0^1 q_i (q_i')^2 dx - b a_1^2 \int_0^1 q_i^2 q_i^{(2)} dx + b \int_0^1 q_i^3 dx - 4a_3 \int_0^1 q_i^3 dx \right] \\
& + u_i^3 \left[6a_2^2 \int_0^1 q_i^3 q_i^{(6)} dx - 6 \int_0^1 q_i^3 q_i^{(4)} dx + 6a_3 \int_0^1 q_i^4 dx \right] \\
& + u_i^4 \left[-4a_2^2 \int_0^1 q_i^4 q_i^{(6)} dx + 4 \int_0^1 q_i^4 q_i^{(4)} dx - 4a_3 \int_0^1 q_i^5 dx \right] \\
& + u_i^5 \left[a_2^2 \int_0^1 q_i^5 q_i^{(6)} dx - \int_0^1 q_i^5 q_i^{(4)} dx + a_3 \int_0^1 q_i^6 dx \right] \\
& + b \int_0^1 q_i dx = 0
\end{aligned} \tag{8}$$

It has been solved by Runge-Kutta method, and the result is discussed at next section.

Numerical results

To validate the present analysis, the dynamic response of a nano-beam clamped at both ends under a step voltage is considered, with the following geometric and material properties [9] $L = 130\text{nm}$, $b = 4\text{nm}$, $h = 3.5\text{nm}$, $g = 10\text{nm}$, $E = 186.6\text{Gpa}$, $e_0 = 8.854 \times 10^{-12}$ Farads/ m.

Figs.1-3 display the time histories of the mid-span deflection of the clamped-clamped nano-beam under different coefficients and step voltages. As shown in Fig.1 with increasing the coefficient a_3 , midpoint deflection of the beam increases and natural frequency decreases. As shown in Fig.2 with

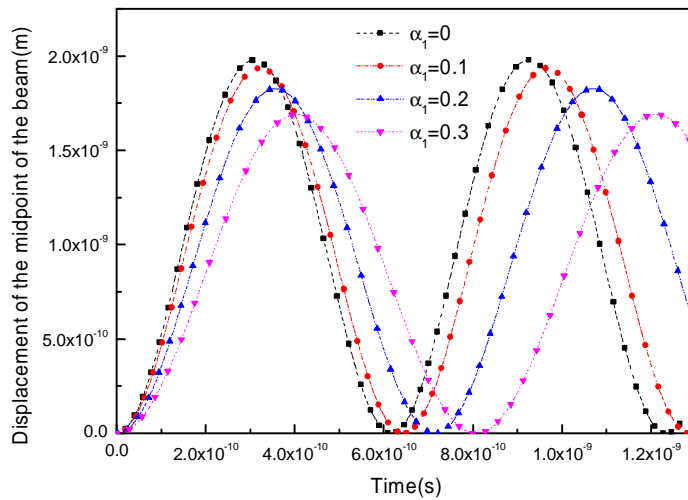


Fig.2 Mid-span deflection for various a_1 values ($a_2 = 0.05$, $a_3 = 10$, Voltage = 4V)

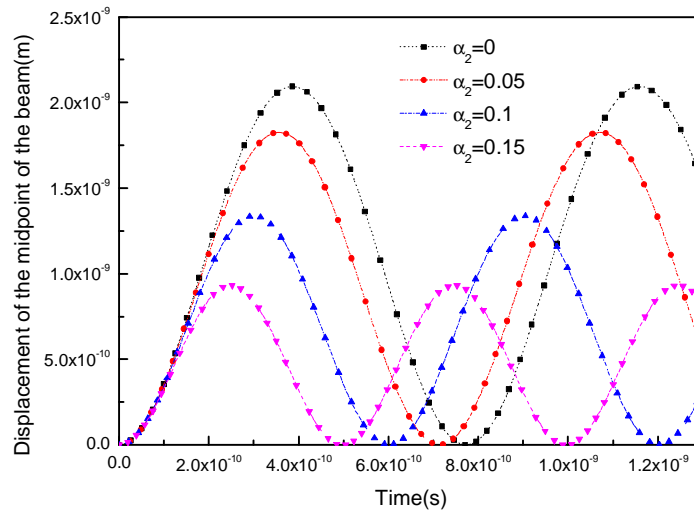


Fig.3 Mid-span deflection for various a_2 values ($a_1 = 0.2, a_3 = 10$, Voltage=4V)

increasing the gradient coefficient a_1 , midpoint deflection and natural frequency of the beam decreases. As shown in Fig.3 with increasing the gradient coefficient a_2 , midpoint deflection of the beam decreases and natural frequency increases.

Conclusions

The dynamic analysis of nano-beams resting on an elastic foundation is investigated in this paper based on the nonlocal-gradient elastic theory. The governing equation is solved by using Galerkin method and Runge-Kutta method. It is found from the numerical results that the midpoint deflection of the beam increases and natural frequency decreases with increasing the coefficient a_3 , the midpoint deflection and natural frequency of the beam decreases with increasing the gradient coefficient a_1 and the midpoint deflection of the beam decreases and natural frequency increases with increasing the gradient coefficient a_2 . Results provide a reference for the choice of design and industrial applications of such nano-beams bonded to an elastic foundation.

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