

Post-peak stress-strain relations of rock mass considering elasto-plastic coupling based on strain-softening model

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Abstract. Based on Hoek-Brown failure criterion, selecting the plastic shear strain as strain-softening parameter and considering elasto-plastic coupling, the approach of solving nonlinear relations between stress and axial strain, hoop strain and volumetric strain in the post-peak region was proposed by modeling. On this basis, combining with numerical examples, the influence of elasto-plastic coupling on post-peak stress-strain curves and deformation is analyzed. The results reveal that both elastic modulus and poisson's ratio coupling have some influence on post-peak stress-strain curves and deformation. In terms of influence extent, the coupling of elastic modulus has larger influence on axial strain than that of poisson's ratio, and the coupling of poisson's ratio has larger influence on hoop strain than that of elastic modulus.

Introduction

In the excavation of underground engineering such as roadway and so on, the surrounding rock mass is often in the post-peak deformation state. It is important for understanding the post-peak mechanical property and predicting the stability of surrounding rock mass to study the post-peak strain-softening behavior of rock mass. In terms of studies on deformation of strain-softening model, it is an important method based on the perspective that the strength parameters gradually evolve with the increasing of strain-softening parameter. The method has been used to calculate deformation of surrounding rock mass in the excavation of geotechnical engineering^[1-4], and when the method is used, the elasto-plastic coupling has been seldom considered so far, but the elasto-plastic coupling actually exists in engineering.

The aim of the present study is to develop a model for describing the strain-softening behavior more accurately, which can consider the elasto-plastic coupling. Firstly, based on Hoek-Brown failure criterion, choosing the plastic shear strain as strain-softening parameter and considering Elasto-plastic Coupling, a approach of solving nonlinear relations between stress and axial strain, hoop strain and volumetric strain in the post-peak region is proposed. Then, on this basis,

combining with examples, the influence of elasto-plastic coupling on post-peak stress-strain curves and deformation is analyzed.

Post-peak stress-strain relationships of rock mass

It is assumed that the confining pressure and axial pressure of rock mass are respective σ_3 and σ_1 . In terms of choosing strain-softening parameter, the plastic shear strain γ_p , which defined as express (1), is widely used in strain-softening literature, so the γ_p is selected as strain-softening parameter in present study.

$$\gamma_p = \varepsilon_{1p} - \varepsilon_{3p} \quad (1)$$

where ε_{1p} refers to the axial strain and ε_{3p} refers to hoop strain. Hoek-Brown failure criterion is selected as failure criterion for our study. The strength parameters m and s in Hoek-Brown failure criterion change with the increasing of γ_p , so the Hoek-Brown failure criterions can be written as

$$\sigma_1 = \sigma_3 + \sigma_c \left(m(\gamma_p) \frac{\sigma_3}{\sigma_c} + s(\gamma_p) \right)^{\frac{1}{2}} \quad (2)$$

where the relations of m , s and γ_p can be determined by laboratory experiments or field estimation. In our approach, it is assumed that m and s can be described by piecewise linear functions of γ_p defined as express (3) and (4), both for the sake of simplicity and due to the fact that an analysis of tests in rock samples indicates a trend towards this linear decrease in the m and s parameters.

$$m(\gamma_p) = \begin{cases} \frac{m^r - m^p}{\gamma_p^r} \gamma_p + m^p, & 0 \leq \gamma_p < \gamma_p^r \\ m^r, & \gamma_p \geq \gamma_p^r \end{cases} \quad (3)$$

$$s(\gamma_p) = \begin{cases} \frac{s^r - s^p}{\gamma_p^r} \gamma_p + s^p, & 0 \leq \gamma_p < \gamma_p^r \\ s^r, & \gamma_p \geq \gamma_p^r \end{cases} \quad (4)$$

where η denotes one of m and s , the superscripts ' p ' and ' r ' denote the peak and residual values

Stress-axial strain relationship

Here Mohr-Coulomb type of criterion is selected as plastic potential function, so that the plastic potential function may be written as

$$G(\sigma_1, \sigma_3, \gamma_p) = \sigma_1 - k(\gamma_p) \sigma_3 \quad (5)$$

where $k(\gamma_p)$ is known as the coefficient of dilation and defined as

$$k(\gamma_p) = \frac{1 + \sin \phi(\gamma_p)}{1 - \sin \phi(\gamma_p)} \quad (6)$$

ϕ in equation (6) is dilatancy angle. Then the plastic flow rule gives the following relation between the axial and hoop strain increments:

$$d\varepsilon_{3p} = -k(\gamma_p)d\varepsilon_{1p} \quad (7)$$

The relation of the axial plastic strain ε_{1p} and plastic shear strain γ_p can be solved from the equation (1) and (7).

$$\varepsilon_{1p} = \frac{\gamma_p [1 - \sin(\phi(\gamma_p))]}{2} \quad (8)$$

When elasto-plastic coupling is considered, some literatures only consider the coupling of elastic modulus E or shear modulus G [1,4,5], some literatures think that both poisson's ratio ν and elastic modulus E have coupling [6,7]. In our approach, the coupling of both E and ν are considered, i.e., both E and ν may vary with the increasing of γ_p . From the system of expresses (2), (3), (4) and (8), the relation of axial strain and stress can be obtained as

$$\varepsilon_1 = \varepsilon_{1p} + \varepsilon_{1e} = \frac{\gamma_p}{2} [1 - \sin(\phi(\gamma_p))] + \frac{\sigma_1 - 2\nu(\gamma_p)\sigma_3}{E(\gamma_p)} \quad (9)$$

Where ε_{1p} and ε_{1e} respective denote axial plastic strain and elastic strain.

If dilatancy effect is not considered, i.e. dilatancy angle is zero, express (9) can be written as

$$\varepsilon_1 = \frac{\gamma_p}{2} + \frac{\sigma_1 - 2\nu(\gamma_p)\sigma_3}{E(\gamma_p)} \quad (10)$$

Stress-hoop strain and stress- volumetric strain relationships

From the expresses (7) and (8), the relation between stress and hoop strain can be obtained as

$$\varepsilon_3 = -\frac{1 + \sin \phi(\gamma_p)}{1 - \sin \phi(\gamma_p)} \varepsilon_{1p} + \frac{[\sigma_3 - \nu(\gamma_p)(\sigma_1 + \sigma_3)]}{E(\gamma_p)} = -\frac{(1 + \sin \phi(\gamma_p))\gamma_p}{2} + \frac{[\sigma_3 - \nu(\gamma_p)(\sigma_1 + \sigma_3)]}{E(\gamma_p)} \quad (11)$$

From the expresses (9) and (12), the relation between stress and volumetric strain can be obtained as

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 = -\frac{\gamma_p}{2} [1 + 3\sin \phi(\gamma_p)] + \frac{[(1 - 2\nu(\gamma_p))\sigma_1 + [2 - 4\nu(\gamma_p)]\sigma_3]}{E(\gamma_p)} \quad (12)$$

For simplicity, here it is assumed that the ϕ , E and ν can be described by piecewise linear functions of γ_p too. According to the assumption of evolving law for ϕ , E and ν and expression (9), (11) and (12), the relations of stress and axial strain, hoop strain, volumetric strain can be obtained, which can consider coupling of E , ν and ϕ . Let $E^r = E^p$, $\nu^r = \nu^p$, the ones without elasto-plastic coupling can be obtained.

Examples and analysis

The parameters of some rock mass are as follows: $\sigma_c = 30$ MPa, $m_p = 1.70$,

$m_r = 0.80$, $s^p = 0.0040$, $s^r = 0.0020$, $\gamma_p^r = 0.0088$, $\phi^p = \phi^r = \frac{\pi}{12}$, $\sigma_3 = 20$ MPa. Here we analyze the effects of elasto-plastic coupling on post-peak softening behavior.

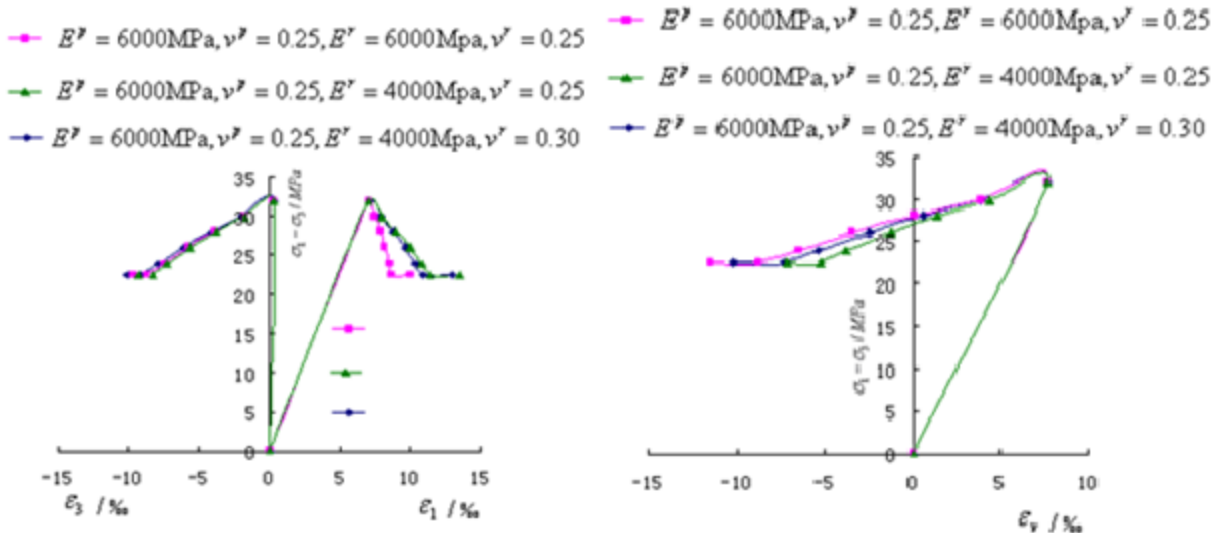


Fig.1 Influence of elasto-plastic coupling on post-peak stress-axial strain and stress-hoop strain curves

Fig.2 Influence of elasto-plastic coupling on post-peak stress-volumetric strain curves

It is stipulated that the strain is positive in the case of compression and negative in the case of dilation. The following conclusions can be easily drawn from express(9), (11) and (12): Both elastic modulus and poisson's ratio coupling have some influence on the three kinds of post-peak stress-strain curves. The greater the poisson's ratio $\nu(\gamma_p)$, the smaller the value of axial strain,

hoop strain and volumetric strain are. The greater the elastic modulus $E(\gamma_p)$, the smaller the value of axial strain and volumetric strain are, but the variation trend of hoop strain is uncertain and in connection with poisson's ratio $\nu(\gamma_p)$. To verify the validity of the above conclusions, the

post-peak stress-axial strain and stress-hoop strain curves are shown in Fig.1, and stress-volumetric strain curves is shown in Fig.2. The two figures both include three cases of curves which are having no elasto-plastic coupling, only having elastic modulus elasto-plastic coupling and having both elastic modulus and poisson's ratio elasto-plastic coupling. From the

two figures, it can be seen that, in the strain-softening region, for the stress-axial strain curves, from top to bottom on the coordinate system the curves are respective the one only considering elastic modulus coupling, the one considering both elastic modulus and poisson's ratio coupling and the one without elasto-plastic coupling, and which reveals that the greater the residual elastic modulus and poisson's ratio, the smaller the axial strain is. For the stress-hoop strain curves, the one considering both elastic modulus and poisson's ratio coupling is above the other two ones which reveals that the greater the residual poisson's ratio, the smaller the value of hoop strain. For the stress-volumetric strain curves, from top to bottom on the coordinate system the curves are respective the one without elasto-plastic coupling, the one considering both elastic modulus and poisson's ratio coupling and the one only considering elasto-plastic coupling of elastic modulus which reveals that the greater the residual elastic modulus and poisson's ratio, the smaller the value of volumetric strain is and the more remarkable the dilatancy effect. In the three cases of having no elasto-plastic coupling, only having elastic modulus coupling and having both elastic modulus and poisson's ratio coupling, at the intersection of strain-softening and residual region, the axial strains are respective 8.7‰、11.4‰ and 10.9‰, the hoop strains are respective -8.7‰、-8.3‰ and -9.1‰; the volumetric strains are respective -8.7‰、-5.2‰ and -7.3‰, and which reveals that, in terms of influence extent, the coupling of elastic modulus have more influence on axial strain than that of poisson's ratio, and the coupling of poisson's ratio have more influence on hoop strain than that of elastic modulus.

Conclusions

- (1)Based on the evolutionary behaviors of strength parameters, using Hoek-Brown failure criterion, selecting the plastic shear strain γ_p as strain-softening parameter and considering elasto-plastic coupling, the approach of solving nonlinear relations between stress and axial strain, hoop strain and volumetric strain in the post-peak region was proposed by modeling.
- (2)Combining with numerical examples, the influence of elasto-plastic coupling on stress-strain curves was discussed. The results reveal that both elastic modulus and poisson's ratio coupling have some influence on post-peak stress-strain curves and deformation of rock mass.
- (3)This study can comprehensively considers the influence of evolution of various parameters on post-peak stress-strain curves, and these parameters include strength parameters m, s , deformation parameters E, ν and dilatancy angle ϕ .

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