

## Flexural vibration reduction of phononic crystal floating bridge

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**Abstract:** The phononic crystal (PC) concept is introduced to the floating bridge, in order to reduce the flexural vibrations. Four kinds of PC floating bridge models, including the identically hinged and two-component rigid connected, section-hinged, cell-hinged cases are defined. The corresponding structures with the actual available bridge section size are constructed. And the frequency dispersion relations and amplitude frequency responses of these structures are calculated and analyzed. The study shows that, the four kinds of PC floating bridges all have good band gap (BG) properties. Due to the restraints of foundation, the PC floating bridges have the first BGs from 0Hz to the corresponding pivotal frequencies. For the identically hinged and two-component cell-hinged cases, the pivotal frequencies are submerged in the adjacent BGs, so they have wide BGs start from 0Hz. Moreover, compared with the rigid connected case, the existence of hinges helps to obtain wider BGs with stronger attenuation. Finally, we design a combined floating bridge from hinged connecting the identically hinged and two-component cell-hinged cells and realize extended wide BGs.

### Introduction

The floating bridge is a kind of ancient facilities for crossing river. Compared with the permanent bridge, the floating bridge could be constructed quickly and not permanently occupy the surrounding place. Many floating bridges have been constructed in USA, UK, Canada, Norway, etc., following the first modern steel floating bridge which was completed in Istanbul at 1912 [1,2]. The floating bridge has very important usage in both civil and military applications, especially for cases of soft river bed, high depth of water, as well as the emergency.

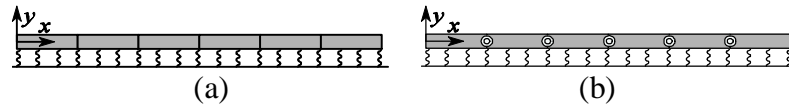
The floating bridge floats on water, and make use of buoyancy of water to support loads. As a kind of structure with relatively good flexibility, flexural vibrations caused by vehicles, wind and wave would affect its reliability. Many researches have been done in order to investigate its dynamic behaviors induced by wave, moving loads and other environmental loads [3-7]. The dynamic behaviors and vibration control methods of these widely used structures are constantly concerned. How to effectively eliminate the flexural vibrations of floating bridge is a meaningful topic.

In recent years, the phononic crystals (PCs) which have periodically arrayed composite materials have caused much attention [8-11]. PC has many attractive properties, one of them is the existence of band gaps (BG), within which there can be no propagation of elastic waves. The BG feature could be applied to many fields, such as acoustic insulation [8] vibration reduction [12,13] and vibration control [14,15], etc. We believe that the combination of PC and the floating bridge is a possible way to eliminate and control flexural vibrations in it, by using the BG properties.

In this paper, we first define four kinds of PC floating bridge models, and briefly introduce the transfer matrix (TM) method for the calculation of frequency dispersion relation and BG ranges. Then we construct the corresponding models with the actual available bridge section size, and analyze the frequency dispersion relations and amplitude frequency responses of them. Finally, we design a combined floating bridge from hinged connecting two kinds of models and realize extended wide BGs.

## Models of the PC floating bridge

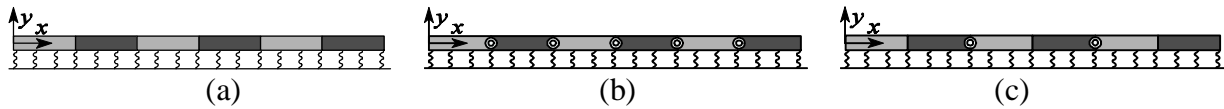
The floating bridge takes advantage of the law of buoyancy of water to support the loads on it. Generally, floating bridge could be simplified as the model of the beam on an elastic foundation. For floating bridge, Winkler model [16,17] is an appropriate elastic foundation model and often adopted, because water cannot withstand shear and the interaction of buoyancy of water could just be considered as lots of springs. The stiffness of foundation of water  $c=\rho g\approx 1.0\times 10^4\text{N/m}^3$ , where  $\rho$  is the density of water, the acceleration of gravity  $g\approx 10\text{m/s}^2$ . For convenient construction, the floating bridge is composed of several same sections. The connection between two adjacent sections could be simplified as two types, which are the rigid connection and hinge. Figure 1 shows the models with two kinds of connections.



**Fig. 1.** The floating bridges modelled by the (a) rigid connected and (b) hinged beams on Winkler foundation.

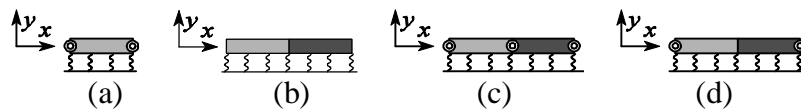
We consider that a bridge section is simplified as a beam composed of identical homogeneous material. So the rigid connected floating bridge is actually an extended homogeneous beam on water. And the hinged floating bridge is a periodical structure, because of the periodically hinged positions. For the periodicity construction along the propagation direction of flexural vibrations from one end to the other, the hinged floating bridge could be considered as a kind of PC.

Besides the periodicity of hinged position, surely, we could also easily introduce the periodicity to the floating bridge from adding another kind of material. Figure 2 shows three kinds of two-component PC floating bridges, including one rigid connected case and two hinged cases, after adding the periodicity of materials. The periodicities of hinged position and materials give two kinds of hinged PC floating bridges, which we call the section-hinged case and cell-hinged case in Fig. 2(b)~(c).



**Fig. 2.** The (a) rigid connected, (b) section-hinged and cell-hinged floating bridges with the periodicity of two kinds of materials.

So we give four kinds of PC floating bridges above. When we discuss the PC problems, the idealized model with infinite periodical cells should be analyzed, in order to obtain the frequency dispersion relation and BGs ranges. For infinite structures, we could just analyze the cell, which contains all the informations of the whole structure. Figure 3 shows the four cells corresponding to the four kinds of PC floating bridges above.



**Fig. 3.** The cells corresponding to the (a) indentically hinged and two-component (b) rigid connected, (c) section-hinged, (d) cell-hinged PC floating bridges.

Although we have different models, actually, each section with identical material is a homogeneous Euler-Bernoulli beam on a Winkler foundation. Based on the TM method for the calculation of the flexural vibration frequency dispersion relation of the rigid connected PC beam-foundation system [18,19], at each cross section, the rigid connected case has four independent degrees of freedom (DOFs), which are the displacement  $v(x)$ , rotation angle  $\theta(x)$ , flexural moment  $M(x)$  and shear force  $Q(x)$ . For a periodic length  $a$ , the transfer relation can be written as

$$\mathbf{F}(a)=\mathbf{TF}(0), \quad (1)$$

where  $\mathbf{F}(a)=[v(a) \theta(a) M(a) Q(a)]^T$ ,  $\mathbf{F}(0)=[v(0) \theta(0) M(0) Q(0)]^T$ .

For the hinged case, the flexural moment could not pass over a hinge and the rotation angles around a hinge are not continuous anymore. So the DOFs just become to the displacement and shear force, for the hinged PC floating bridges. Applying these two conditions, the transfer relation can be rewritten as

$$\mathbf{F}'(a)=\mathbf{T}'\mathbf{F}'(0), \quad (2)$$

where  $\mathbf{F}'(a)=[v(a) Q(a)]^T$ ,  $\mathbf{F}'(0)=[v(0) Q(0)]^T$ .

The eigenvalue problems which contain the frequency dispersion relations of the indently hinged and two-component rigid connected, section-hinged and cell-hinged PC floating bridges could be given, after using the Bloch's theorem [20]. Solving the eigenvalue problems, the corresponding frequency dispersion relations and BGs ranges could be obtained.

## Results and Discussion

### Band gaps

We consider the models of PC floating bridges with the actual available section size. The section length  $l=8.0\text{m}$ . The cross section is simplified as a rectangular, which has the width  $b=8.0\text{m}$  and height  $h=1.5\text{m}$ . So we give the cross sectional moment of inertia  $I=bh^3/12=2.25\text{m}^4$ . We choose two kinds of materials, aluminum and epoxy resin. The density and elastic modulus of aluminum are  $\rho_{Al}=2730\text{kg/m}^3$ ,  $E_{Al}=7.76\times 10^{10}\text{Pa}$ , and these of epoxy resin are  $\rho_{Ep}=1180\text{kg/m}^3$ ,  $E_{Ep}=4.35\times 10^9\text{Pa}$ .

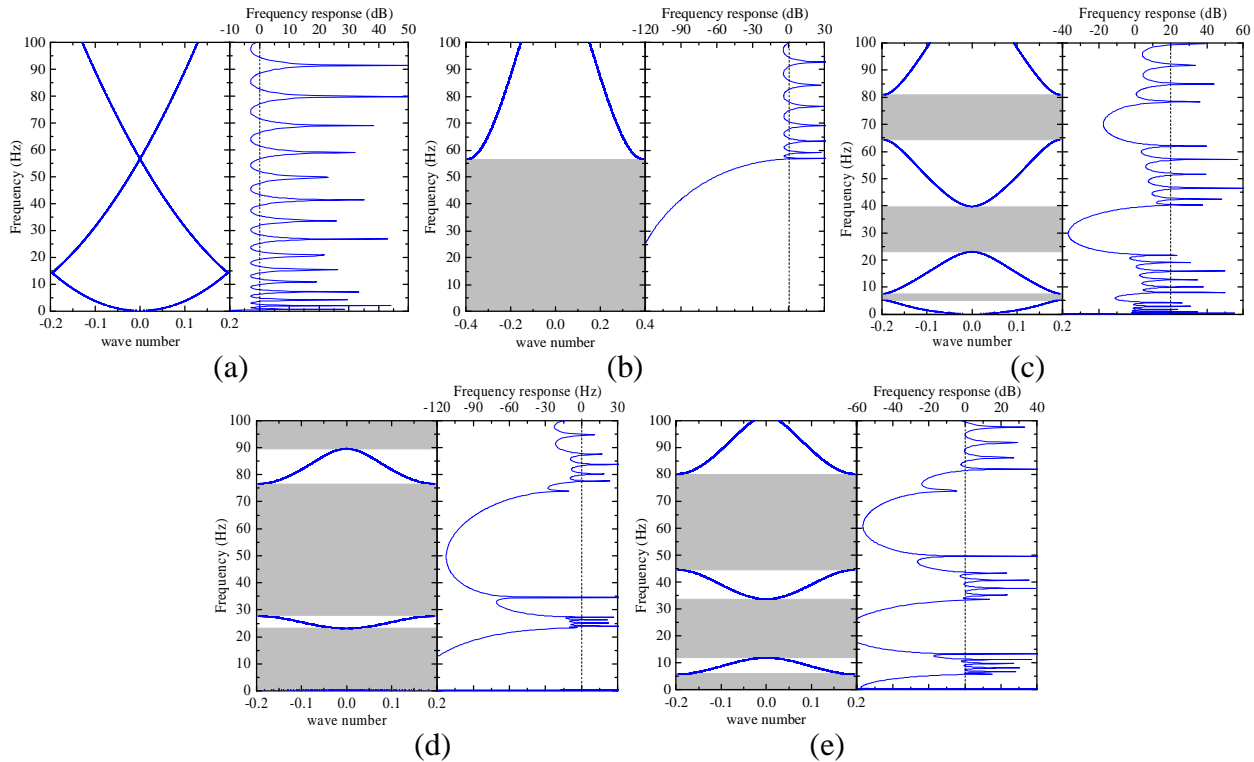
Then, we construct the rigid connected and identically hinged floating bridges with aluminum. The identically hinged floating bridge has the periodic length  $a=8.0\text{m}$ . We also could consider that the identically rigid connected floating bridge has the same periodic length, so its frequency dispersion relation could be similarly obtained for comparison. Next, we add epoxy resin to the models, and give the two-component rigid connected, section-hinged and cell-hinged PC floating bridges. For two-component cases, the periodic length  $a=16.0\text{m}$ .

We calculate the frequency dispersion relations of the five cases in the frequency range of 0~100Hz and wave number range of  $-2\pi/a\sim 2\pi/a$ . which are shown in the left panels of Fig. 4(a)~(e). For the identically rigid connected case, one BG exists, the range is 0~0.25Hz. For the identically hinged case, also one BG exists, the range is 0~56.66Hz. For the two-component rigid connected case, there are four BGs, which are 0~0.30Hz, 5.31~7.37Hz, 23.09~39.60Hz and 64.58~80.58Hz. For the two-component section-hinged case, there are also four BGs, which are 0~0.27Hz, 0.33~23.10Hz, 27.65~76.58Hz and 89.55~100Hz. For the two-component cell-hinged case, there are three BGs, which are 0~5.73Hz, 11.89~33.59Hz and 44.57~80.03 Hz.

We also consider the actual finite structures. Based on the same numbers of bridge sections, we calculate the amplitude frequency responses corresponding to the above five cases, which are shown in the right panels of Fig. 4(a)~(e). The five cases are all composed of 12 sections. We apply the harmonic transverse displacement impulses which sweeps over the range of 0~100Hz to one end of the finite structure, and then get the frequency response at the other end. One can see that, in the BGs ranges, the flexural vibrations have distinct attenuations. And evidently, the BGs ranges obtained by frequency dispersion relations and frequency response results have a good agreement. So the BGs results could be reciprocally verified.

For an identical beam on Winkler foundation, because of the restraints of elastic foundations, a pivotal frequency exists. There is no flexural vibration between 0 Hz and the pivotal frequency. So for the identically rigid connected case, one BG with a range of 0~0.25Hz exists. For the PC cases, the pivotal frequencies also exist. But due to the different connections and materials, the pivotal frequencies are different. Especially for the identically hinged and cell-hinged cases, the pivotal frequencies are even submerged in the adjacent BGs. So the two cases have wide first BG which starts from 0Hz. Besides the first BG which determined by the foundation constraint, the hinged cases have much wider BGs than the rigid connected case. The reason is that the existence of hinge relaxes constraints which damages the integrality of the beam in the flexural motion and decreases the flexural

motion modes. This causes each dispersion curve being narrow, so the BGs ranges are wider. In addition, considering the frequency response results, in the BGs, the attenuations of the hinged cases are surely much stronger. That means, in the applications, the hinged PC floating bridge composed of just few cells could also have clear BG properties.

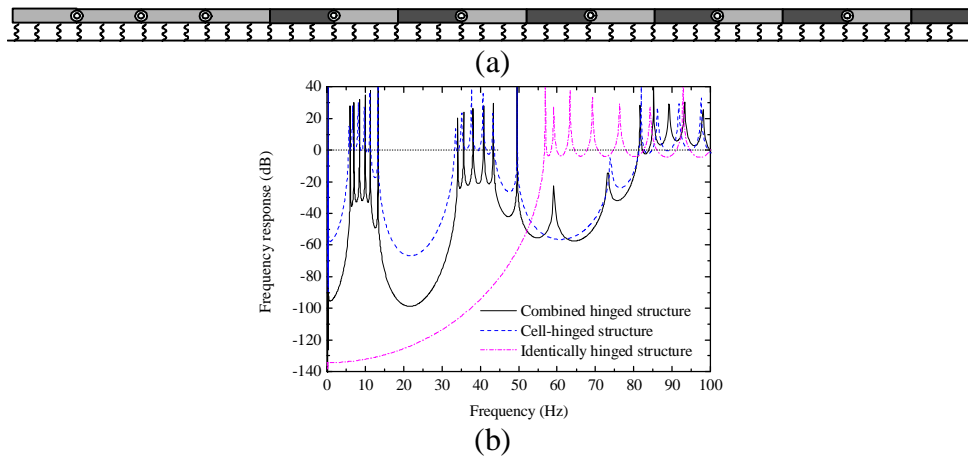


**Fig. 4.** The frequency dispersion relations (left) and amplitude frequency responses (right) of the (b) identically hinged and two-component (c) rigid connected, (d) section-hinged, (e) cell-hinged PC floating bridges, as well as the (a) identically rigid connected case. Aluminum is used in the one-material cases, aluminum and epoxy resin are used in the two-component cases. The BGs are marked as the shadow areas.

### Combined hinged PC floating bridge with extended BG

It is possible to realize wider BGs, from combining different PC floating bridges with complementary BGs [21,22]. According to the BGs ranges of the four PC floating bridge models, while the BGs of the identically hinged and two-component cell-hinged cases combine together, we could get an extended wide BG which just cover 0~80.03 Hz. So we design a combined floating bridge from hinged connecting the three identically hinged cells and six two-component cell-hinged cells, which is shown in Fig. 5(a).

We obtain the amplitude frequency response at the right end, from applying the harmonic transverse displacement impulses from 0~100Hz at the left end, which is shown in Fig. 5(b). We also give the above frequency responses results of the identically hinged and two-component cell-hinged cases for comparison. One can see that, the combined hinged structure has relatively significant BG property. Almost all of the flexural vibration impulses in 0~80Hz cannot pass through the structure, except few frequencies. The nonoverlapping BGs of the two substructures obviously decrease the opponent's flexural vibrations respectively. So the original flexural vibrations which could propagate in 0~80Hz all get attenuations to a certain degree. The extended wide BG would effectively eliminate the flexural vibrations in combined hinged PC floating bridge in a wide frequency range.



**Fig. 5.** (a) The floating bridge model constructed from hinged connecting the three identically hinged cells and six two-component cell-hinged cells and (b) its flexural vibration frequency responses. The frequency responses of the identically hinged and two-component cell-hinged cases are also given for comparison.

## Conclusion

We introduce the PC concept into the floating bridge in order to find the feasible way to eliminate and control the flexural vibrations in the floating bridge. We propose four kinds of PC floating bridges, including the identically hinged and two-component rigid connected, section-hinged, cell-hinged cases, and study the flexural vibration characteristics of the corresponding structures with the actual available bridge section size. The frequency dispersion relations and amplitude frequency responses are both analyzed. We find that the four kinds of PC floating bridges all have good BG properties. Because of restraints of foundation, the PC floating bridges have the first BG from 0Hz to the corresponding pivotal frequencies. However, for the identically hinged and cell-hinged cases, the pivotal frequencies are submerged in the adjacent BGs. So the two cases have wide BGs start from 0Hz. Furthermore, compared with the rigid connected case, the existence of hinge helps to obtain wider range of BGs in which the attenuation is also much stronger. According to the BGs ranges of different cases, we design a combined floating bridge from hinged connecting the identically hinged and two-component cell-hinged cells and realize extended wide BGs. Our study provides a possible effective way to eliminate flexural vibrations in the floating bridge.

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