Dynamic analysis for shallowly buried elliptic inclusion near interfacial crack impacted by SH waves in bi-material half space

Xiao-hao DING[a], Hui QI[b,*]

College of Aerospace and Civil Engineering, Harbin Engineering University, Harbin, China

[a]email:dingxiaohao_2012@sina.cn, [b]email:qihui205@sina.com

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Abstract. Base on elastodynamics, complex function method with mapping function and Green's function method are used to investigate the scattering of SH waves and dynamic stress concentration in bi-material half space with elliptic inclusion and interfacial crack. Firstly, the bi-material half space is divided into two parts along the vertical interface. Secondly, "image" method and conformal mapping method are employed to construct the scattering wave field in part I. Then the Green's function needed is obtained. Thirdly, the interfacial crack is created by using crack dividing technique With the aid of interface "conjunction" technique, a series of integral equations for determining the unknown force system could be set up through continuity conditions on the interface and Green's function. Finally, some examples for the dynamic stress concentration factor (DSCF) around the elliptic elastic inclusion are presented. Numerical results show that the interfacial crack and vertical interface are indeed capable of affecting the distribution of DSCF.

Introduction

It is normal that various defects exist in nature or artificial materials, which always attracts researchers and engineers. In the study of the scattering of elastic plane waves by complex terrain[1-10], canyons[1], cavities[2,5], hills[4] and inclusions[6,7] are the most common types. Many researches on complex terrain have been carried out and many valuable results have been obtained to satisfy the theoretical and engineering needs in the recent decades. The research work of scattering problem of elastic waves has important application value in many subjects and fields, such as geological prospecting, soil dynamics, civil engineering, seismic engineering, nondestructive testing, etc.

In the field of civil engineering or seismic engineering, interface between the different soils is common and inevitable. Many researches on interfacial dynamic and elliptical inclusion belong to full space problem of interface [8] and isotropic half-space problem [9,10]. Fewer published papers are given to the scattering problem by elliptical inclusions near a vertical interface crack in bi-material half space. In this paper, the problem has been investigated.

Theoretical model

Fig.1 shows a bi-material half-space with an elliptical inclusion of half-macroaxis a and half-brachyaxis b and an interfacial crack impacted by SH waves. Its material properties are given by the shear modulus m and mass body density r. The distances from the center of elliptical inclusion to the vertical interface and horizontal surface are d and h, respectively. The length of interfacial crack is 2A, the depth of crack center point o'' is l, the incident angle is a_0.

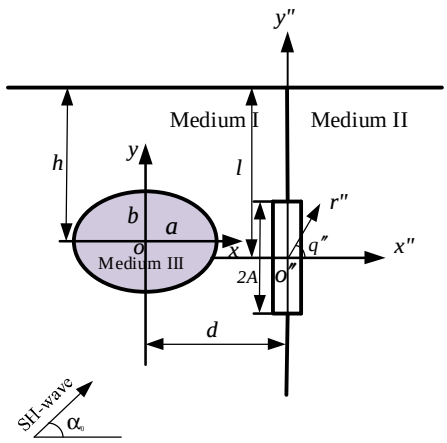


Fig.1 Theoretical model

Green's function

Green's function used in this paper is an essential solution of displacement field for a quarter-plane with an elliptical elastic inclusion by anti-plane harmonic line source $d(z-z_0)$ loading at vertical surface. The displacement function G must satisfy the governing equation with omitted the time harmonic factor $\exp(-iwt)$ of the following form:

$$\frac{\partial^2 G}{\partial z \partial \bar{z}} + \frac{1}{4}k^2 G = 0 \qquad (1)$$

In which, z and $\bar{z}$ are the complex variables, $k = w / c_s$ is wave number, w and $c_s = \sqrt{m / r}$ are the disturbing circular frequency and the shear velocity of the media, r and m are the mass body density and the shear modulus of the media, respectively.

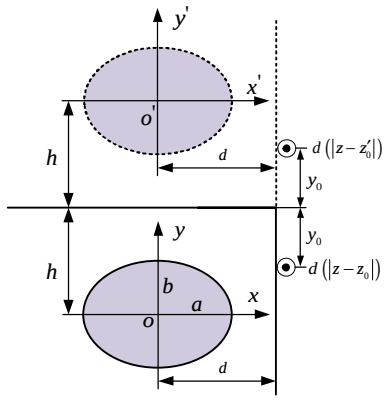


Fig.2 Model of imaginary point source

The stresses corresponding to Eq.(1) can be written as:

$$t_{rz} = m\left(\frac{\partial G}{\partial z}e^{iq} + \frac{\partial G}{\partial \bar{z}}e^{-iq}\right), t_{qz} = im\left(\frac{\partial G}{\partial z}e^{iq} - \frac{\partial G}{\partial \bar{z}}e^{-iq}\right) \qquad (2)$$

For wave scattering problems involving elliptical inclusion in the complex $(z,\bar{z})$ plane, it is possible to map the internal/external region of the elliptical inclusion into the inside/outside region of the circle (in the $(h,\bar{h})$ plane)[3].

Introducing the mapping function:

$$Z = w(h) = R(h + m/h)\ h = Re^{iq} \qquad (3)$$

The above mapping function map the outside of the inclusion in the $(z,\bar{z})$ plane into the region $|h| > 1$. Consequently, the corresponding governing Eq.(1) in $(h,\bar{h})$ plane takes on the following form:

$$\frac{1}{w'(h)\overline{w'(h)}}\frac{\partial^2 G}{\partial h \partial \bar{h}} + \frac{1}{4}k^2 G = 0 \qquad (4)$$

In $(h,\bar{h})$ plane, Eq.(2) can be written as:

$$t_r = \frac{m}{R|w'(h)|}\left(h\frac{\partial G}{\partial h} + \bar{h}\frac{\partial G}{\partial \bar{h}}\right),\ t_q = \frac{im}{R|w'(h)|}\left(h\frac{\partial G}{\partial h} - \bar{h}\frac{\partial G}{\partial \bar{h}}\right) \qquad (5)$$

The boundary conditions can be written as:

$$\begin{cases} \Gamma_1 : t_{xz}\big|_{x=d} = d(|z-z_0|) \\ \Gamma_2 : t_{yz}\big|_{y=h} = 0 \\ \Gamma_3 : t^{\mathrm{I}}_{rz}\big|_{|h|=1} = t^{\mathrm{III}}_{rz}\big|_{|h|=1}, G^{\mathrm{I}}\big|_{|h|=1} = G^{\mathrm{III}}\big|_{|h|=1} \end{cases} \qquad (6)$$

In which, $d(\bullet)$ is Dirac-Delta function and z_0 is the point where the line force is acting, G represents Green's function.

With the aid of image method, as Fig.2 shows, the displacement field produced by the line source force $d(z-z_0)$ on the vertical surface in a complete elastic quarter-plane can be written as the follow in the $(h,\bar{h})$ plane:

$$G^{(i)}(h,\bar{h}) = \frac{i}{2m}\left[H_0^{(1)}(k_1|w(h)-w(h_0)|) + H_0^{(1)}(k_1|w(h)-\overline{w'(h_0)}|)\right] \qquad (7)$$

Where, $H_0^{(1)}(\bullet)$ is the first kind Hankel function of 0 order , $w(h_0)=d+\mathrm{i}(h-y_0)$, $w'(h_0)=d+\mathrm{i}(h+y_0)$. $w(h_0)$ and $w'(h_0)$ represent the position of the vector of the source point and the image point respectively.

For the existence of the vertical surface, it is difficult to construct the scattering wave field to satisfy the stress-free conditions on the straight boundaries directly. In order to overcome the difficulty, image method and multi-polar coordinate transformation are employed to construct the scattering wave field which has the following form:

$$G^{(s)}(h,\bar{h})=\sum_{n=-\infty}^{\infty}A_n\sum_{j=1}^{4}S_n^{(j)} \tag{8}$$

in which,

$$S_n^{(1)}=H_n^{(1)}[k|w(h)|][\frac{w(h)}{|w(h)|}]^n\ ,\ S_n^{(2)}=H_n^{(1)}[k|w(h)-2ih|][\frac{w(h)-2ih}{|w(h)-2ih|}]^{-n}$$

$$S_n^{(3)}=(-1)^nH_n^{(1)}[k|w(h)-2d|][\frac{w(h)-2d}{|w(h)-2d|}]^{-n}\ ,\ S_n^{(4)}=(-1)^nH_n^{(1)}[k|w(h)-2ih-2d|][\frac{w(h)-2ih-2d}{|w(h)-2ih-2d|}]^{n}$$

And A_n is an unknown coefficient determined by the boundary conditions.

The standing wave field in elliptical inclusion can be written as:

$$G^{(t)}(h,\bar{h})=\sum_{n=-\infty}^{\infty}B_nJ_n(k_3|w(h)|)[\frac{w(h)}{|w(h)|}]^n \tag{9}$$

Where, B_n is an unknown coefficient determined by the boundary conditions, $k_3=w/c_3$ is shear wave number in inclusion, w and $c_3=\sqrt{m_3/r_3}$ are the disturbing circular frequency and the shear velocity of the medium III, and $\mathrm{Jn}(\bullet)$ is Bessel function of first kind.

In the $(h,\bar{h})$ plane, the Green's function in area I and area II can be expressed as follows:

$$\begin{cases}G_1=G^{(i)}+G^{(s)}\\ G_2=G^{(i)}\end{cases} \tag{10}$$

Analysis

Considering the existence of the free boundary and the interface in bi-material media, the image method is employed to transform right angle space to full space. Consider the full space model as an equivalent model with multiple wave sources. And the equivalent incident wave can be expressed as follows:

$$\begin{aligned}W^{(i,e)}=W_0\exp\{&\frac{\mathrm{i}k_1}{2}[(w(h)-\mathrm{i}h)e^{-\mathrm{i}a_0}+(\overline{w(h)}+\mathrm{i}h)e^{\mathrm{i}a_0}+(w(h)-\mathrm{i}h-2d)e^{-\mathrm{i}g_0}\\&+(\overline{w(h)}+\mathrm{i}h-2d)e^{\mathrm{i}g_0}]\}\end{aligned} \tag{11}$$

Where, $g_0=p-a_0$, a_0 and W_0 are incident angle and the amplitude of the incident wave respectively.

Similarly, the equivalent reflected wave and refracted wave are:

$$\begin{aligned}W^{(r,e)}=W_1\exp\{&\frac{\mathrm{i}k_1}{2}[(w(h)-ih)e^{-\mathrm{i}a_1}+(\overline{w(h)}+\mathrm{i}h)e^{\mathrm{i}a_1}+(w(h)-ih-2d)e^{-\mathrm{i}g_1}\\&+(\overline{w(h)}+\mathrm{i}h-2d)e^{\mathrm{i}g_1}]\}\end{aligned} \tag{12}$$

$$\begin{aligned}W^{(f,e)}=W_2\exp\{&\frac{\mathrm{i}k_2}{2}[(w(h)-\mathrm{i}h)e^{-\mathrm{i}a_2}+(\overline{w(h)}+\mathrm{i}h)e^{\mathrm{i}a_2}+(w(h)-\mathrm{i}h-2d)e^{-\mathrm{i}g_2}\\&+(\overline{w(h)}+\mathrm{i}h-2d)e^{\mathrm{i}g_2}]\}\end{aligned} \tag{13}$$

$g_1=p-a_1, g_2=p-a_2$, a_1 and a_2 are the reflection angle and refraction angle respectively.

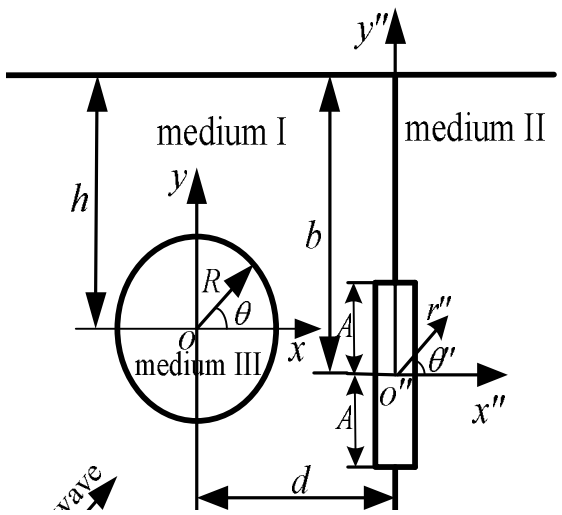


Fig.3 Conjunction model

As shown in Fig.3, the theoretical model is divided into two parts along the vertical interface. A pair of opposite forces with the amplitude $[-t^{(\mathrm{I})}_{q''z''}]$ and $[-t^{(\mathrm{II})}_{q''z''}]$ are applied to the left and right side of the section of the region where the crack will appear[6], resulting in a stress-free section as interfacial crack. Meanwhile, unknown force systems f_1 and f_2 are loaded on the sections outside the crack-section to satisfy the continuity conditions on the interface. So a series of Fredholm integral equations for determining the unknown forces can be set up.

The total displacements $W^{(\mathrm{I})}$, $W^{(\mathrm{II})}$ and total stresses $t^{(\mathrm{I})}_{q''z''}$, $t^{(\mathrm{II})}_{q''z''}$ in the two parts are:

$$\begin{cases} W^{(\mathrm{I})} = W^{(i,e)} + W^{(r,e)} + W^{(s)}, \ W^{(\mathrm{II})} = W^{(f,e)} \\ t^{(\mathrm{I})}_{q''z''} = t^{(i,e)}_{q''z''} + t^{(r,e)}_{q''z''} + t^{(s)}_{q''z''}, \ t^{(\mathrm{II})}_{q''z''} = t^{(f,e)}_{q''z''} \end{cases} \tag{14}$$

Where, $W^{(s)}$ is the displacement of the scattering wave at the link section and has the same form as scattering wave in Green's function. $t^{(s)}_{qz}$ is the stress of the scattering wave.

The stress continuity condition at the linking section can be expressed as:

$$t^{(\mathrm{I})}_{q''z''}\sin q''_0 + f_1(r''_0, q''_0) = t^{(\mathrm{II})}_{q''z''}\sin q''_0 + f_2(r''_0, q''_0) \tag{15}$$

Where r''_0 and q''_0 are the polar coordinates at the linking section in the global coordinate system $x''o''y''$ and $z'' = r''\exp(\mathrm{i}q)$, $z'' = z - d$. When $q''_0 = b_1 = -p/2$, $A \le r''_0 \le \infty$, when $q''_0 = b_2 = p/2$, $A \le r''_0 \le h$.

According to $t^{(i,e)}_{q''z''} + t^{(r,e)}_{q''z''} = t^{(f,e)}_{q''z''}$, we can get:

$$f_1(r''_0, q''_0) = f_2(r''_0, q''_0), q_0 = b_1, b_2 \tag{16}$$

The displacement continuity conditions at the linking section can be written as:

$$W^{(\mathrm{I})} + W^{(f_1)} + W^{(c_1)} = W^{(\mathrm{II})} + W^{(f_2)} + W^{(c_2)} \tag{17}$$

According to $W^{(i,e)} + W^{(r,e)} = W^{(f,e)}$, we can obtain:

$$W^{(\mathrm{s})} + W^{(f_1)} + W^{(c_1)} = W^{(f_2)} + W^{(c_2)} \tag{18}$$

Where, $W^{(f_1)}$ is the displacement field caused by force system f_1, $W^{(c_1)}$ is the displacement field caused by $[-t^{(\mathrm{I})}_{q''z''}]$, and $W^{(f_2)}$ is the displacement field caused by force system f_2, and $W^{(c_2)}$ is the displacement field caused by $[-t^{(\mathrm{II})}_{q''z''}]$.

According to the continuity condition at the linking section and the Green's function we have obtained, the integral equations with unknown anti-plane forces can be expressed as:

$$\begin{aligned} &\int_A^l f_1(r''_0, b_2)[G_1(r'', b_1; r''_0, b_2) + G_2(r'', b_1; r''_0, b_2)]dr''_0 + \int_A^\infty f_1(r''_0, b_1)[G_1(r'', b_1; r''_0, b_1) + G_2(r'', b_1; r''_0, b_1)]dr''_0 \\ &= [-\mathrm{W}^{(\mathrm{S})}]_{q''_0 = b_1} + \int_0^A t^{(\mathrm{I})}_{q''z''}(r''_0, b_2)G_1(r'', b_3; r''_0, b_2)dr''_0 - \int_0^A t^{(\mathrm{I})}_{q''z''}(r''_0, b_3)G_1(r'', b_3; r''_0, b_2)dr''_0 + \\ &\int_0^A t^{(\mathrm{II})}_{q''z''}(r''_0, b_2)G_2(r'', b_3; r''_0, b_2)dr''_0 - \int_0^A t^{(\mathrm{II})}_{q''z''}(r''_0, b_3)G_2(r'', b_3; r''_0, b_2)dr''_0 \end{aligned} \tag{19}$$

$$\begin{aligned} &\int_A^l f_1(r''_0, b_2)[G_1(r'', b_2; r''_0, b_2) + G_2(r'', b_2; r''_0, b_2)]dr''_0 + \int_A^\infty f_1(r''_0, b_1)[G_1(r'', b_2; r''_0, b_1) + G_2(r'', b_2; r''_0, b_1)]dr''_0 \\ &= [-\mathrm{W}^{(\mathrm{S})}]_{q''_0 = b_2} + \int_0^A t^{(\mathrm{I})}_{q''z''}(r''_0, b_2)G_1(r'', b_2; r''_0, b_2)dr''_0 - \int_0^A t^{(\mathrm{I})}_{q''z''}(r''_0, b_3)G_1(r'', b_2; r''_0, b_3)dr''_0 + \\ &\int_0^A t^{(\mathrm{II})}_{q''z''}(r''_0, b_2)G_2(r'', b_2; r''_0, b_2)dr''_0 - \int_0^A t^{(\mathrm{II})}_{q''z''}(r''_0, b_3)G_2(r'', b_2; r''_0, b_3)dr''_0 \end{aligned} \tag{20}$$

In which, G_1 and G_2 are the Green's functions in area I and II respectively.

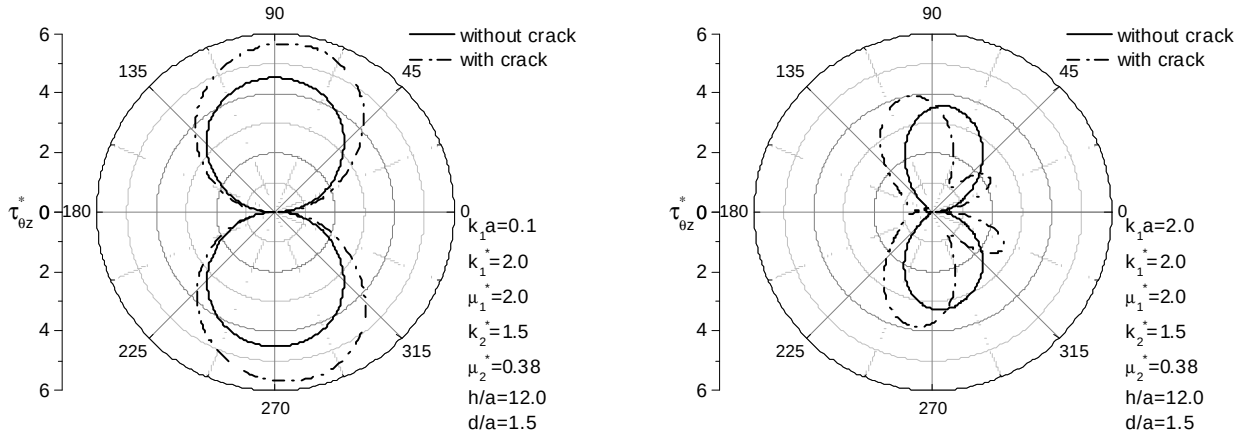
The dynamic stress concentration factor around the inclusion can be described as:

$$t^*_{q''z''} = \left| t^{(\bullet)}_{q''z''} / t_0 \right| \tag{21}$$

Where, $t^{(\bullet)}_{q''z''}$ is dimensionless stress representing DSCF and t_0 is the amplitude of the incident stress.

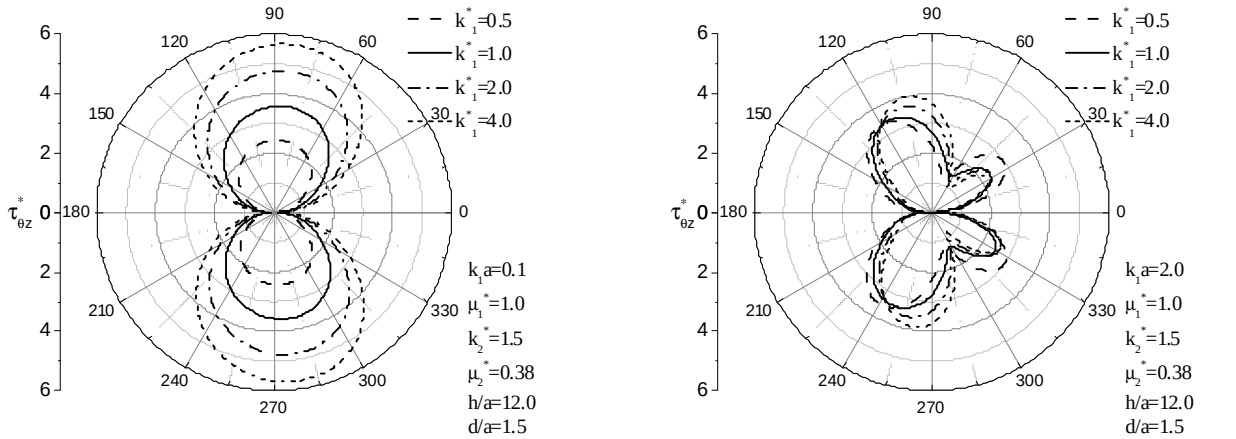
Examples and Results

For the numerical calculation, set the ratio *b/a*=0.8, the number of the series n=7. Here the dimensionless parameters $m_1^* = m_2 / m_1$, $m_2^* = m_3 / m_1$, $k_1^* = k_2 / k_1$, $k_2^* = k_3 / k_1$ are the ratio related to the parameters of medium I, II, III, and $l = d$, $k_1 a$ is the incident wave number.



(a) (b)

Fig.4 Distribution of t^*_{qz} effected by the crack disturbed by SH wave horizontally



(a) (b)

Fig.5 Distribution of t^*_{qz} with k_1^* disturbed by SH wave horizontally

Fig.4 discuss the effect of the existence of the crack on the distribution of DSCF around the inclusion. When the crack exists, A=1.0a. When the incident wave number $k_1 a$ =0.1, this is the case of quasi-static, the maximum value of DSCF increases 25.3% and the location trends to the direction of crack. When the model disturbed by SH waves in high frequencies, the maximum value of DSCF increases 12.1%. The result indicates that the existence of the crack has influence on the distribution DSCF around the inclusion.

Fig.5 discuss the effect of the rate of shear wave number on the distribution of DSCF around the inclusion. As we can see from the comparison between (a) and (b), the maximum value of DSCF appears when SH waves impact the model in low frequency. When $k_1^* < 1.0$, which means medium I is softer than medium II, the value of DSCF is less than the value in the situation $k_1^* > 1.0$ which

means medium I is harder than medium II, because the soft medium absorbed some energy of the SH waves.

Conclusion

Base on elastodynamics, the scattering of elastic waves and dynamic stress concentration problem in bi-material half space with elliptical inclusion and interfacial crack is investigated by using complex function method with conformal mapping method and Green's function. According to numerical results and analysis above, the following conclusions are drawn:

1. Compared with the situation of no interfacial crack, the DSCF of elliptic inclusion are intensified 12.1%-25.3% by the interfacial crack.

2. The ratios related to the parameters of medium I and II have significant effects on the DSCF of the inclusion, the soft medium absorbs some energy, which results in the decline of DSCF value.

The results provide certain reference meaning for construction of underground structure near the interface between different soils.

References

[1] WONG H L, TRIFUNAC M D. Surface motion of a semi-elliptical alluvial valley for incident plane SH waves, J. Bulletin of the Seismological Society of America. 1974, 64(5): 1389–1403.

[2] Lee V W, Manoogian M E. Surface motion above an arbitrary shape underground cavity for incident SH wave, J. European Earthquake Engineering. 1995, 8(1): 3-11.

[3] LIU Gang, HAN Feng, LIU Dian-kui, et al. Conformal mapping for the Helmholtz equation: Acoustic wave scattering by a two dimensional inclusion with irregular shape in an ideal fluid, J. The Journal of the Acoustical Society of America. 2012, 131(2): 1055-1065

[4] Lee V W, Amornwongpaibun A. Scattering of anti-plane (SH) waves by a semi-elliptical hill: I—Shallow hill, J. Soil Dynamics and Earthquake Engineering. 2013, 53: 116-125

[5] LUO Hao, V W Lee, LIANG Jian-wen. Anti-plane (SH) waves diffraction by an underground semi-circular cavity: analytical solution, J. Earthquake engineering and engineering vibration. 2010, 9(3): 385-396.

[6] QI Hui, YANG Jie. Dynamic analysis for circular inclusion of arbitrary positions near interfacial crack impacted by SH-wave in half-space, J. European Journal of Mechanics /A Solids. 2012, 36: 18-24.

[7] QI Hui, YANG Jie, SHI Yong, TIAN Jia-yong. Dynamic Analysis for Circular Inclusion near Interfacial Crack Impacted by SH-Wave in Half Space, J. Journal of Mechanics, 2012, 28(1): 143-151.

[8] CHEN Zhi-gang. Far field solution to SH-wave scattered by an interface cavity, J. Journal of Vibration and Shock. 2014, 33(12): 138-144. (in Chinese)

[9] YANG Zai-lin, YAN Pei-lei, LIU Dian-kui. Scattering of SH-waves and ground motion by an elastic cylindrical inclusion and a crack in half space, J. Chinese Journal of Theoretical and Applied Mechanics. 2009, 41(2): 229-235. (in Chinese)

[10] YANG Zai-lin, XU Hua-nan, HEI Bao-ping. Dynamic response of elliptical inclusion and crack neat interface under vertically incident SH-wave from above, J. Rock and Soil Mechanics. 2013, 34(8): 2378-2384. (in Chinese)